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Experimental observation of the small-scale self-focusing of a beam in the nondestructive regime*

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***Abstract*. Two methods for the experimental study of the nonlinear interaction of the fundamental beam and noise component of high-power laser radiation under conditions of small-scale self-focusing in the nondestructive regime are proposed and realised. In the case of the nonlinear phase shift $B = 1$, the theoretically predicted circular structure of the spatial spectra of the field and laser beam intensity (pulse energy 20 J and peak power 20 GW) propagated through a glass sample was experimentally observed.**

Keywords: *small-scale laser beam self-focusing, nondestructive regime*.

1. Introduction

A fundamental parameter restricting the power of modern solid-state pulsed lasers is the radiation resistance – the threshold radiation intensity at which damages appear in the volume or on the surface of laser elements. Laser systems are calculated so that the linear intensity of propagating light would not achieve the threshold value at which the material is damaged. However, because the refractive index depends on the light intensity as $n(I) = n_0 + \gamma I$ (n_0 is the linear refractive index, γ is the parameter of a nonlinear medium), the self-focusing of light can be observed. The increase in the light intensity due to self-focusing can cause the damage of laser elements. In this sense, self-focusing is said to restrict the power of solid-state nanosecond lasers [1 – 3].

There exist two limiting cases of self-focusing – large-scale and small-scale self-focusing (SSSF). The first type of self-focusing is manifested most strongly for the beams with a strong radial dependence of the intensity. During the propagation of such beams in a nonlinear medium, peripheral beams are pressed against the axis, resulting in the self-focusing of the beam as a whole.

However, for nanosecond solid-state lasers the most dangerous is SSSF [4]. Bespalov and Talanov showed [5] that small-scale amplitude and phase spatial inhomogeneities in a cubic nonlinear medium, which always exist in a beam, can be amplified in the presence of a high-power wave. This leads to the decomposition of the beam into separate filaments, i.e. to SSSF. The degree of self-focusing is usually characterised by the B-integral describing the nonlinear phase shift in a medium of length L:

$$B(r) = \frac{2\pi}{\lambda}\gamma\int_0^L I(r,z)\mathrm{d}z, \qquad (1)$$

where λ is the wavelength of light. The prominent filament structure of the beam is usually observed for $B > 3$.

To find out the conditions under which SSSF can occur, it is necessary to analyse in detail the process of enhancement of small-scale perturbations in a nonlinear medium. In addition, it is always desirable to analyse the enhancement of perturbations at radiation intensities safe for laser elements. This problem is far from trivial, and we are not aware of any experimental studies of SSSF in the nondestructive regime.

The SSSF theory developed in papers [5 – 7] predicts that after the propagation of light through a nonlinear medium, the localised noise with the initially broad uniform spatial spectrum acquires the amplitude-modulated spectrum even when $B \approx 1$. Such a modulation of the noise spectrum leads to the formation of rings in the far-field intensity distribution. These rings characterise the amplification of noise and, therefore, the development of SSSF. In this paper, we observed such rings for the first time at the initial stage of the SSSF development.

In addition, we proposed and realised an alternative method for the observation of SSSF for $B \approx 1$. The method is based on the transfer of the image of a noise-source plane to the detector (CCD camera) plane through a layer of a nonlinear medium. The mathematical analysis of the Fourier transform of the image obtained from the CCD camera also showed the presence of the circular structure, which was confirmed experimentally.

The value of the B-integral in experiments did not exceed 1.2. For such B it is difficult to determine the beginning of the formation of the filament-like structure in the beam, which caused the damage of a dielectric. The methods of monitoring SSSF used in our study allow us to observe self-focusing at the initial stage, thereby preventing the damage of optical elements in laser setups.

