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Thermal regimes and limiting pump intensities of a disk laser with the one-dimensional temperature distribution inside the disk

A.N. Alpat'ev, V.A. Smirnov, I.A. Shcherbakov

Abstract. **Analytic expressions are derived for one-dimensional distributions of temperature and thermoelastic stresses in an active element (AE) upon stationary multipass pumping equal to or exceeding the threshold. Three events caused by the AE heating are considered, each of the events damaging the laser: a decrease in the efficiency due to the change in the spectral and luminescent properties of the active media, violation of the normal regime of the AE cooling, and destruction of the AE due to the increase in the thermomechanical stresses. The domains of the parameters (such as the AE absorption coefficient, the AE thickness, etc.) are determined at which one of the above events first occurs as the pump intensity and correspondingly the AE temperature are increased. Expressions for estimating the limiting pump intensities for each set of the parameters are obtained.**

Keywords: *disk lasers, lasing efficiency, limiting pump intensity.*

1. Introduction

In designing high-power lasers subjected to significant thermal loads, there appear problems related to the operation of the active medium under conditions of strong heating. The factors restricting the laser operation efficiency or simply damaging the laser include, for example, the change in the spectral and luminescent properties of the active medium during heating, violation of the normal cooling regime (in the case of water cooling – boiling of water), breakdown of the active element (AE) caused by the increase in the temperature stresses. In this case, depending on the AE parameters and the cooling system, there exists a certain sequence of the above events as the pump intensity is increased. The knowledge of this sequence allows one to select, for each specific situation, the laser parameters optimal from the viewpoint of realisation of the active medium potentials.

The study of thermal regimes became especially important in the development of high-power disk lasers [1 – 3] and of the concept of optically dense media as applied to disk lasers [4, 5]. In this paper, we derived analytic expressions for a one dimensional distribution of temperature and thermoelastic stresses in a disk AE in the case of the stationary multipass pump whose intensity is equal or exceeds the threshold intensity. This makes it possible to assume, in the first approximation, the absorption coefficient to be constant over the disk thickness. We determined the domains of the parameters (in particular, the absorption coefficient at the pump wavelength, the AE thickness, etc.) at which, as the pump intensity and correspondingly the AE temperature increase, there first occurs one of the three above events making the laser inoperative. We also obtained expressions making it possible to estimate for each set of the parameters the corresponding limiting pump intensities.

A.N. Alpat'ev, V.A. Smirnov, I.A. Shcherbakov A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia; e-mail: lisdenis@mail.ru

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2. Optical pump scheme

Consider the absorption of longitudinal stationary uniform (with respect to the disk radius) pump by a disk crystal with the absorption coefficient k leading to the excitation of the working laser level of the activator and to achievement of the threshold inverse population. We will consider two variants of the pump scheme. The first variant is a conventional (with a minimal number of mirrors) optical scheme (Fig. 1a) when pump radiation is incident on the AE through the resonator mirror of the laser and then, either emerges from the resonator after a single pass in the AE or returns back after reflection from the second resonator mirror. In this scheme, the total absorption is increased by increasing the concentration of absorbing ions, which, as a rule, are working laser ions. The second variant is a multipass optical pump scheme (Fig. 1b), which employs any number of additional mirrors (apart from the resonator mirrors) repeatedly returning the pump unabsorbed during the previous passes to the AE.

Consider the propagation of pump radiation in the AE (Fig. 1) taking into account reflection from all the mirrors.

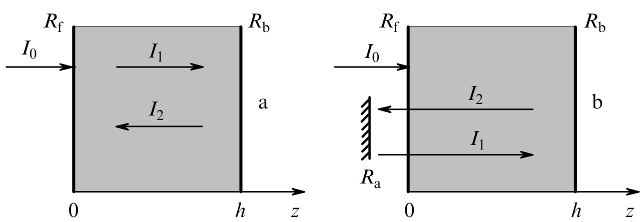

Figure 1. Conventional optical scheme of disk pumping (a) and multipass optical scheme of disk pumping with additional mirrors (b) (only one mirror is shown).

We assume that the pump absorption coefficient by the activator in the crystal is identical at any point of the AE (this is possible when the pump is above the threshold). After all the passes, the total pump intensity inside the AE at the point z is

$$I(z) = I_1(z) + I_2(z) = I_0 A_1 e^{-kz} + I_0 A_1 A_2 e^{kz}, \quad (1)$$

where $I_1(z)$ is the total intensity of the waves propagating to the right; $I_2(z)$ is the total intensity of the waves propagating to the left; I_0 is the input intensity.

In the case of conventional pumping of the disk (Fig. 1a),

$$A_1 = \frac{T_f}{1 - R_b R_f e^{-2kh}}$$

is the summing coefficient (the sum of infinite geometric progression) of all the intensities $I_{1i}(0)$ of the waves on the surface $z = 0$ (where i is the number of the round trip) returning to the disk after reflection from the mirror front surface; R_f and R_b are the reflection coefficients of the front and back mirrors, respectively (the mirrors can be deposited directly either on the disk surface or be mounted at some distance from it); $T_f = 1 - R_f$ is the transmission coefficient of the pump by the front mirror surface.

In the case of the multipass pump with additional extracavity mirrors (Fig. 1b),

$$A_1 = \frac{T_f[1 - (R_b R_0)^N e^{-2Nkh}]}{1 - R_b R_0 e^{-2kh}}$$

is the summing coefficient of all the intensities $I_{1i}(0)$ of the waves on the surface $z = 0$ returning to the disk after reflection from the combined additional mirror with the effective reflection coefficient $R_0 = R_a T_f^2$; R_a is the reflection coefficient of the additional extracavity mirror; $2N$ is the number of passes; N is the number of additional extracavity mirrors.

