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Experimental observation of stochastic resonance in a solid-state ring laser in the absence of bistability

I.I. Zolotoverkh, N.V. Kravtsov, E.G. Lariontsev, V.V. Firsov, S.N. Chekina

***Abstract*. The appearance of a stochastic resonance is found in a solid-state ring laser in the absence of bistability in it. It is shown that in this laser operating in the self-modulation regime of the first kind, the noise modulation of the pump can cause a significant increase in the laser response to the periodic modulating signal. These effects observed are explained qualitatively.**

Keywords: *ring laser, nonlinear dynamics, stochastic resonance, relaxation frequency, noise modulation.*

1. Introduction

The stochastic resonance is one of the most interesting and important phenomena appearing in dynamic nonlinear systems subjected to noise. In particular, the constructive role of noise is manifested in this phenomenon when the energy of stochastic oscillations is transformed into the energy of a coherent signal [1, 2]. This leads to amplification of a weak periodic signal in a certain region of noise intensities, which can favour more efficient signal selected against the noise background.

As a rule, the stochastic resonance appears due to bistability in a nonlinear dynamic system. The most popular models of the stochastic resonance are nonlinear overdamped [1 – 3] or weakly decaying oscillators in a bistable potential well [4, 5]. Noise-induced transitions between bistable states lead to the amplification of the periodic signal, when the frequency of these transitions is close to the signal frequency. A number of papers [6 – 8] discussed other mechanisms of the stochastic resonance in nonlinear dynamic systems. In particular, the stochastic resonance was studied in a nonlinear oscillator with a monostable potential [6]. The problem of existence of the stochastic resonance in monostable states of dynamic systems has been poorly studied so far.

The stochastic resonance in a ring dye laser was first discovered and studied in paper [9] under conditions when two bistable states corresponding to unidirectional lasing regimes in opposite directions prove to be stable. These states are stable only in the case of very weak coupling of counterpropagating waves. Investigation of the stochastic resonance in these bistable states is also of interest for solid-state ring lasers. However, the research performed previously showed that the fulfilment of the weak coupling condition, which provides the stability of the unidirectional lasing in solid-state ring lasers, is difficult to realise in practice [10].

Of undoubtful interest is the study of the stochastic resonance in a two-directional solid-state ring laser (both in the presence of bistability and in monostable state). One of the most interesting lasing regimes in an autonomous solid-state ring laser is the self-modulation regime of the first kind, which is characterised by the out-of-phase sinusoidal intensity modulation of counterpropagating waves. We studied the stochastic effects in such a laser in papers [11 – 13]. In particular, we found in these papers the influence of the noise intensity on the frequency, the spectrum width and the intensity of noise-excited relaxation oscillations of laser radiation.

The aim of this paper is to study experimentally and theoretically the possible appearance of the stochastic resonance in a solid-state ring laser operating in the self-modulation regime of the first kind. The research performed demonstrated for the first time the possibility of appearance of the stochastic resonance in a monostable solid-state laser.

2. Experimental setup

We studied a 1.06-μm Nd : YAG chip laser with a nonplanar resonator. The geometrical perimeter L of the ring cavity was 28 mm and the nonplanarity angle was 85°. The Nd : YAG laser was pumped by a 0.810-μm laser diode. The pump was modulated by periodic and noise electric signals fed to the laser diode (Fig. 1). Noise signals were produced by a noise generator and had a continuous (almost constant in intensity) spectrum covering the range from zero to 200 kHz.

During the experiments, the intensity of the pump noise could be changed by varying the output voltage of the noise generator from zero to the maximum value determined by the maximum intensity of the electric noise at the noise generator output, which was 10^{-6} W Hz^{-1}. The experimental data were processed by normalising the pump noise intensity to the generator output of 10^{-7} W Hz^{-1}. Signals were detected with an analogue-to-digital converter (ADC) and a broadband Tektronix TDS-2014 digital oscilloscope.