2. Small-scale self-focusing

The SSSF theory considers the development of harmonic small-scale perturbations with the transverse wave number

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$\varkappa$ against the background of an intense linearly polarised plane wave propagating through a cubic nonlinear medium. The boundary of the instability region of such perturbations and the maximum increment were found by using the linearized theory [5]. The boundary-value problem of the development of such perturbations was considered in the paper of Rozanov and Smirnov [7]. The complex amplitude of the perturbation at the output from a nonlinear medium of length L is determined from the transformation matrix [7, 8]

$$U = \begin{pmatrix} \cosh(\chi L) & \dfrac{-\sinh(\chi L)}{\sqrt{(\alpha_{\rm cr}/\alpha)^2 - 1}} \\ -\sinh(\chi L)\sqrt{(\alpha_{\rm cr}/\alpha)^2 - 1} & \cosh(\chi L) \end{pmatrix}, \qquad (2)$$

$$\chi = k_0 n_0 \frac{\alpha}{2}\sqrt{\alpha_{\rm cr}^2 - \alpha^2},$$

where $\alpha = \pm\varkappa/(k_0 n_0)$ is the transverse propagation angle of each of the harmonics in the medium with respect to the fundamental wave; $k_0 = 2\pi/\lambda$; and $\alpha_{\rm cr} = [4B/(k_0 n_0 L)]^{1/2}$ is the boundary of the perturbation instability band.

A change in the modulus of the complex perturbation amplitude during the propagation of radiation through a nonlinear medium can be conveniently described with the help of the intensity transfer coefficient $G(\alpha, \varphi_0)$, which is equal to the ratio of the squares of the moduli of complex amplitudes at the output from and input in the nonlinear medium:

$$G(\alpha, \varphi_0) = (U_{11}^2 + U_{21}^2)\cos^2\varphi_0 + (U_{12}^2 + U_{22}^2)\sin^2\varphi_0 + (U_{11}U_{12} + U_{21}U_{22})\sin 2\varphi_0. \qquad (3)$$

Together with the modulus of the complex amplitude, its phase also changes. The perturbation phase φ at the output of the nonlinear medium is expressed in terms of the phase φ_0 at the input as [8]

$$\tan\varphi(\alpha, \varphi_0) = \frac{\tan\varphi_0 - \tanh(\chi L)\sqrt{(\alpha_{\rm cr}/\alpha)^2 - 1}}{1 - \tan\varphi_0 \tanh(\chi L)\Big/\sqrt{(\alpha_{\rm cr}/\alpha)^2 - 1}}. \qquad (4)$$

In the limit $B \to 0$, the perturbation acquires only the linear phase shift $\varphi(\alpha) = \varphi_0 + (\alpha^2/2)k_0 n_0^2 L$.

In this paper, we consider the model of the localised amplitude perturbation (noise) whose source is located at some distance l from the nonlinear medium. Note that the SSSF theory considers the propagation of one spatial harmonic [5]. Because the noise, having a set of wave numbers $\varkappa$, can be expanded in plane waves, the use of the results of this theory is valid.

Each of the noise harmonics propagating in air between the source and nonlinear medium acquires the phase $\varphi_0 = \varkappa^2 l/(2k_0)$ with respect to the intense plane wave. If the phase of the plane wave is set equal to zero, the initial perturbation phase $\varphi_0(\alpha)$ at the input to the medium is defined uniquely:

$$\varphi_0 = \frac{\alpha^2}{2}k_0 n_0^2 l. \qquad (5)$$

In this case, the enhancement or attenuation of the perturbation harmonic will be determined already by the function of one variable $G(\alpha)$ obtained from (3) by using (5).

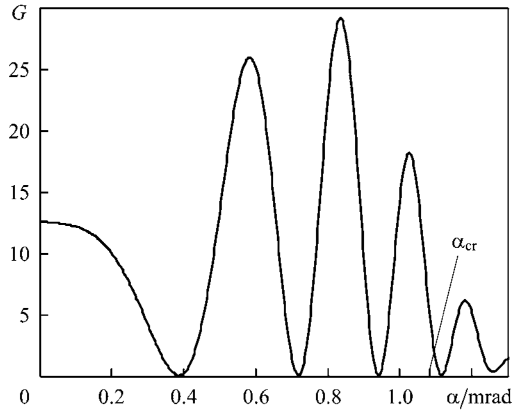

Figure 1. Dependence $G(\alpha)$ for $\varphi_0 = (\alpha^2/2)k_0 n_0^2 l$ and $B = 1.7$, $l = 110$ cm, $L = 63$ cm, $n_0 = 1.534$, and $\lambda = 1054$ nm.