The coefficient $A_2 = R_b e^{-2kh}$ characterises the decrease in the intensity $I_{1i}(0)$ due to one round trip in the disk and is the same both for the conventional pump and for the pump with additional extracavity mirrors.

3. Temperature of the disk AE in the case of the multipass uniform stationary pump

Consider the disk as a thermalphysic object which has internal heat release sources and, hence, thermal flows producing a temperature field. This field can lead both to the destruction of the disk itself and to the undesirable effect on the cooling medium.

We will assume that the side surfaces of the disk are heat-insulated, the pump illuminates the entire disk plane and the end faces are cooled so that the temperature gradient in the disk be perpendicular to them, i.e. the temperature distribution is homogeneous along the disk radius. This situation is reduced to a one-dimensional heat conduction problem when the temperature depends only on one coordinate z [6–8].

Let the disk be in the coordinate system shown in Fig. 1. The stationary heat conduction equation, in which the thermal flows are directed only along the z axis and the heat release power is identical over the disk radius and is the function of z, has the form

$$\frac{dT^2(z)}{dz^2} = -\frac{q_v(z)}{\lambda}, \quad (2)$$

where λ is the heat conductivity; $q_v(z) = \xi I_0 A_1 k(e^{-kz} + A_2 e^{kz})$ is the total volume power density of the heat source for the intensities of all the pump waves inside the disk propagating in the forward and backward directions; ξ is the part of the absorbed pump power transformed to heat.

The boundary conditions of the third kind on both surfaces of the disk have the form:

$$\left.\frac{dT}{dz}\right|_{z=0} = \frac{\alpha_1}{\lambda}[T(0) - T_{f1}], \quad \left.\frac{dT}{dz}\right|_{z=h} = -\frac{\alpha_2}{\lambda}[T(h) - T_{f2}], \quad (3)$$

where α_1, $T(0)$ and T_{f1} are the heat exchange coefficient of the disk with the cooling medium, the disk surface temperature at $z = 0$, and the temperature of the cooling medium to the left of the AE (Fig. 2); α_2, $T(h)$ and T_{f2} is the heat exchange coefficient between the disk surface and the cooling medium, the temperature of the disk surface at $z = h$, and the temperature of the cooling medium to the right of the AE (the cooling medium, for example, water is assumed to cool directly either the disk end face or the substrate on which the disk is located, the diameters of the disk and surface being equal and the quantity α_2 representing the effective heat exchange coefficient of the disk with the cooling water through the substrate).

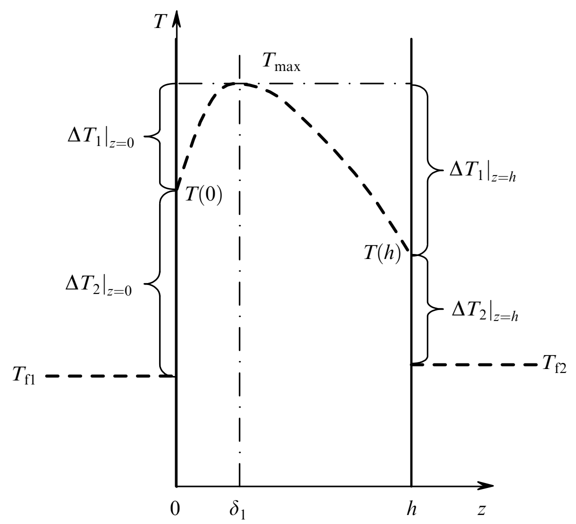

Figure 2. Qualitative pattern of the temperature distribution in the disk.

The solution of equation (2) with boundary conditions (3) has the form

$$T(z) = I_0 \xi A_1 [f_1(z) - A_2 f_2(z)] - \Delta T(z) + T_{f1}, \quad (4)$$

where

$$f_1(z) = \frac{1}{\lambda}\left[\frac{\lambda}{\alpha_1} - \frac{a_1}{b}\left(\frac{\lambda}{\alpha_1} + z\right) + \frac{(1 - e^{-kz})}{k}\right];$$

$$f_2(z) = \frac{1}{\lambda}\left[\frac{\lambda}{\alpha_1} - \frac{a_2}{b}\left(\frac{\lambda}{\alpha_1} + z\right) - \frac{(1 - e^{kz})}{k}\right];$$

$$a_1 = \frac{\lambda}{\alpha_1} + \frac{1}{k} + \mathrm{e}^{-kh}\left(\frac{\lambda}{\alpha_2} - \frac{1}{k}\right);$$

$$a_2 = \frac{\lambda}{\alpha_1} - \frac{1}{k} + \mathrm{e}^{kh}\left(\frac{\lambda}{\alpha_2} + \frac{1}{k}\right); \quad b = \frac{\lambda}{\alpha_1} + \frac{\lambda}{\alpha_2} + h;$$

$$\Delta T(z) = \frac{\Delta T_{\mathrm{f}}}{b}\left(\frac{\lambda}{\alpha_1} + z\right); \quad \Delta T_{\mathrm{f}} = T_{\mathrm{f1}} - T_{\mathrm{f2}}.$$

Figure 2 shows the qualitative pattern of the temperature distribution and notations. The temperature drops with respect to the temperatures of the cooling media $\Delta T_2|_{z=0} = T(0) - T_{\mathrm{f1}}$ and $\Delta T_2|_{z=h} = T(h) - T_{\mathrm{f2}}$ take place on the disk surfaces.