I.I. Zolotoverkh, N.V. Kravtsov, E.G. Lariontsev, V.V. Firsov, S.N. Chekina D.V. Skobel'tsyn Institute of Nuclear Physics, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119991 Moscow, Russia; e-mail: e.lariontsev@yahoo.com

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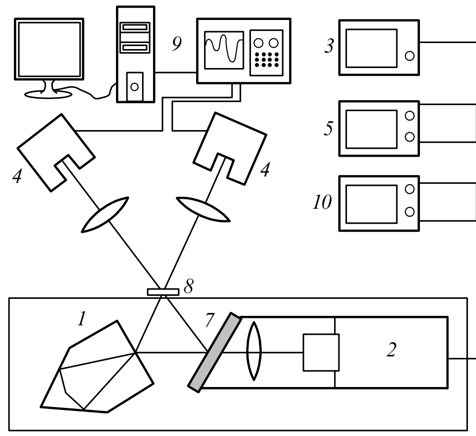

Figure 1. Scheme of the experimental setup: (*1*) ring chip Nd : YAG laser; (*2*) pump diode; (*3*) power supply unit of the laser diode; (*4*) LFD-2 photodetectors; (*5*) GZ-124 generator of stochastic oscillations; (*6*, *11*) transformer coils; (*7*) selective mirror with the reflection coefficient close to 100 % at the laser wavelength and the transmission coefficient close to 90 % at the pump wavelength (this mirror allows measuring simultaneously radiation of two counterpropagating waves); (*8*) IR filter suppressing pump radiation; (*9*) ADC-based oscilloscope – spectrum analyser complex; (*10*) GZ-56 generator of periodic oscillations.

3. Experimental results

The parameters of the bidirectional chip laser under study were chosen so that the ring laser operated in the self-modulation regime of the first kind in the absence of the noise modulation of the pump. In this regime, we studied the power spectra of laser radiation. Main attention was paid to the spectral characteristics in the range of frequencies that are close to the fundamental relaxation frequency. Investigations were performed for two values of the excess of the pump over the threshold: $\eta_1 = 0.08$ and $\eta_2 = 0.22$. The frequency of self-modulation oscillations in this case was equal to 210 kHz and the fundamental relaxation frequency (in the absence of the noise modulation of the pump) – 61.5 kHz. The signal frequency ν_s modulating the pump was selected equal to 56 kHz. At the fixed frequency ν_s, the output power spectrum was used to measure the laser response to the periodic signal and noise at different noise intensities. While processing the experimental results, we performed statistical averaging over 20 realisations.

In the experiments performed we found the resonance in the noise intensity dependence of the laser response to the periodic signal, the resonance being typical of the stochastic resonance. Figure 2 shows experimental emission spectra for two values of the noise intensity. The research showed that when the pump noise increases, the intensity of the periodic signal first increases and achieves the maximum value exceeding the initial value by several times, and then starts decreasing. In the spectrum under study two spectral components are observed against the noise background: one corresponds to the signal frequency $\nu_s = 56$ kHz, while the other – to the peak at the fundamental relaxation frequency broadened due to the noise action. When the noise intensity increases, the central frequency of this peak decreases and approaches the signal frequency. At a relative noise intensity $D = 0.56$ (Fig. 2a), the intensity of the spectral component at the signal frequency exceeds almost

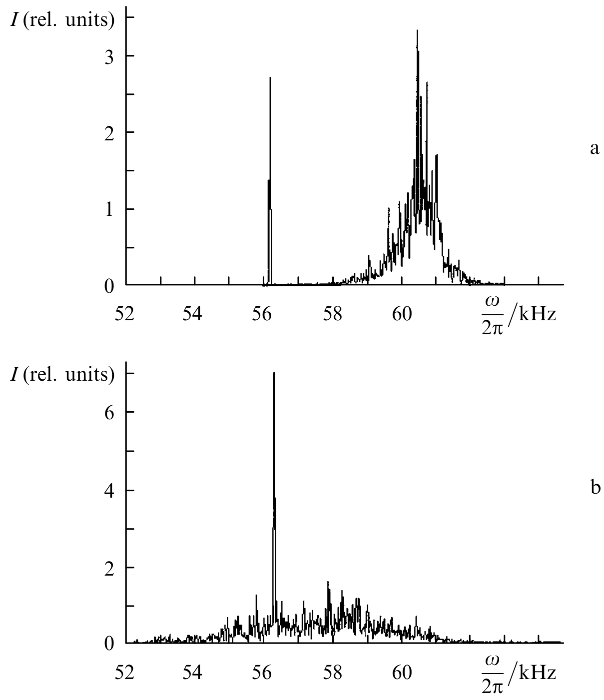

Figure 2. Experimentally measured spectra $I(\omega)$ of the radiation intensity of one of the waves in the case of the excess of the pump over the threshold $\eta_1 = 0.08$ and at the relative noise intensity $D = 0.56$ (a) and 2 (b).