It is easy to see that the transfer coefficient $G(\alpha)$ at the output from the nonlinear medium is amplitude-modulated (Fig. 1). Thus, the initially broad uniform perturbation spectrum acquires modulation after the propagation of radiation through the nonlinear medium. Therefore, the far-field perturbation intensity distribution has a circular structure because the problem under study is independent of the azimuthal angle. The appearance of such a structure indicates the presence of the localised noise in the system and the beginning of the SSSF development. One of the experimental methods for observing the SSSF development realised in our paper is based on obtaining the far-field perturbation intensity distribution.

3. Spatial intensity spectrum in the image plane of a perturbation source

The observation of SSSF from the far-field intensity distribution is a direct experimental method. In this paper, we propose to use an alternative method for studying SSSF based on the Fourier analysis of the noise source image. The image transfer was performed through nonlinear medium (*3*) (Fig. 2a). The Fourier spectrum of the

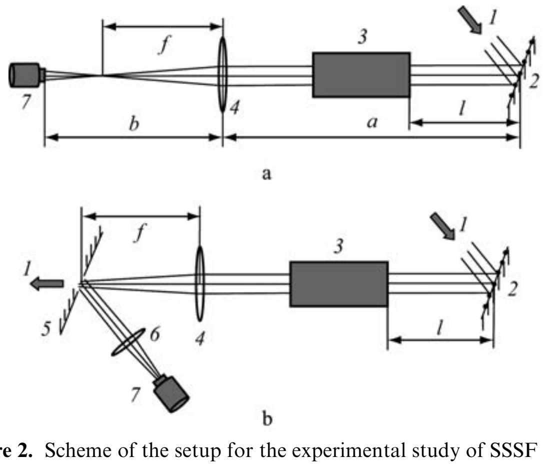

Figure 2. Scheme of the setup for the experimental study of SSSF by the near-field (a) and far-field (b) intensity distributions of the noise component: (*1*) laser beam; (*2*) noise source; (*3*) nonlinear medium of length $L = 63$ cm with optical parameters $n_0 = 1.534$ and $\gamma = 3.2\times10^{-7}$ cm^2 GW^{-1}; (*4*) lens with the focal distance $f = 73$ cm; (*5*) reflecting screen with a hole of diameter 0.65 mm; (*6*) CCD camera objective; (*7*) CCD camera.

image in the CCD camera plane is the convolution of the Fourier transforms of the field and, therefore, it should contain information about a change in the noise spectrum during the propagation of light through the nonlinear medium. Let us verify this fact analytically.

The spatial spectrum of the filed in the noise source plane consists of the sum of the spectra of the fundamental beam and perturbation component:

$$S_0^{\text{field}}(\boldsymbol{\varkappa}) = S_0^{\text{beam}}(\boldsymbol{\varkappa}) + S_0^{\text{noise}}(\boldsymbol{\varkappa}). \tag{6}$$

We considered theoretically and experimentally only the localised amplitude noise with the spectrum having the property

$$S_0^{\text{noise}}(\boldsymbol{\varkappa}) = (S_0^{\text{noise}}(-\boldsymbol{\varkappa}))^*. \tag{7}$$