Because the AE is cooled from the end faces, the temperature distribution along the disk thickness will have a maximum. Let this maximum be located at a distance δ_1 from the left-hand side of the disk surface. Then, when the derivative $\mathrm{d}T/\mathrm{d}z|_{z=\delta_1}$ is equal to zero, we obtain the expression

$$-I_0\xi A_1 \frac{1}{\lambda}(\mathrm{e}^{-k\delta_1} - A_2\mathrm{e}^{k\delta_1}) = C, \tag{5}$$

where

$$C = -I_0\xi A_1 \frac{1}{b\lambda}(a_1 - a_2A_2) - \frac{\Delta T_{\mathrm{f}}}{b}.$$

Let $x = \mathrm{e}^{-k\delta_1}$, equation (5) can be reduced to the form:

$$x^2 + \frac{C\lambda}{\xi I_0 A_1}x - A_2 = 0.$$

The solution of the last equation has the form

$$x = -\frac{C\lambda}{2\xi I_0 A_1} + \left[\left(\frac{C\lambda}{2\xi I_0 A_1}\right)^2 + A_2\right]^{1/2},$$

or

$$\delta_1 = -\frac{1}{k}\ln x.$$

The maximum temperature in the disk is

$$T_{\max} = \Delta T_1|_{z=0} + \Delta T_2|_{z=0} + T_{\mathrm{f1}} = \Delta T_1|_{z=h} + \Delta T_2|_{z=h} + T_{\mathrm{f2}},$$

where

$$\Delta T_1|_{z=0} = T_{\max} - T(0); \quad \Delta T_1|_{z=h} = T_{\max} - T(h).$$

4. Limiting internal and external temperatures

Consider the case when the cooling media on both surfaces of the disk have identical temperature equal to room temperature ($T_{\mathrm{f1}} = T_{\mathrm{f2}} = 300$ K). Then, the expressions for the temperature drops $\Delta T_i|_z$ (where $i = 1, 2$; $z = 0, h$) are written in the form

$$\Delta T_1|_{z=0} = \xi I_0 A_1 F_{10} = \frac{\xi I_0 A_1}{k\lambda} \times \left[1 - x + A_2\left(1 - \frac{1}{x}\right) - \frac{k\delta_1}{b}(a_1 - a_2A_2)\right],$$

$$\Delta T_2|_{z=0} = \xi I_0 A_1 F_{20} = \frac{\xi I_0 A_1}{\alpha_1}\left[1 - A_2 - \frac{1}{b}(a_1 - a_2A_2)\right],$$

$$\Delta T_1|_{z=h} = \xi I_0 A_1 F_{1h} = \frac{\xi I_0 A_1}{k\lambda}\left[\mathrm{e}^{-kh} - x + A_2 \times \left(\mathrm{e}^{kh} - \frac{1}{x}\right) + \frac{k(h-\delta_1)}{b}(a_1 - a_2A_2)\right], \tag{6}$$

$$\Delta T_2|_{z=h} = \xi I_0 A_1 F_{2h} = \frac{\xi I_0 A_1}{\alpha_2} \times \left[-\mathrm{e}^{-kh} + A_2\mathrm{e}^{kh} + \frac{1}{b}(a_1 - a_2A_2)\right],$$

where

$$x = \mathrm{e}^{-k\delta_1} = \frac{a_1 - a_2A_2}{2b}\left\{1 + \left[1 + \frac{4b^2A_2}{(a_1 - a_2A_2)^2}\right]^{1/2}\right\}.$$

One can see that the temperature drops are proportional to the pump intensity, the number of pump passes in the disk and the fraction of the absorbed pump power transformed to heat as well as to the function $F_{iz}(\lambda, k, h, \alpha_1, \alpha_2, R_{\mathrm{b}})$, whose arguments are responsible for the heat exchange with the cooling medium. The ratios of the temperature drops to each other are independent of the input pump intensity and the number of pump passes in the disk $[A_1(N)]$ with additional mirrors, but depend only on the parameters of the active medium (h, k, λ) and the cooling conditions (α_1 and α_2).

It is obvious that for the temperature drops there exist some critical values whose excess damages the laser, which results in limiting the pump intensity. Here, we will consider some cases of the limiting temperature drops. We will call the drops ΔT_{10} and ΔT_{1h}, whereby either the disk is damaged or $T_{\max}$ achieves a specified critical value, the internal critical temperature drops. We will call the drops, whereby the disk surface being cooled begins to affect the cooling medium so that the normal cooling regime is violated, the external critical temperature drops. For example, water starts to boil or low-melting metal starts to melt. For definiteness, we assume that the temperature of the disk end faces being cooled should not exceed, in these cases, the boiling temperature of water under normal conditions or the melting temperature of the substrate. Table 1 presents expressions describing these critical events.

5. Thermoelastic stress and limiting pump intensity damaging thermally the disk

The expression for the thermoelastic stresses at an arbitrary temperature $T(z)$ inside a free thin disk is presented, for example, in [9, 10] and has the form

$$\sigma(z) = \gamma\left[-T(z) + \bar{T} + \tilde{T}(z)\left(z - \frac{h}{2}\right)\right], \tag{7}$$

where

$$\sigma_{xx}(z) = \sigma_{yy}(z) = \sigma(z); \ \sigma_{zz} = 0;$$

Table 1. Temperature drops at an arbitrary pump intensity and limiting pump intensities

