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Control of filamentation of femtosecond laser pulses in a turbulent atmosphere

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***Abstract*. Various methods for controlling the onset of filamentation of a high-power laser pulse in extended vertical atmospheric paths are analysed. It is shown that the increase in the structure constant of the atmospheric turbulence on average leads to the earlier formation of 'hot' spots when the initial pulse power exceeds the critical self-focusing power by an order of magnitude and more. It is found in the numerical experiments that the use of broad focused beams is preferable for achieving the minimal standard deviation of a distance for the filament onset.**

Keywords: *filamentation, femtosecond laser pulses, self-focusing, atmosphere, turbulent atmosphere, filamentation control.*

1. Introduction

Problems of controlling laser radiation in the atmosphere have attracted the attention of researchers beginning from the 1970s. These problems include the compensation of distortions of laser beams in atmospheric paths caused by the nonlinear interaction of radiation with the components of air and turbulent fluctuations of the refractive index. In the case of quasi-cw radiation, the lowest energy threshold among nonlinear effects is inherent in thermal self-action, which is irregularly manifested under the conditions of atmospheric turbulence. S.A. Akhmanov et al. discussed in review [1] the methods for compensating the thermal self-action of laser radiation in the atmosphere based on the optimisation of its aperture, and programmable and adaptive control of the wave front. In the case of femtosecond pulses, the type of nonlinear-optical interaction of radiation with a medium and related self-action effects change qualitatively [2]. In the case of high-power ultrashort pulses with a high peak intensity, the dominating nonlinear effect in the atmosphere is self-action caused by the cubic nonlinearity of gas components in air and by the nonlinearity of the induced laser plasma. A new direction developed in the last years – the femtosecond nonlinear spectroscopy of atmosphere, encompasses a broad scope of problems of formation of extended filaments and plasma channels in high-power pulses, the supercontinuum and conical emission generation, the interaction of laser pulses with aerosols [3], and practical applications of femtosecond laser radiation in atmospheric optics systems. A high concentration of laser radiation energy in a filament producing its nonlinear-optical transformation is caused by the spatial and temporal redistribution of the radiation power. Accordingly filamentation can be controlled in two ways: either spatially, by changing the aperture and the wave front of the laser beam, or temporarily, by varying the pulse duration and the initial phase modulation of the pulse.

The aperture control of the filament parameters in a regular medium by the scaling of the beam diameter was considered in [4], while the use of the elliptic intensity distribution was discussed in [5]. The possibility of controlling the distance to the filament onset by varying the angular divergence of radiation was demonstrated in [6]. The production of two spatially separated filaments in pulses with the wave-front astigmatism was reported in [7]. In [8], the nonuniform focusing in the beam cross section was proposed to use for increasing the filamentation region. The spatial regularisation of many filaments in a pulse with random intensity perturbations was considered by applying the amplitude [9] and phase [10] modulation of the light field in the pulse cross section. The first experiments on controlling filaments with the help of a spatial modulator in the form of waves specially produced on a liquid surface were performed in [11]. The distance to a filament onset was increased by using the phase modulation of the pulse in [12] and this effect was studied theoretically in [13].

The efficiency of the phase modulation of pulses for producing filaments of length of a few kilometres in horizontal [14] and vertical paths [15] and for increasing the distance of probing by supercontinuum radiation appearing upon filamentation was demonstrated in full-scale experiments [16]. The influence of turbulence on the filamentation of phase-modulated pulses in an atmospheric path was studied numerically in [17].

In this paper, we considered the complex control of the aperture, focusing and phase modulation of a high-power femtosecond laser pulse for the spatial positioning of its filament in the case of atmospheric turbulence in a vertical path. The analysis is performed with the help of a numerical experiment.

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2. Numerical calculation method

A mathematical model of formation of filaments in femtosecond laser pulses is based on the quasi-static approximation [18], which neglects the contribution from the self-induced laser plasma to the formation of a nonlinear focus. Indeed, threshold pulse intensities for the photoionisation of air molecules are achieved near the focus whose spatial position has been already determined. Therefore, the spatial positioning of filaments at the initial filamentation stage can be analysed by neglecting the contribution from plasma. The equation for the complex amplitude $E(x, y, z, t)$ of the light field has the form

$$2ik\frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + kk_2\frac{\partial^2 E}{\partial t^2} + \frac{2k^2}{n_0}\left[n_2|E^2| + \Delta\tilde{n}(\boldsymbol{r})\right]E. \quad (1)$$

Here, k is the wave number; $\boldsymbol{r}$ is the radius vector with coordinates x, y, z; k_2 is the group velocity dispersion coefficient in air; n_0 is the refractive index of the unperturbed medium; $\Delta\tilde{n}$ is a random addition to the refractive index caused by the atmospheric turbulence; n_2 is the Kerr nonlinearity coefficient in air estimated as $(1.92-5.57)\times 10^{-19}$ cm^2 W^{-1} [19–21].