It can be shown that the field spectrum in the noise source plane is described by the expression

$$S^{\text{field}}(\boldsymbol{\varkappa}) = \frac{1}{4\pi^2}\exp\left(\frac{\mathrm{i}b}{2k_0}\boldsymbol{\varkappa}^2\right)\int\left[S_0^{\text{beam}}(\boldsymbol{\varkappa}) + S_0^{\text{noise}}(\boldsymbol{\varkappa})\right]\exp\left(\frac{\mathrm{i}a}{2k_0}\boldsymbol{q}^2\right)$$
$$\times\exp\left\{\frac{\mathrm{i}k_0}{2f}\left[\boldsymbol{r} - \frac{f}{k_0}(\boldsymbol{\varkappa} - \boldsymbol{q})\right]^2 - \frac{\mathrm{i}f}{2k_0}(\boldsymbol{\varkappa} - \boldsymbol{q})^2\right\}\mathrm{d}^2\boldsymbol{q}\,\mathrm{d}^2\boldsymbol{r}, \tag{8}$$

where a and b are optical paths in front of and behind a lens, respectively, satisfying the formula of a thin lens $f = ab/(a+b)$, where f is the focal distance of the lens (Fig. 2a).

The SSSF theory considers a noise with a broad spectrum of spatial harmonics, which is weak compared to the fundamental wave. We followed these conditions in our experiments, which are described by the inequalities

$$\left|S^{\text{beam}}(\boldsymbol{\varkappa})\right| \gg \left|S^{\text{noise}}(\boldsymbol{\varkappa})\right|, \quad \Delta\boldsymbol{\varkappa}^{\text{beam}} \ll \Delta\boldsymbol{\varkappa}^{\text{noise}}. \tag{9}$$

Relations (9) allow us to simplify analytic calculations by assuming that $S^{\text{beam}}(\boldsymbol{\varkappa}) = \delta(\boldsymbol{\varkappa})$, where $\delta(\boldsymbol{\varkappa})$ is the Dirac delta-function, i.e. by replacing the fundamental beam by a plane wave. By using (8), we write expressions for the change in the spectra $S^{\text{beam}}(\boldsymbol{\varkappa})$ and $S_0^{\text{noise}}(\boldsymbol{\varkappa})$ during the transfer of the localised noise plane image:

$$S^{\text{beam}}(\boldsymbol{\varkappa}) = \frac{\mathrm{i}f}{2\pi k_0}\exp\left[\frac{\mathrm{i}(b-f)}{2k_0}\boldsymbol{\varkappa}^2\right],$$
$$S^{\text{noise}}(\boldsymbol{\varkappa}) = -\frac{f}{a-f}\,S_0^{\text{noise}}\left(-\frac{f}{a-f}\,\boldsymbol{\varkappa}\right). \tag{10}$$

When the perturbation source plane image is transferred through a nonlinear medium, the factor $\sqrt{G(\varkappa)}\exp(\mathrm{i}\Delta\varphi_{\text{NL}})$ will appear in the expression for the noise component, according to the SSSF theory [7]:

$$S^{\text{noise}}(\boldsymbol{\varkappa}) = -\frac{f}{a-f}\,S_0^{\text{noise}}\left(-\frac{f}{a-f}\,\boldsymbol{\varkappa}\right)$$
$$\times\sqrt{G(\varkappa)}\exp\left(\mathrm{i}\Delta\varphi_{\text{NL}}\right), \tag{11}$$

where $G(\varkappa)$ is the dependence of the transfer coefficient on the wave number $\varkappa$, which is uniquely related to the angle $\alpha = \varkappa/(k_0 n_0)$; $\Delta\varphi_{\text{NL}} = \varphi(\alpha) - (\alpha^2/2)k_0 n_0^2 l - (\alpha^2/2)k_0 n_0^2 L$.