Region	Temperature drops	Limiting pump intensities
A	$\Delta T_{20} = I_0 \xi A_1 F_{20}$	$I_0^{\mathrm{cr}} = \dfrac{\Delta T_{20}^{\mathrm{cr}}}{\xi A_1 F_{20}}$
B	$\Delta T_{2h} = I_0 \xi A_1 F_{2h}$	$I_0^{\mathrm{cr}} = \dfrac{\Delta T_{2h}^{\mathrm{cr}}}{\xi A_1 F_{2h}}$
C	$\Delta T_{10} = I_0 \xi A_1 F_{10}$ $\Delta T_{1h} = I_0 \xi A_1 F_{1h}$	$I_0^{\mathrm{dst}} = \dfrac{\lambda k}{\xi A_1 \Phi(0)} \dfrac{\sigma_{\mathrm{s}}}{\gamma}$
D	$T_{\max} - T_{\mathrm{f1}} = I_0 \xi A_1 (F_{10} + F_{20})$ $T_{\max} - T_{\mathrm{f2}} = I_0 \xi A_1 (F_{1h} + F_{2h})$	$I_0^{T_{\max}} = \dfrac{T_{\max} - T_{\mathrm{f1}}}{\xi A_1 (F_{10} + F_{20})}$ $I_0^{T_{\max}} = \dfrac{T_{\max} - T_{\mathrm{f2}}}{\xi A_1 (F_{1h} + F_{2h})}$

Notes: region A is the achievement of the critical external temperature drop (for example, water boiling) at $z = 0$ ($\Delta T_{20}^{\mathrm{cr}} + T_{\mathrm{f1}} = 373$ K); region B is the achievement of the critical external temperature drop (for example, water boiling or substrate melting) at $z = h$ ($\Delta T_{2h}^{\mathrm{cr}} + T_{\mathrm{f2}} = 373$ K or $\Delta T_{2h}^{\mathrm{cr}} + T_{\mathrm{f2}} = T_{\mathrm{melt}}$); region C is the crystal disk destruction; region D is the limitation of the maximum internal temperature by the specified quantity of $T_{\max}$.

$$\bar{T} = \frac{1}{h}\int_0^h T(z)\mathrm{d}z; \quad \tilde{T}(z) = \frac{12}{h^3}\int_0^h T(z)\left(z - \frac{h}{2}\right)\mathrm{d}z;$$

γ is the coefficient proportional to the thermal expansion coefficient, inverse to the elastic compliance coefficient, and changing as a function of the disk orientation with respect to the crystallographic axis of the crystal [4].

By substituting temperature (4) into expression (7), we obtain

$$\sigma(z) = \gamma I_0 \xi A_1 \frac{1}{\lambda k}\Phi(z),$$

where

$$\Phi(z) = \Phi_1(z) - A_2\Phi_2(z);$$

$$\Phi_1(z) = -\frac{1 - \mathrm{e}^{-kh}}{kh} + \mathrm{e}^{-kz} + \frac{12}{kh^2}\left(z - \frac{h}{2}\right) \times \left(\frac{1 + \mathrm{e}^{-kh}}{2} - \frac{1 - \mathrm{e}^{-kh}}{kh}\right);$$

$$\Phi_2(z) = -\frac{1 - \mathrm{e}^{kh}}{kh} - \mathrm{e}^{kz} + \frac{12}{kh^2}\left(z - \frac{h}{2}\right) \times \left(\frac{1 + \mathrm{e}^{kh}}{2} + \frac{1 - \mathrm{e}^{kh}}{kh}\right).$$

According to [4], the inequality

$$\sigma_{\mathrm{s}} \geqslant \gamma I_0 \xi A_1 \frac{\Phi(z)}{\lambda k} \tag{8}$$

should be fulfilled not to damage the disk during heating. Here, σ_{s} is the limiting stress in the case of brittle fracture. If the right-hand side of (8) does not exceed the left-hand side at none of the values of z, the disk is not damaged. Two nonsymmetric unloading planes parallel to the disk ends exist inside a disk in a stressed state. Between the unloading planes the stress is negative (compression) and between the unloading planes and the disk end faces – positive (tension). The maximum stress at any parameters k and h appears on the surface $z = 0$; in this case, the expression for $\Phi(z)$ assumes the form

$$\Phi(0) = \Phi_1(0) - A_2\Phi_2(0),$$

where

$$\Phi_1(0) = -\frac{1 - \mathrm{e}^{-kh}}{kh} + 1 - \frac{6}{kh}\left(\frac{1 + \mathrm{e}^{-kh}}{2} - \frac{1 - \mathrm{e}^{-kh}}{kh}\right);$$

$$\Phi_2(0) = -\frac{1 - \mathrm{e}^{kh}}{kh} - 1 - \frac{6}{kh}\left(\frac{1 + \mathrm{e}^{kh}}{2} + \frac{1 - \mathrm{e}^{kh}}{kh}\right).$$

Thus, in the case of the limiting pump intensity, we obtain the expression

$$I_0^{\mathrm{dst}} = \frac{\sigma_{\mathrm{s}}}{\gamma}\frac{\lambda k}{\xi A_1 \Phi(0)}. \tag{9}$$

It follows from this expression that as k increases, the pump intensity damaging the sample increases, i.e. the sample damage threshold increases. This effect is experimentally confirmed, for example, in paper [4].

6. Regions of critical events (A, B, C, D) in the parameters *k* and *h*

By regions are meant such domains of the parameters k and h in which, as the pump intensity increases, one of the following events is achieved first: boiling of the cooling water, melting of the substrate made of low-melting metal (used, for example, in Peltier refrigerators), a significant deterioration of the laser parameters owing to changes in the spectral and luminescent properties of the active medium during heating, disk damage or melting. These regions can correspond not only to critical events but also to regions with temperatures specified in advance inside the disk and on the disk surface, which allows simulation of laser systems. Thus, estimates show that the slope efficiency of a Nd:YAG disk at ~ 473 K decreases by more than 10 % due to changes in the spectral and luminescent properties of the active medium during heating, while the threshold increases by several times. However, in lasers with a high output power such an increase in the threshold (for a four-level scheme) is insignificant.