Turbulent fluctuations of the refractive index in the atmosphere were simulated by using the model of phase screens, which were generated by the modified spectral method [22]. We considered screens with the modified Karman spectrum including the internal (l_0) and external (L_0) turbulence scales.

The dispersion of phase-modulated pulses in long atmospheric paths was taken into account by using the semi-analytic model [17, 23]. The filamentation of pulses in vertical paths of length more than 1 km was analysed taking into account the altitude dependence of the concentration of air molecules, which was obtained from the standard atmosphere model [24], and the altitude dependence of the structure constant of the atmospheric turbulence (the Hafnegel–Walley model [25]).

We considered a focused Gaussian pulse with the quadratic phase modulation with the complex amplitude

$$E(x, y, z=0, t) = E_0 \exp\left(-\frac{x^2+y^2}{2a_0^2}\right)\exp\left(k\frac{x^2+y^2}{2R_\mathrm{f}}\right)$$
$$\times \exp\left[-\frac{1}{2}\left(\tau_{\delta 0}^{-2} + \mathrm{i}\delta\right)t^2\right], \quad (2)$$

where a_0 is the beam radius; δ is the phase modulation parameter; and R_f is the focal distance of a lens. The laser radiation wavelength is 800 nm. The peak amplitude E_0 of the electric field depends on the energy W_0 and duration $\tau_{\delta 0}$ of a phase-modulated pulse according to the expression

$$E_0 = \frac{2}{a_0\sqrt[4]{\pi}}\sqrt{\frac{2W_0}{c\tau_{\delta 0}}}. \quad (3)$$

The statistical parameters of filaments in kilometre atmospheric paths were obtained in numerical experiments by the Monte-Carlo method. For this purpose, a sequence of random phase screens was synthesised with specified statistic parameters describing random fluctuations of the refractive index along the entire path. The nonlinear process of propagation of femtosecond pulse (2) in the turbulent atmosphere was numerically simulated for each set of the screens by using equation (1) and the coordinates x, y, z of the formation of 'hot' spots corresponding to the generation of filaments. Figure 1 presents the typical picture of formation of many filaments. The laser beam decomposes into many 'hot' spots due to the modulation instability of a high-power light field in a medium with the Kerr nonlinearity [26]. In this case, fluctuations of the refractive index in the atmosphere initiate a random formation of nonlinear foci in the cross section of the high-power pulse.

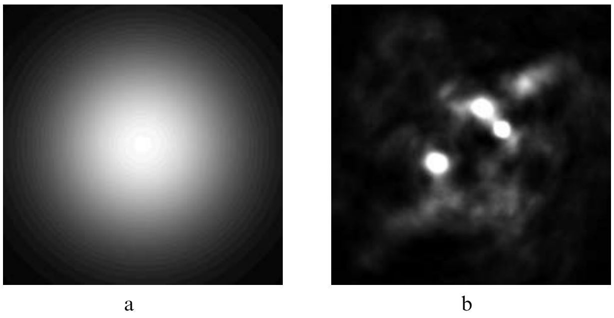

Figure 1. Typical fluence distribution in the pulse cross section at the output aperture $z = 0$ (a) and at the altitude $z = 2.5$ km, when the radiation intensity at one of the local maxima achieves the level $I/I_0 = 50$ (b); I_0 is the peak intensity of a transform-limited pulse of duration $\tau_0 = 250$ fs; the pulse energy is $W = 200$ mJ, and the beam radius is $a_0 = 10$ cm.

By performing repeatedly this calculation of the filamentation onset for new statistically independent sets of phase screens and averaging the results, we calculated the statistical characteristics of the filament position in the vertical path.