The field intensity spectrum in the noise source plane can be written, accurate to the first-order smallness terms, in the form

$$S_I(\boldsymbol{\varkappa}) = \int S^{\text{field}}(\boldsymbol{q})\left(S^{\text{field}}(\boldsymbol{q} - \boldsymbol{\varkappa})\right)^*\mathrm{d}^2\boldsymbol{q}, \tag{12}$$

$$S_I(\boldsymbol{\varkappa}) = \delta(\boldsymbol{\varkappa}) + S^{\text{noise}}(\boldsymbol{\varkappa}) + \left(S^{\text{noise}}(-\boldsymbol{\varkappa})\right)^*. \tag{13}$$

Because the functions $G(\varkappa)$ and $\Delta\varphi_{\text{NL}}(\varkappa)$ are even, we obtain finally the expression

$$\left|S_I(\varkappa > \Delta\varkappa^{\text{beam}})\right|^2 \propto \left|S_0^{\text{noise}}(\boldsymbol{\varkappa})\right|^2 G(\varkappa)\cos^2\Delta\varphi_{\text{NL}} \tag{14}$$

for the modulus of the intensity spectrum outside a small central spectral region $\Delta\varkappa^{\text{beam}}$ (corresponding to the fundamental beam).

The quantity $\left|S_I(\boldsymbol{\varkappa})\right|$ is proportional to the intensity transfer coefficient $G(\varkappa)$ and, therefore, contains information on the amplitude modulation of the spectrum. Thus, the modulus of the spatial spectrum of the intensity distribution in the perturbation source image plane has the form similar to the far-fled intensity distribution of the noise component.

Let us assume that an inaccurate adjustment to the noise source image plane is performed and the image of a plane located at a distance of z from the noise source plane is transferred. Then, the spectrum $S_0^{\text{noise}}(\boldsymbol{\varkappa})$ will acquire the linear phase shift with respect to the intense beam, which is related to its propagation in air and has the form $S_0^{\text{noise}}(\boldsymbol{\varkappa})\exp\left[\mathrm{i}\boldsymbol{\varkappa}^2 z/(2k_0)\right]$, while the argument of the cosine in (14) will be replaced by the sum $\Delta\varphi_{\text{NL}} + \varkappa^2 z/(2k_0)$. This will lead to the appearance of rings not related to nonlinear effects, and the intensity distribution $\left|S_I(\boldsymbol{\varkappa})\right|$ will be blurred. Therefore, an accurate adjustment to the noise source image plane is very important in this method. We controlled it by the absence of rings in the distribution $\left|S_I(\boldsymbol{\varkappa})\right|$ for $B = 0$.

4. Experimental observation of the SSSF development

We analysed the near-field intensity distribution by using the scheme presented in Fig. 2a. As the intense beam, linearly polarised radiation from a 1054-nm laser emitting 20-J pulses with a peak power of 20 GW was used. The radiation intensity distribution in the laser beam cross section was close to the super-Gaussian ($\sim\exp[-(r/\rho)^{2N}]$) with the exponent $N = 6.4$ [1].

The amplitude noise appeared in intense laser beam (*1*) (Fig. 2a) reflected from dielectric mirror (*2*) with defects of size 100 – 200 μm arbitrarily distributed over its surface. Total losses caused by these defects were 2.4% of the fundamental radiation power. The beam passed through nonlinear medium (*3*) located at a distance of 50 – 150 cm from the plane of defects. As a nonlinear medium we used a neodymium silicate glass rod of length 63 cm and diameter 6 cm with optical parameters $n_0 = 1.534$ and $\gamma = 3.2\times10^{-7}$ cm^2 GW^{-1} [9]. After propagation through nonlinear medium (*3*) and attenuation during reflection from wedges (not shown in Fig. 2a), all radiation was incident on lens (*4*) ($f = 73$ cm), which transferred the noise source plane on CCD camera (*7*).

We obtained near-field intensity distributions of the noise component for two distances ($l = 87$ and 135 cm) between the