Of interest is the account for the temperature dependence of the heat conductivity λ obtained in the expressions. This dependence for the Nd:YAG crystal is presented, for example, in papers [1, 11] and has the form

$$\lambda(T) = \lambda_0\left(\frac{204\ \mathrm{K}}{T - 96\ \mathrm{K}}\right)^{0.63}, \tag{10}$$

where λ_0 if the heat conduction coefficient at $T_0 = 300$ K. Paper [12] presents for the same crystal the first-order approximation

$$\lambda(T) = \lambda_0\frac{T_0}{T}. \tag{11}$$

Up to $T = 1000$ K, approximations (10) and (11) differ by no more than 25 %.

We will analyse in the first approximation the possible critical events presented in Table 1 with the help of expression (10) up to temperatures damaging the disk. Then, to estimate the region boundaries of extreme events (A, B, C, D), we will assume that the heat conductivity corresponding to expression (10) is identical over the entire disk thickness at the maximum temperature $T_{\max}$. It is obvious that the disk temperature average over the thickness is smaller than the maximum temperature and, hence, the heat conductivity average over the disk thickness will be somewhat greater. We believe that absorption in the AE weakly depends on temperature, which can be partially achieved by matching the pump spectrum with the absorption spectrum of neodymium ions with increasing temperature.

Based on the above results and assumptions, we can estimate the region boundaries of extreme events and thus determine the sequence of these events. Inside each of the regions A, B, C, D, temperatures can achieve only their critical values, the regions being rigorously separated.

Consider, for example, a disk AE made of the Nd : YAG crystal with a cubic symmetry and the crystallographic axis oriented parallel to the pump beam propagation ($z \,||\, [100]$) for different types of cooling and identical temperatures of the cooling media $T_{\rm f1} = T_{\rm f2} = 300$ K. We used the following parameters for the Nd : YAG crystal: $\lambda_0 = 0.13$ W cm^{-1} K^{-1}, $\sigma_{\rm s} = 2008$ kg cm^{-2}, $\gamma = 33.95$ kg cm^{-2} K^{-1} [4], $\xi = 0.241$ (Stokes losses during pumping at $\lambda_{\rm p} = 0.808$ μm and lasing at $\lambda_{\rm g} = 1.064$ μm).

6.1 Method for determining the boundary of regions B – C

The region boundary separates two events – achievement of the critical external temperature drop $\Delta T_{2h}^{\rm cr}$ at $z = h$ (region B) and damage of the disk (region C).

6.1.1 Edge of the boundary of regions B – C

To determine the edge of the region boundary, it is necessary to find the values $h_x, \lambda_x(T_{\max})$. We will use from Table 1 (regions B and C) the expressions for the pump intensities at which the critical external temperature drop is achieved and the disk is damaged, and will equate them to each other. We transform the derived expression for the case $k \to 0$:

$$\lim_{k\to 0} \frac{\lambda k F_{2h}(k,h,\lambda)}{\Phi(0,k,h)} = \frac{12\lambda}{h\alpha_2}\frac{b_1(h,\lambda)}{b(h,\lambda)} = \frac{\gamma}{\sigma_{\rm s}}\Delta T_{2h}^{\rm cr}, \tag{12}$$

where $b_1(h,\lambda) = \lambda/\alpha_1 + h/2$. Then, we will use from Table 1 (region C), for example, the expression for the internal temperature drop at the destruction moment. We transform the derived expression for the case $k \to 0$:

$$\lim_{k\to 0} \Delta T_{1h}^{\rm dst}(k,h,\lambda) = 6\frac{\sigma_{\rm s}}{\gamma}\frac{b_1^2(h,\lambda)}{b^2(h,\lambda)}$$

$$= T_{\max}(\lambda) - \Delta T_{2h}^{\rm cr} - T_{\rm f2}. \tag{13}$$

We obtain, respectively, from (12) and (13)

$$h = -\frac{C(\lambda)}{2} + \left[\frac{C^2(\lambda)}{4} + \frac{12\lambda^2\sigma_{\rm s}}{\alpha_1\alpha_2\gamma\Delta T_{2h}^{\rm cr}}\right]^{1/2}, \tag{14}$$

$$h = \frac{12\lambda}{\alpha_2\Delta T_{2h}^{\rm cr}}\left\{\frac{\sigma_{\rm s}}{6\gamma}\left[T_{\max}(\lambda) - \Delta T_{2h}^{\rm cr} - T_{\rm f2}\right]\right\}^{1/2}, \tag{15}$$

where

$$C(\lambda) = \lambda\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} - \frac{6\sigma_{\rm s}}{\alpha_2\gamma\Delta T_{2h}^{\rm cr}}\right).$$

By equating (14) and (15) to each other, we obtain λ_x, $T_{\max}$ and h_x.

For the air – disk – water system ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.75$ W cm^{-2} K^{-1}, $\Delta T_{2h}^{\rm cr} = 73$ K), for the disk thickness $h_x = 0.8$ cm, the water starts boiling and the disk is damaged at the same pump intensity (at $k \to 0$). In this case, $T_{\max} = 664$ K and $\lambda_x = 0.0682$ W cm^{-1} K^{-1}. For comparison: when the $\lambda(T)$ dependence is used according to (11), we obtain $h_x = 0.7$ cm, $T_{\max} = 666$ K, and $\lambda_x = 0.0586$ W cm^{-1} K^{-1}.

Thus, because λ is temperature dependant, the region boundary separating two extreme events – water boiling and disk destruction, corresponds to a 0.8-cm-thick disk (at small k). In this case, the maximum disk temperature at small k should be equal, at least, to 664 K and increase along the region boundary (with increasing h and k).