3. Phase modulation

The most popular method of controlling filamentation of high-power femtosecond laser pulses is their phase or frequency modulation. By varying the frequency modulation of a pulse, we can change its duration and, hence, its peak power, thereby controlling a distance to the filament onset. In this case, a pulse with the negative phase modulation propagating in air is 'focused' in time and the energy localised in a filament increases. To check the possibility to control the distance z_fil to the filament onset with the help of phase modulation in a long vertical path in the atmosphere, we performed a series of computational experiments in which the filamentation onset (the appearance of the first 'hot' spot) was controlled by the increase in the pulse peak intensity by 50 times compared to its initial value. To obtain filaments at an altitude of 2.5 km, broad Gaussian beams of radius $a_0 = 11$ cm were used. The pulse energy W was 200 mJ and the pulse duration was varied from 250 fs for a transform-limited pulse to 1400 fs in the case of strong phase modulation. Depending on the initial phase modulation, the pulse peak intensity was from 200 (for a transform-limited pulse) up to 36 critical self-focusing powers P_cr for the same pulse energy W. We considered the atmospheric turbulence with the internal and external scales $l_0 = 3$ mm and $L_0 = 10$ m, respectively, and the structure constant of the atmospheric turbulence C_n^2 on the Earth surface changing in the range $(0.125-6.0)\times 10^{-15}$ cm$^{-2/3}$.

Figure 2 presents the dependences of the distance to the filament onset on the phase-modulated pulse duration in the cases of weak and strong turbulences in the path. The standard deviations of distances to the filament onset from the average value are also indicated. One can see that, by increasing the pulse duration due to phase modulation, it is possible to obtain filaments, on average, at higher altitudes. In this case, the increase in the C_n^2 values leads, on average, to the earlier formation of the first 'hot' spot. Therefore, while to obtain a filament at an altitude of 2.5 km in the case of a weak turbulence, it is necessary to stretch a laser pulse from 250 to 630 fs, in the case of a strong turbulence, the pulse should be already stretched up to 1.25 ps. Note that the same pulses in a regular medium form filaments at an altitude more than four kilometres (the solid curve in Fig. 2), i.e. in the case of multiple filamentation of high-power pulses, the turbulence considerably reduces the distance to the first filament. At the same time, during the formation of one filament, the distance to its onset increased with increasing turbulence [27]. In the case of formation of two filaments in the cross section of the pulse, the increase in the distance $z_{\rm fil}$ to the filament onset changed to its decrease at larger values of C_n^2 [28].

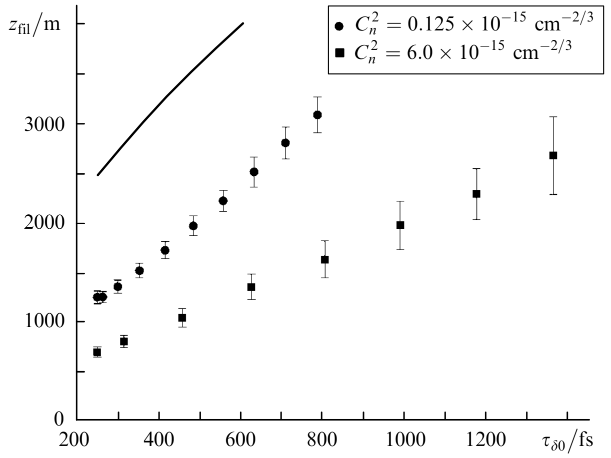

Figure 2. Dependences of the average distance $z_{\rm fil}$ to the first initial 'hot' spot on the phase-modulated (chirped) pulse duration for different structure constants of the atmospheric turbulence on the Earth surface. The solid curve is the distance to the nonlinear focus in a regular medium; $W = 200$ mJ, $a_0 = 11$ cm.

The results obtained for high-power laser pulses show that the use of phase-modulated pulses allows one to displace the formation of the first 'hot' spot to a distance of a few kilometres for a broad range of values of the structure constant of atmospheric turbulence. However, along with the average distance to the first filament, its statistical scatter also increases under turbulent conditions. Thus, for the laser pulse parameters considered above in the case of a weak turbulence ($C_n^2 = 0.125 \times 10^{-15}$ cm$^{-2/3}$), the standard deviation of the filament onset at an altitude of 2.5 km is 150 m, while in the case of a strong turbulence ($C_n^2 = 6.0 \times 10^{-15}$ cm$^{-2/3}$), this deviation exceeds 300 m. This means that the 'shot-to-shot' scatter in the longitudinal position of the first filament will be hundreds of metres. Therefore, it is of interest to analyse other methods for controlling filamentation with the aim of decreasing this scatter.

4. Beam telescoping

The authors of [29, 4, 6] proposed to control the distance to a filament onset in the atmosphere by beam telescoping or changing the size of the beam cross section. As a rule, telescoping preserves the residual geometrical focusing, which allows the use of narrower beams. In the next series of computational experiments, the focal distance $R_{\rm f}$ of a lens was 5000 m, while the initial phase modulation of the pulse was absent. By varying the beam width, we can obtain the filament onset at the specified altitude in the turbulent atmosphere, as illustrated in Fig. 3. Here, the dependences of the distance $z_{\rm fil}$ to the filament onset on the beam radius are presented for two values of C_n^2 in the vertical path.