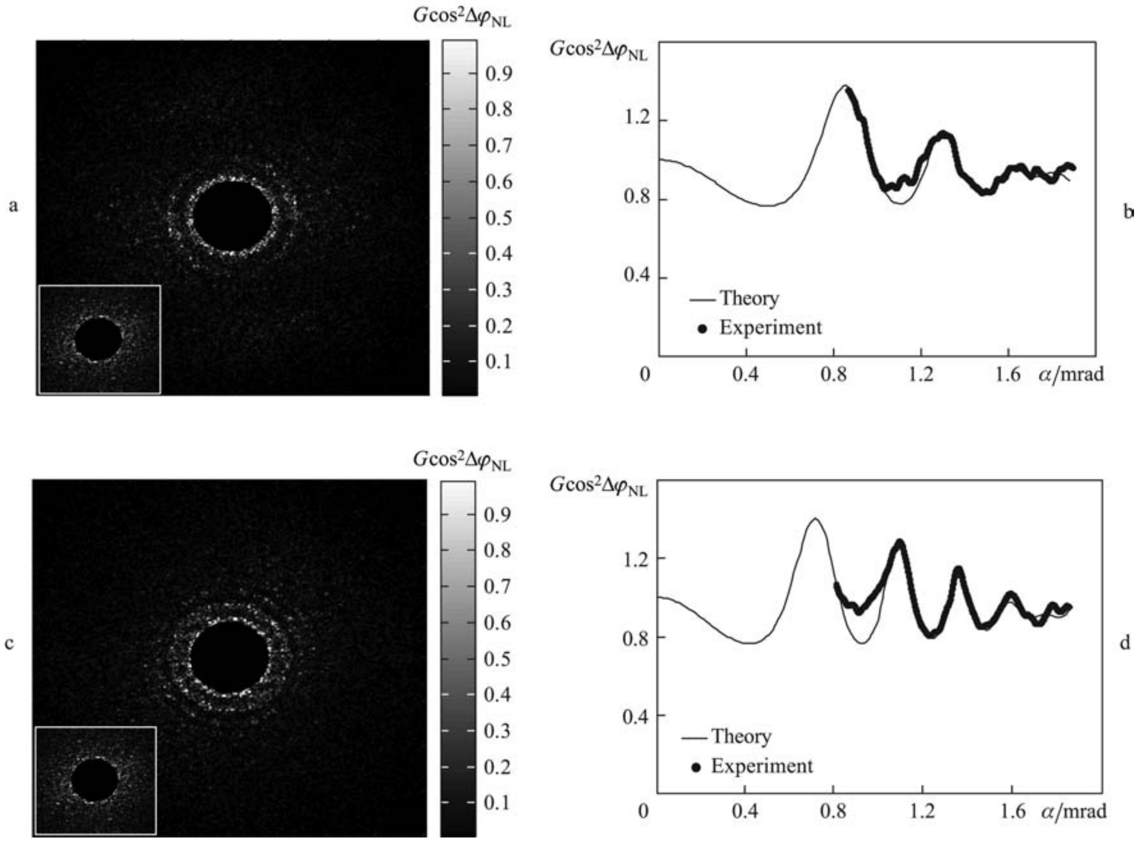

Figure 3. Two-dimensional (a, c) and one-dimensional (b, d) spatial spectra of the near-field intensity distribution of the noise component for $B = 1.05$ and distances between the noise source and nonlinear medium $l = 87$ (a, b) and 135 cm (c, d). The insets in Fig. 3a, c show the near-field intensity distributions of the noise component for $B = 0$.

noise source and nonlinear medium. The intensity distributions were measured for $B = 0$ and 1.05 for each distance. The analysis of the Fourier transform of the obtained images revealed the circular structure of the modulus of the spatial intensity spectrum for $B = 1.05$ (Figs 3a, c). The images were processed by averaging over the azimuthal angle and the dependence of the radiation intensity on angle α normalised to the radiation intensity for $B = 0$ was plotted for a nonzero value of B. Figures 3b and d present the theoretical function $G(\alpha)\cos^2\Delta\varphi_{\rm NL}$ for the specified parameters and the radial dependence of the azimuthal-angle-averaged modulus of the Fourier transform of the near-field intensity distribution for the noise component. One can see that the theoretical and experimental functions are in good agreement.