For the air – disk – low-melting metal system ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.1$ W cm^{-2} K^{-1}, $\Delta T_{2h}^{\rm cr} + T_{\rm f2} = 473$ K – approximate melting temperature of the metal), for the disk thickness $h_x = 2.1$ cm, the substrate melting temperature and the disk breakdown temperature are simultaneously achieved (at $k \to 0$). In this case, $T_{\max} = 678$ K and $\lambda_x = 0.0671$ W cm^{-1} K^{-1}. For comparison: when the $\lambda(T)$ dependence is used according to (11), we obtain $h_x = 1.8$ cm, $T_{\max} = 679$ K, and $\lambda_x = 0.0574$ W cm^{-1} K^{-1}.

6.1.2 Boundary of regions B – C at $T_{\max}(k) > T_{\max}$ $(k \to 0)$. By using the parameters $T_{\max}(\lambda_x)$ and h_x found in section 6.6.1, we can determine the region boundary at any k. To do this, we will use from Table 1 (regions B and C) the expressions for the pump intensities at which the critical external temperature drop is achieved and the disk is damaged, and will equate them to each other. We solve the derived expression for the parameters k and h, by setting the prior quantity $T_{\max} > T_{\max}(k \to 0)$ $[T_{\max} = T_{\max}(k \to 0) + \Delta T]$. Figure 3a shows the results of calculations of the region boundary for the air – disk – water system. One can see that when $T_{\max}$ increases, the boundary of the water boiling and the disk destruction has a 'break' at $h \approx 0.62$ cm and $k \approx 8$ cm^{-1} (at $T_{\max} \approx 733$ K). This allows us to conclude that the disk can be damaged thermomechanically only at thicknesses larger than 0.62 cm. At smaller thicknesses, it is water that first starts boiling.

For the air – disk – low-melting metal system, the behaviour of the boundary separating the substrate melting and disk destruction will be analogous. However, at the melting temperature $T_{\rm melt} = 473$ K, we have a similar 'break' at $h \approx 1.59$ cm and $k \approx 3$ cm^{-1} ($h_x = 2.1$ cm at $k \to 0$) and at $T_{\rm melt} = 523$ K, the 'break' point is displaced towards smaller thicknesses: $h \approx 1.24$ cm at $k \approx 4$ cm^{-1} ($h_x = 1.58$ cm at $k \to 0$).

6.2 Method for determining the boundary of regions B – D

The region boundary separates two events – achievement of the maximum internal prior temperature $T_{\max}$ of the disk AE up to which the laser operates without any noticeable deterioration of the lasing parameters (region D), and achievement of the critical external temperature drop $\Delta T_{2h}^{\rm cr}$ at $z = h$ (region B).

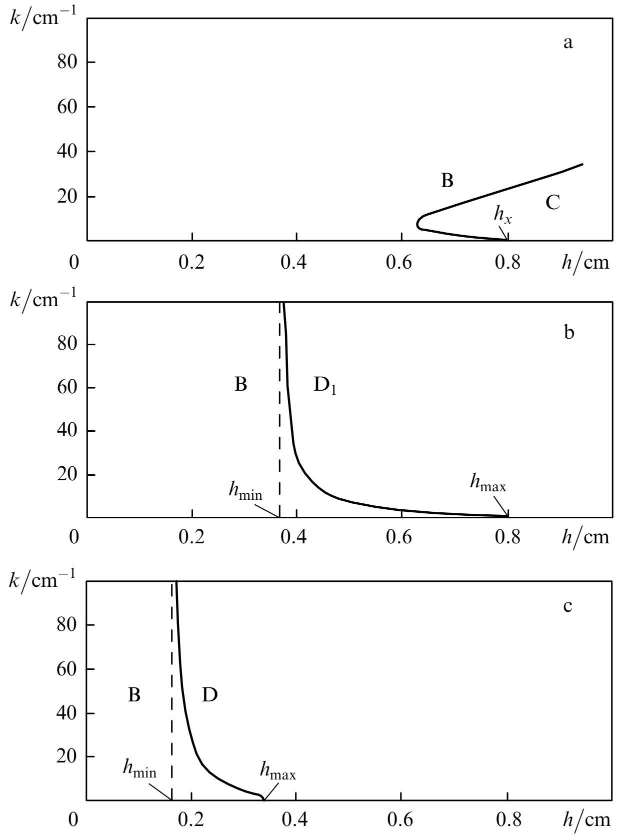

Figure 3. Region boundaries (solid curves) for a disk made of the Nd:YAG crystal with the $z||[100]$ orientation upon cooling in the air–disk–water system ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.75$W cm^{-2} K^{-1}). Regions: (B) water boiling; (C) disk destruction; (D) and (D_1) achievement of $T_{max} = 473$ K and 664 K, respectively. Dashed lines are asymptomatics at $k \to \infty$.

We assume that the maximum temperature T_{max} at which the Nd:YAG laser operates without a noticeable deterioration of lasing parameters is 473 K. We will use the proposed method of analysis to determine the disk parameters at which its maximum temperature does not exceed T_{max}. Simultaneously, we will follow the sequence of other events, namely, water boiling and substrate melting (depending on the cooling type) as the pump intensity is increased.