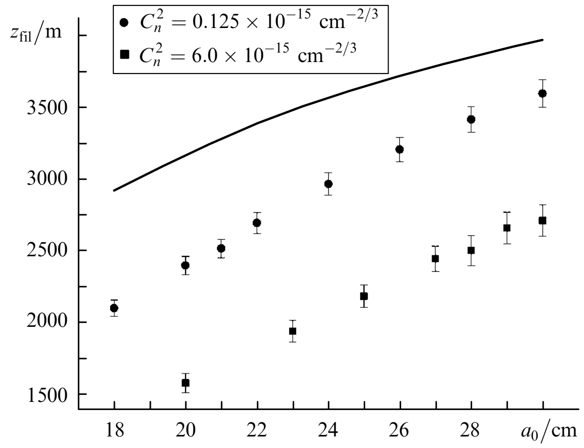

Figure 3. Dependences of the average distance $z_{\rm fil}$ in a transform-limited pulse with $\tau_0 = 250$ fs on the initial beam radius a_0 for strong (■) and weak (●) atmospheric turbulence. The solid curve is the dependence for a regular medium; $W = 200$ mJ, the peak power P exceeds the critical self-focusing power $P_{\rm cr}$ by 200 times, $R_{\rm f} = 5000$ m.

As the beam radius is increased, the first 'hot' spot is formed at a larger distance. This can be explained in the following way. Multiple filamentation appears due to the small-scale self-focusing in the pulse. The beam broadening in the case of the constant pulse energy reduces the intensity. For the same values of C_n^2 and equal spatial frequency ranges of the atmospheric turbulence, the peak power contained in one 'filamentation zone' in a broader beam is lower. This results in the formation of a filament at a larger distance. Thus, by increasing the beam radius, we can efficiently move away the filamentation onset in the vertical path in the case of random fluctuations of the refractive index. Thus, to obtain the first filament at an altitude of 2.5 km in the case of weak turbulence, a laser beam of radius 21 cm is needed, while in the case of strong turbulence, the beam radius should be 28 cm. The standard deviation of the average altitude in the first and second cases is 60 and 110 m, respectively. Note that the standard deviation decreased considerably compared to that for control with the help of phase modulation. This can be explained by the fact that we considered beams focused at the distance $R_{\rm f} = 5000$ m. Therefore, we will consider below the influence of beam focusing on the standard deviation of the distance to the filamentation onset from the average value $z_{\rm fil}$.

5. Pulse focusing

To analyse the influence of focusing on the accuracy of positioning of the first filament in the case of a strong turbulence ($C_n^2 = 6.0 \times 10^{-15}$ cm$^{-2/3}$), we performed a series of computational experiments in which the focal distance was changed from 3000 to 5000 m for the fixed beam radius $a_0 = 28$ cm. The aim was to maintain constant the average distance to the filament onset $z_{\rm fil} = 2.5$ km. This was achieved by selecting the proper phase modulation coefficient δ. The dependence of the standard deviation of the distance from the filament onset on the focal distance of a lens is presented in Fig. 4. One can see that the standard deviation decreases in the case of tighter focusing. A similar dependence is observed in the case of a weak turbulence (for $C_n^2 = 0.125 \times 10^{-15}$ cm$^{-2/3}$).

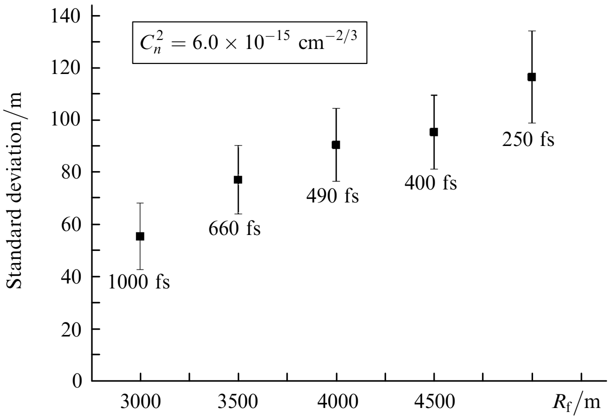

Figure 4. Standard deviation of the distance to the filament onset from the average value $z_{\rm fil} = 2.5$ km as a function of the focal distance $R_{\rm f}$ in the case of strong turbulence; $a_0 = 28$ cm. The confidence intervals and durations of a chirped 200-mJ pulse are shown.