Aside from the experimental method described above, we also observed SSSF by the far-field intensity distribution (Fig. 2b). After propagation through nonlinear medium (*3*), laser beam (*1*) was focused by lens (*4*) on reflecting screen (*5*) with a hole of diameter 0.65 mm. The central part of the

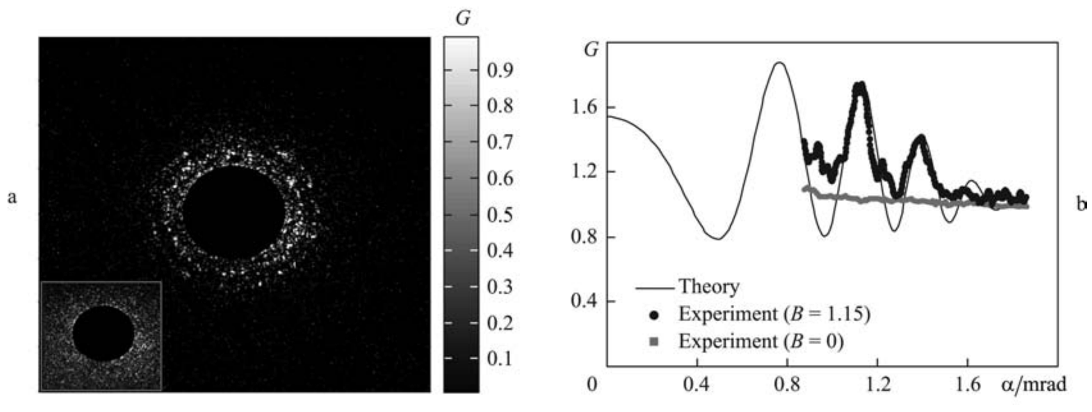

Figure 4. Two-dimensional (a) and one-dimensional (b) far-field intensity distributions of the noise component for $B = 1.15$ and a distance between the noise source and nonlinear medium $l = 135$ cm. The inset in Fig. 4a shows the intensity distribution of the noise component for $B = 0$.

spatial spectrum corresponding to the fundamental beam propagated through the hole in the screen, while the high-frequency harmonics corresponding to the noise reflected from the screen and were focused by objective (*6*) to CCD camera (*7*).

The intensity distribution recorded with CCD camera (*7*) at a distance of 135 cm between the noise source and non-linear medium is shown in Fig. 4a. For $B = 0$, the noise intensity is distributed uniformly. For $B = 1.15$, a circular structure is distinctly observed. The obtained images were processed by averaging over the azimuthal angle and then the dependence of the radiation intensity on angle α normalised to the radiation intensity for $B = 0$ was plotted for a nonzero value of the B-integral. The experimental curve obtained in this way can be compared with the known function $G(\alpha)$ (Fig. 4b). The experimental dependence of the transfer coefficient on angle α is in good agreement with the experimental dependence, the positions of their maxima and minima being coincident.

5. Conclusions

We have obtained the experimental dependences of the intensity transfer coefficients on angle α under the SSSF conditions for $B \approx 1$, i.e. in the nondestructive regime by two methods proposed in our study. In each of the methods realised for the first time we observed the instability of a small amplitude perturbation with a broad uniform spatial spectrum against a linearly polarised narrowband intense laser beam. Analysis of the angular distributions of the transfer coefficient $G(\alpha)$ obtained by both methods confirms the presence of the amplitude modulation of the spectrum of spatial harmonics suggesting the appearance of SSSF. The positions of maxima and minima of the theoretical and experimental dependences of $G(\alpha)$ are in good agreement (Figs 3b, d and 4b). The dif-ference in the amplitudes of maxima can be explained by the presence of a noise source in the nonlinear medium itself.

A comparison of the experimental methods used in the study shows that the method based on the transfer of the noise source image plane through the nonlinear medium offers the following advantages. First, this method has a higher sensitivity because it allows one to discern clearly a greater number of rings with close parameters (Fig. 3d) than the direct method (Fig. 4b). Second, in our opinion, the former method is simpler realised.

After the appropriate improvement, these methods can be used for detecting SSSF in modern laser setups for $B \leqslant 1$, i.e. before the damage of optical elements.

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