By equating the pump intensity taken from Table 1 (regions B and D), at which the external temperature drop achieves a critical value ΔT_{2h}^{cr}, to the pump intensity, at which the maximum temperature achieves T_{max}, we obtain a characteristic expression, that is transformed for the cases $k \to \infty$ and $k \to 0$:

$$\lim_{k\to\infty} \frac{F_{1h}(k,h,\lambda)}{F_{2h}(k,h,\lambda)} = \frac{\alpha_2 h}{\lambda}, \tag{16}$$

$$\lim_{k\to 0} \frac{F_{1h}(k,h,\lambda)}{F_{2h}(k,h,\lambda)} = \frac{\alpha_2 h}{2\lambda}\frac{b_1(h,\lambda)}{b(h,\lambda)}. \tag{17}$$

Note that in this case, the temperature inside the disk is maximal at distances $\delta_2 \to h$ и $\delta_2 \to hb_1/b$ at $k \to \infty$ and $k \to 0$ respectively, where $\delta_2 = h - \delta_1$ is the distance from the right-hand side of the disk to the plane inside the disk with the temperature T_{max}. By using (16) and (17), we obtain the minimal and maximal disk thicknesses:

$$h_{min} = \frac{\lambda}{\alpha_2}\left(\frac{T_{max} - T_{f2}}{\Delta T_{2h}^{cr}} - 1\right), \quad h_{max} \approx 2h_{min}. \tag{18}$$

Because $\alpha_1 \ll \alpha_2$, the accuracy for the second expression is $\sim 10\,\%$ (at $h < 1$ cm). Depending on k, both events are possible in the interval of the thicknesses $h_{min} < h < h_{max}$, i.e. achievement of both the critical external temperature drop ΔT_{2h}^{cr} and the internal maximum temperature T_{max}.

For the air–disk–water system (Fig. 3c) ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.75$ W cm^{-2} K^{-1}), in the case of direct contact of water with the disk, the internal maximum temperature of 437 K at $z = h$ is achieved when the water starts boiling (as the pump intensity increases) for the disk thickness $h < h_{cr} \in [h_{min} = 0.161$ cm, $h_{max} = 0.34$ cm] (depending on k). In this case, if $h < h_{cr}$, the water starts boiling before the maximum internal temperature of 473 K is achieved, while if $h > h_{cr}$, this temperature is achieved before the water starts boiling.

To demonstrate the sequence evolution of the events with the prior critical temperatures, Fig. 3b shows a curve for which $h_{max} = h_x$ and region D_1 corresponds to the constant maximum temperature $T_{max} = 664$ K. One can see that the disk thickness for the boundary B–D_1 is greater than that for the boundary B–D.

For the air–disk–low-melting metal system, the parameters are $\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.1$ W cm^{-2} K^{-1}, $\Delta T_{2h}^{cr} + T_{f2} = T_{melt}$. If $T_{melt} = 373$ K, the substrate starts melting at $h < h_{cr} \in [h_{min} = 1.21$ cm, $h_{max} = 3.24$ cm], if $T_{melt} = 423$ K – at $h < h_{cr} \in [h_{min} = 0.36$ cm, $h_{max} = 0.87$ cm] (Fig. 4b), and if $T_{melt} = 453$ K – at $h < h_{cr} \in [h_{min} = 0.12$ cm, $h_{max} = 0.27$ cm] (Fig. 4a). For $T_{melt} \geqslant T_{max} = 473$ K, the first event, as the pump intensity is increased, will obviously be the achievement of the temperature $T_{max} = 473$ K, this temperature being achieved

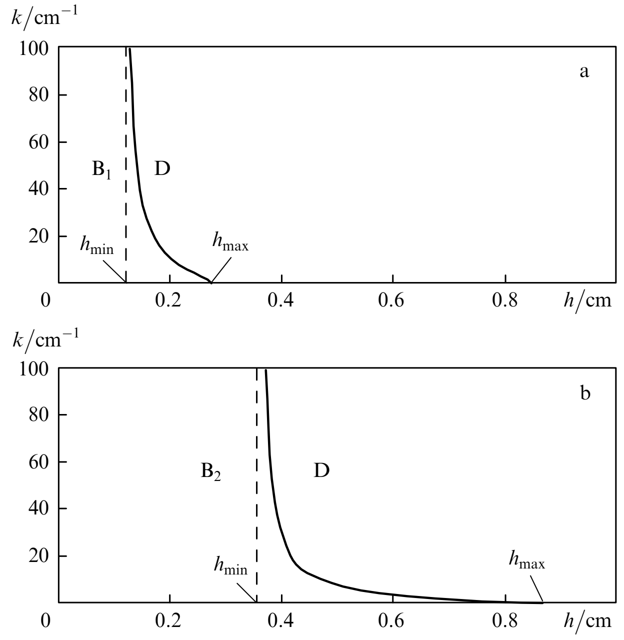

Figure 4. Region boundaries (solid curves) for a disk made of the Nd:YAG crystal with the $z||[100]$ orientation upon cooling in the air–disk–low-melting metal system ($\alpha_1 = 0.015$W cm^{-2} K^{-1}, $\alpha_2 = 0.1$ W cm^{-2} K^{-1}). Regions: (B) and (B_1) substrate melting with the melting temperatures 453 K and 423 K, respectively; (D) achievement of $T_{max} = 473$ K. Dashed lines are asymptomatics at $k \to \infty$.

in the interval of thicknesses $0 \leqslant h \leqslant 12.4$ cm. At $h > 12.4$ cm (as shown below), the first event, as the pump intensity is increased, will be the disk destruction.

6.3 Method for determining the boundary of regions D – C

The region boundary separates two events – achievement of the maximum internal preliminarily specified temperature T_{max} of the disk AE at which the laser operates without any noticeable deterioration of the lasing parameters (region D), and the disk damage at the same maximum temperature (region C).

We will use from Table 1 (regions D and C) the expressions for the pump intensities at which the internal temperature drop becomes critical for lasing and the disk is damaged, and will equate them to each other. We transform the derived expression for the case $k \to 0$:

$$\lim_{k\to 0} \frac{\lambda k[F_{1h}(k,h,\lambda) + F_{2h}(k,h,\lambda)]}{\Phi(0,k,h)}$$

$$= 6\,\frac{b_1^2(h,\lambda)}{b^2(h,\lambda)} + \frac{12\lambda}{h\alpha_2}\frac{b_1(h,\lambda)}{b(h,\lambda)} = \frac{\gamma}{\sigma_s}[T_{max}(\lambda) - T_{f2}]. \quad (19)$$

For the air – disk – air system, we have $\alpha_1 = \alpha_2 = 0.015$ W cm^{-2} K^{-1}. We obtain from equation (19) that $h_x = 24.7$ cm. Therefore, as the pump intensity is increased, the maximum temperature inside the disk (473 K) is achieved before other possible extreme temperatures (responsible for the destruction, melting) at $h < 24.7$ cm and any k. However, at $h > 24.7$ cm and the corresponding k, the disk destruction can first take place when the internal maximum temperature is even smaller than 473 K.