by three times its value in the absence of the noise modulation of the pump. The spectrum in Fig. 2b corresponds to the case, when the frequency of the relaxation peak becomes almost equal to the signal frequency ($D = 2$). In this case, the intensity of the spectral component at the signal frequency increases approximately sevenfold.

Amplification of the laser response to the periodic signal in the presence of noise is conveniently expressed with the help of the stochastic gain factor S, which is determined as the ratio of the modulation amplitude of the laser intensity $A(D)$ at the given noise intensity D to the modulation amplitude in the absence of the external noise $A(0)$:

$$S = A(D)/A(0).$$

If the noise does not affect the laser response to the periodic modulation, the stochastic gain factor is $S = 1$. Figure 3a presents the experimentally measured dependence of the central frequency of the relaxation peak on the noise intensity and Fig. 3b – the dependence of S^2 on D. One can see that the signal intensity achieves the maximum when the signal frequency approaches the central frequency of the relaxation peak. Note that the signal intensity achieves a maximum when the signal frequency differs from the central frequency of the relaxation peak approximately by 1 kHz.

The results of similar investigations performed at $\eta_2 = 0.22$ are demonstrated in Fig. 4. In this case, the fundamental relaxation frequency was 102.8 kHz and the signal frequency was 99 kHz. The experimental dependences of the stochastic gain factor of the signal S^2 on the noise intensity did not change qualitatively; however, the signal intensity achieves a maximum at a significantly lower noise intensity and the signal frequency, which substantially

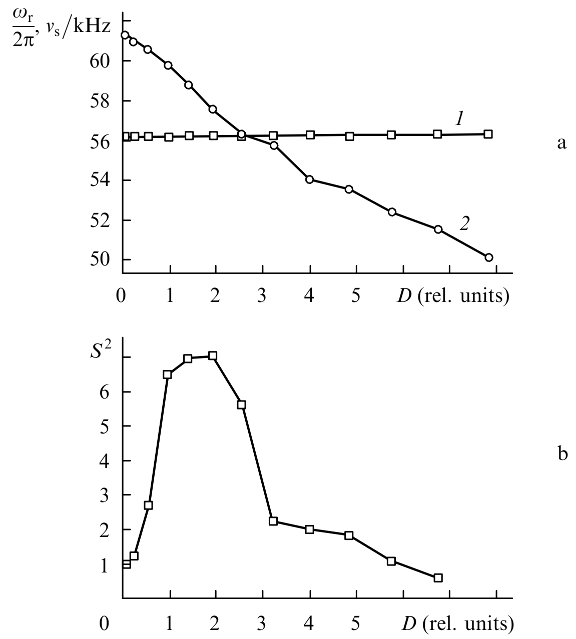

Figure 3. Experimentally measured dependences of the central frequency of the relaxation peak (a) and the stochastic gain factor of the signal S^2 (b) on the noise intensity D in the case of the excess of the pump over the threshold $\eta_1 = 0.08$. Curve (*1*) corresponds to the periodic signal frequency ν_s, which is constant and independent of the noise intensity, while curve (*2*) – to the central frequency of the relaxation peak $\omega_r/2\pi$.