Figure 5 presents the dependences of the standard deviation of the distance from the filament onset on the focal distance $R_{\rm f}$ of a lens for different beam radii. The beam radius a_0 was changed from 10 to 50 cm, and $R_{\rm f}$ was changed from 2600 to 5000 m. The phase modulation was selected so that $z_{\rm fil}$ was equal to 2.5 km. These results show that to reduce the standard deviation of the distance to the filament onset, it is desirable to use focusing as tight as possible. The value of $R_{\rm f}$ is bounded below because a filament is produced in front of the focal plane of a lens and, therefore, the focal distance of the lens should be larger than $z_{\rm fil}$. As the upper value of $R_{\rm f}$ approaches $z_{\rm fil}$, it is necessary to perform a stronger phase modulation of the pulse and to stretch the pulse in time (to maintain the constant average distance $z_{\rm fil}$ to the filament onset). When the pulse is stretched, its peak power decreases and, hence, its excess over the critical self-focusing power also decreases. As a result, in the presence of random fluctuations of the refractive index in the path, filaments are formed not always because the pulse power can be sometimes insufficient [28]. Note also that strong turbulence can sometimes destruct filaments in high-power beams as well [30].

Thus, for $a_0 = 10$ cm and $R_{\rm f} = 2600$ m, to obtain $z_{\rm fil} = 2500$ m, it is necessary to increase a phase-modulated pulse duration $\tau_{\delta 0}$ up to 20 ps, i.e. to stretch it by 80 times compared to the transform-limited pulse. In this case, the initial peak intensity of the phase-modulated pulse exceeds the critical self-focusing power only by 2.5 times, which is insufficient for obtaining 'hot' spots in all realisations. For turbulence parameters specified in our calculations, filamentation appeared approximately in 65 % of pulses. To form filaments stably in each pulse, it is necessary to maintain the excess of the pulse peak power over the critical self-focusing power approximately by an order of magnitude.

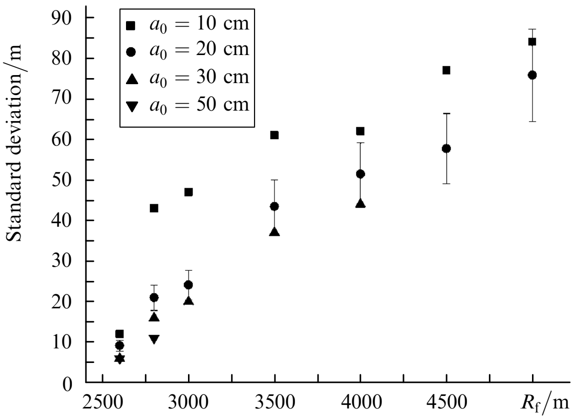

Figure 5. Dependences of the standard deviation of the distance to the filament onset on the focal distance of a lens for $C_n^2 = 0.125 \times 10^{-15}$ cm$^{-2/3}$ and different beam radii a_0.

Note also that, to reduce the standard deviation of the distance to the filament onset for the same value of $R_{\rm f}$, it is preferable to use beam telescoping than its phase modulation. Thus, in the case of focusing by a distance of 3000 m, the increase in the beam width from 10 to 30 cm reduces the standard deviation more than by half (Fig. 5).

6. Conclusions

By using computational experiments, we have studied the possibility of the complex control of filamentation in high-power femtosecond laser pulses propagating in long vertical atmospheric paths of an altitude 2.5 km in the case of random fluctuations of the refractive index.

It has been shown that the increase in the structure constant C_n^2 of the atmospheric turbulence leads, on average, to the earlier formation of the first 'hot' spot and to the increase in the statistical scatter of the distance at which this point is formed, irrespective of the method used to control the filamentation onset at the initial pulse power exceeding the critical self-focusing power by an order of magnitude and more.

By using phase-modulated pulses and beam telescoping, it is possible to produce the first filament at kilometre altitudes, in particular, at an altitude of 2.5 km, for $C_n^2 = (0.125 - 6) \times 10^{-15}$ cm$^{-2/3}$. The results of computational experiments have shown that in the case of a strong turbulence, it is necessary to increase stronger the initial duration of the phase-modulated pulse or the beam width for obtaining a 'hot' spot at the same altitude.

It has been shown that, to reduce the standard deviation of the distance from the filament onset, it is desirable to use focusing as tight as possible. In this case, the focal distance is bounded below by the desirable distance to the first filament onset and by the necessity of retaining the peak pulse power sufficient for the stable formation of filaments.

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