For the air – disk – heat-resistant metal system ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.1$ W cm^{-2} K^{-1}), we obtain from equation (19) that $h_x = 12.4$ cm. The thermal processes and their consequences are similar to those considered above for the air – disk – air cooling system.

Thus, the pump intensities $I_0^{T_{max}}$ presented in Table 1 (region D) are limiting for both cooling methods. By using these intensities, one can calculate the generation parameters of a laser at any k and $h < h_x$ when the maximum internal temperature does not exceed the prescribed value restricting the laser efficiency.

7. Maximum limiting lasing intensity restricted by the critical event B

Based on the determined pump intensities (Table 1) and the critical regions in the parameters k and h, we can estimate the maximum power of the disk laser. Thus, we can determine the minimum size of the AE in order to obtain the desired output power. We will not consider the output beam quality as well as the resonator dimensions and the curvature of the mirrors. The resonator is assumed to be short, i.e. the mirrors are either located near the AE end faces or deposited on them.

The output intensity of a four-level scheme laser during the stationary pump is described by the expression

$$I_g = \frac{a}{a + 2\chi h}\frac{\hbar\omega_g}{\hbar\omega_p}\eta_{abs}(I_0 - I_{th}), \quad (20)$$

where $a = \ln R^{-1}$; R is the reflection coefficient of the resonator mirror at the laser wavelength (the back mirror is highly reflecting); χ is the passive loss coefficient of the resonator; $\hbar\omega_g$ and $\hbar\omega_p$ are the laser and pump photon energies, respectively; I_{th} is the threshold pump intensity;

$$\eta_{abs} = A_1(1 - e^{-kh})(1 + A_2 e^{kh}) \quad (21)$$

is the portion of the absorbed pump energy.

Obviously, the quantities η_{abs} and I_{th} depend on temperature due to changes in the spectroscopic properties of the AE during heating (such as the Boltzmann population of Stark levels, temperature broadening of pump absorption lines and the luminescence line of the working laser level, etc.). As the AE temperature increases, η_{abs} decreases while I_{th} increases, which affects the lasing efficiency.

For each disk thickness h, expression (20) allows one to determine the maximum lasing intensity for the given reflection coefficient of the output mirror, if we substitute into it the maximum possible pump intensity limited by one of the critical events considered above. When the critical even B takes place first (water boiling), the limiting lasing intensity increases with decreasing the disk thickness and in the limit (at $h \to 0$) achieves the quantity

$$I_g^{max} = \frac{\hbar\omega_g}{\hbar\omega_p}\lim_{h\to 0}(\eta_{abs} I_0^{cr}) = \frac{\hbar\omega_g}{\hbar\omega_p}\frac{\Delta T_{2h}^{cr}}{\xi}(\alpha_1 + \alpha_2). \quad (22)$$

The derived expression shows the potentials of the disk laser in which the thermal flows are directed only along the z axis.

A disk laser of this type allows one to increase the output power by simply increasing the diameter of the pumped region, because in this case both the power and heat extraction will increase proportionally. For example, for the above values of the thermal parameters of the air- and water-cooled disk, the intensity is $I_g^{max} = 176$ W cm^{-2} for the Nd : YAG crystal ($\alpha_1 = 0.015$ W cm^{-2} K^{-1}, $\alpha_2 = 0.75$ W cm^{-2} K^{-1}). In this case, to obtain, for example, the output power of 1, 10, and 100 kW, the disk diameter should be no less than 2.7, 8.5, and 27 cm, respectively.

8. Conclusions

In this paper, we have obtained analytic expression for the homogeneous distribution of the temperature and thermoelastic stresses in a longitudinally uniformly steady-state pumped disk AE. Calculations have been performed for Nd : YAG crystals. We have determined the parameter values at which, as the pump intensity is increased, there first occurs one of three events violating the laser operation: disk destruction, violation of the normal cooling regime, achievement, inside the AE, of some specified temperature whose excess noticeably deteriorates the laser output characteristics due to a change in the spectral and luminescent properties of the active medium. Estimates of the existing maximum admissible pump intensities have been presented.

If the pump does not illuminate the entire disk plane, the heat can 'spread' over the unexcited region of the crystal, and the heat is extracted by the coolant (for example, flowing water) not only from the end faces of the pumped region but also from the entire disk. In this case, radial temperature gradients appear and the problem is not one-dimensional any longer. Note, however, that the temperature distribution along the disk axis will be the closer to the distribution obtained without 'spreading', the higher is the ratio of the pumped region diameter d of the disk to its thickness h. Thus, for example, for the Nd : YAG crystal at $d/h = 20$ (which satisfies the condition $d/h \leqslant 20 - 25$ when parasitic modes and superluminescence can be neglected [5]), the calculated temperatures on the disk axis with and without this 'spreading' differ by less than 5 % [in this case we have selected such a power of heat release sources (2000 W cm^{-3}) at which the crystal surface being cooled has the temperature of ~ 373 K and its thickness is 0.03 cm].

Therefore, the conclusions of this paper about the violation of the disk cooling regime and deterioration of its lasing properties during heating remain valid when 'spreading' of the heat over the crystal takes place at real values of d/h.

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