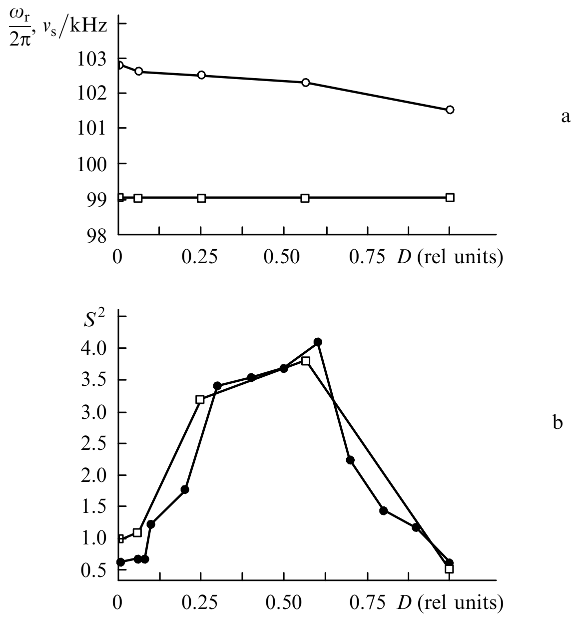

Figure 4. Same as in Fig 3 at $\eta_2 = 0.22$; ○, □ is the experiment, ● is the numerical simulation.

differs (approximately by 3 kHz) from the central frequency of the relaxation peak.

4. Results of numerical simulation

The phenomena under study were numerically simulated by using the vector model of the solid-state ring laser [14], which takes into account the effect of the pump noise. In this case, the initial system of equations of the vector model has the form

$$\frac{d\tilde{E}_{1,2}}{dt} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm i\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{i}{2}\tilde{m}_{1,2}\tilde{E}_{2,1} + \frac{\sigma l}{2T}\left(N_0\tilde{E}_{1,2} + N_{\mp}\tilde{E}_{2,1}\right),$$

$$\frac{dN_0}{dt} = \frac{1}{T_1}\left[N_{\rm th}(1+\eta+\eta_s) - N_0 - N_0 a\left(|E_1|^2 + |E_2|^2\right) - N_+ aE_1E_2^* - N_- aE_1^*E_2\right] + g_w, \tag{1}$$

$$\frac{dN_+}{dt} = -\frac{1}{T_1}\left[N_+ + N_+ a\left(|E_1|^2 + |E_2|^2\right) + \beta N_0 aE_1^*E_2\right],$$

$$N_- = N_+^*.$$

As the dynamic variables we use the complex field amplitudes of the counterpropagating waves $\tilde{E}_{1,2}(t) = E_{1,2}\exp(i\varphi_{1,2})$ and spatial harmonics N_0, $N_\pm$ of the inverse population N determined by the expressions

$$N_0 = \frac{1}{L}\int_0^L N dz, \quad N_\pm = \frac{1}{L}\int_0^L \boldsymbol{e}_1^*\boldsymbol{e}_2 N\exp(\pm i2kz)\,dz. \tag{2}$$

Here, $\omega/Q_{1,2}$ are the resonator bandwidths; ω is the optical frequency; k is the wave number; $Q_{1,2}$ are the resonator Q factors for counterpropagating waves; $T = L/c$ is the round-trip transit time for light in the resonator; T_1 is the time of longitudinal relaxation; l is the length of the active element; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; σ is the laser transition cross section; $\eta = P/P_{\rm th} - 1$ is the excess of the pump power over the threshold; $\eta_s = A_s \times \sin(2\pi\nu_s t)$ is the periodic signal modulating the pump; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the resonator; ω_1, ω_2 are the resonator eigenfrequencies for counterpropagating waves; $N_{\rm th}$ is the threshold inverse population. The pump rate is presented in the form $N_{\rm th}(1+\eta+\eta_s)/T_1$. The linear coupling of counterpropagating waves is determined by the phenomenologically introduced complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(i\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-i\vartheta_2), \tag{3}$$

where $m_{1,2}$ are the moduli of the coupling coefficients and $\vartheta_{1,2}$ are their phases. The field polarisations of counterpropagating waves are characterised by arbitrary unit vectors $\boldsymbol{e}_{1,2}$. The polarisation factor β is defined by the expression $\beta = (\boldsymbol{e}_1\boldsymbol{e}_2)^2$. Note that expressions (1) are written for the case of generation at the centre of the gain line.

The noise modulation of the pump is described with the help of the source of white Gaussian noise g_w with the following statistic characteristics:

$$\langle g_w(t)\rangle = 0, \tag{4}$$

$$\langle g_w(t)g_w(s)\rangle = D\delta(t-s), \tag{5}$$

where D is the noise intensity; $\delta(t)$ is the Dirac delta function.

In the numerical simulation some parameters were set equal to the experimentally measured parameters of the laser under study. The resonator bandwidth ω/Q, where

$Q = (Q_1 + Q_2)/2$, was determined by the relaxation frequency $\omega_r = [\eta\omega/(QT_1)]^{1/2}$ and was equal to 4.4×10^8 s^{-1}. The amplitude nonreciprocity of the ring resonator $\Delta = (\omega/Q_2 - \omega/Q_1)/2$ and the periodic signal frequency were set equal to 5000 s^{-1} and 99.5 kHz, respectively. The value of the polarisation parameter $\beta = 0.75$ was found by using the experimentally measured dependence of the additional relaxation frequency ω_{r1} on the frequency nonreciprocity Ω of the resonator [14].

For simplicity the coupling coefficients were assumed to be complex conjugated ($\vartheta_1 - \vartheta_2 = 0$). The results of the numerical simulation presented in Fig. 4a were obtained for the moduli of the coupling coefficients $m_1/2\pi = 130$ kHz, $m_2/2\pi = 318$ kHz and the self-modulation frequency $\omega_m/2\pi = 220$ kHz. One can see that the results of the numerical simulation well agree with the experimental results.

5. Discussion of the results

The physical mechanism determining the appearance of the stochastic phenomena observed in this paper is similar to that described in paper [6], which demonstrated the possibility of appearance of the stochastic resonance in a nonlinear oscillator with a monostable potential. The relaxation oscillations excited in a solid-state laser can be treated as oscillations of some nonlinear oscillator, for example, Toda oscillator (see, for example, [15]). The nonlinearity of this oscillator is manifested in the fact that the frequency of its oscillations proves to depend on their amplitude (anisochronicity).

In the Toda oscillator, the oscillation frequency decreases with increasing the amplitude. The effect of the noise modulation of the pump results in an increase in the amplitude of relaxation oscillations and in a decrease in their frequency due to anisochronicity. If the frequency of the periodic signal (in the absence of noise) is smaller than the fundamental relaxation frequency, a resonance can appear at some noise intensity: the average frequency of noise-excited nonlinear relaxation oscillations becomes equal to the signal frequency. The physical mechanism considered makes it possible to explain qualitatively the results obtained in this paper.

One should expect from the qualitative assumptions that the maximum signal amplification should be observed when the signal frequency coincides with the central frequency of the relaxation peak; however, it follows from the experimental results and from the results of numerical simulation shown in Figs 3 and 4 that the maximum signal gain takes place at the signal frequencies, which are smaller than the central frequency of relaxation peak. The shift of the signal maximum with respect to the relaxation peak maximum increases with increasing the excess of the pump over the threshold. This inconsistency is obviously related to the parametric interaction of self-modulation and relaxation oscillations. At $\eta_2 = 0.22$, the fundamental relaxation frequency proves to be close to half the self-modulation frequency, which indicates the proximity to the parametric resonance (see, for example, [10]). A rigorous substantiation of the results obtained requires further investigations.

6. Conclusions

Thus, we have found in this paper the appearance of the stochastic resonance in a monostable solid-state ring laser operating in the self-modulation regime of the first kind. We have shown that the noise modulation of the pump can lead to a significant increase in the laser response to the periodic modulating signal. The phenomena observed have been explained qualitatively. These investigations can be generalised to the case when the bistability appears in the ring laser. It has been shown in [13] that the bistability of self-modulation oscillations can appear in the solid-state ring laser: under the same conditions, apart from the self-modulation regime of the first kind, there exists and proves to be stable the quasi-periodic self-modulation regime. The frequencies of relaxation oscillations in these bistable regimes can somewhat differ, which should, undoubtedly, lead to the appearance of some peculiarities in the stochastic resonance. This case will be studied elsewhere.

Note that the stochastic resonance discovered in this paper should take place not only in solid-state lasers but also in any class B lasers (in particular, in semiconductor and CO_2 lasers).

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