Quantum Electronics **39** (11) 1045 – 1046 (2009)

PACS numbers: 42.55.Rz; 42.60.Fc; 42.60.Mi
DOI: 10.1070/QE2009v039n11ABEH014129

Non-isochronism of the radiation frequency of a solid-state laser with a homogeneously broadened gain line

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***Abstract*. The dependence of the radiation frequency of a solid-state laser on the laser radiation field amplitude is studied theoretically. It is shown the non-isochronism of the laser frequency appears only when the contribution of spontaneous emission to the generated resonator mode is taken into account and is manifested most strongly at small excesses of the pump over the threshold.**

Keywords: *nonlinear self-oscillation system, non-isochronism, solid-state laser with a homogeneously broadened gain line.*

1. Oscillations in nonlinear dynamic systems, as a rule, are non-isochronous, i.e. the oscillation frequency depends on the oscillation amplitude. A well-studied and simplest example is the non-isochronism of the nonlinear oscillator (see, for example, [1 – 3]). The non-isochronism of oscillations in self-oscillation systems was studied in a number of papers (see, for example, [4 – 7]). In particular, it was established that the non-isochronism of self-oscillations leads to the asymmetry of the spectral line shape. Lasers are one of the well-known self-oscillation systems. In the case of solid-state lasers, as far as we know, the problem of non-isochronism has not been studied so far.

The aim of this paper is to study theoretically the dependence of a solid-state laser frequency on the output laser intensity (amplitude).

2. Consider a single-frequency solid-state laser with a homogeneously broadened gain line (linear or ring unidirectional laser). The non-isochronism of optical oscillations can be studied by using a self-consistent semi-classical system of equations for the complex field amplitude $E = |E|\exp(\mathrm{i}\varphi)$ inside the resonator and the inverse population of the active medium N [8]. This system of equations has the form:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\omega}{2Q}E + \mathrm{i}(\omega_n - \omega)E + \frac{\sigma l}{2T}N(1 + \mathrm{i}\alpha)E,$$
$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{T_1}\left[N_{\mathrm{th}}(1 + \eta) - N - aN|E|^2\right]. \qquad (1)$$

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Received 27 April 2009
Kvantovaya Elektronika **39** (11) 1045 – 1046 (2009)
Translated by I.A. Ulitkin

Here, ω is the laser radiation frequency; Q is the resonator Q factor; ω_n is the eigenfrequency of the resonator mode; σ is the laser transition cross section; l is the active element length; T is the round-trip transit time of light in the resonator; T_1 is the relaxation time of inverse population; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter. The pump rate is represented in the form $N_{\mathrm{th}}(1 + \eta)/T_1$, where N_{th} is the threshold inverse population and $\eta = P/P_{\mathrm{th}} - 1$ is the excess of the pump power P over the threshold. The amplitude – phase coupling factor $\alpha = \alpha_1 + \alpha_2$ takes into account the medium polarisability caused by the resonance amplification processes at the working transition (α_1) and nonresonance transitions from the working levels (α_2) [9]. For resonance transitions in the case of homogeneous gain line broadening, the factor α_1 is determined by the expression $\alpha_1 = (\omega_0 - \omega_n)/\gamma$, where ω_0 and γ are the central frequency and the half-width of the gain line.

We obtain from (1) the following equations for the dimensionless intensity of the intracavity radiation field $I = a|E|^2$ and the optical radiation phase:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\frac{\omega}{Q}I + \frac{\sigma l}{T}NI, \qquad (2)$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega_n - \omega + \frac{\sigma l}{2T}N\alpha. \qquad (3)$$

The stationary solution of system (1) has the form

$$I_0 = \eta, \qquad (4)$$

$$N_0 = N_{\mathrm{th}}, \quad \frac{\sigma l}{T}N_0 = \frac{\omega}{Q}, \qquad (5)$$

$$\omega = \omega_n + \frac{\omega}{2Q}\alpha. \qquad (6)$$

It follows from the above solution that the laser radiation frequency is independent of the radiation amplitude (intensity I_0). In the stationary lasing regime the frequency is shifted with respect to the mode eigenfrequency by a constant quantity $(\omega/2Q)\alpha$ independent of the field amplitude. Thus, within the framework of the approximation under study, a solid-state laser is an isochronous self-oscillation system. The absence of non-isochronism is caused by the fact that the inverse population is saturated by laser radiation at a constant level $N_0 = N_{\mathrm{th}}$, which is independent of the laser radiation intensity (amplitude).

We will show that when the quantum fluctuations are taken into account, the non-isochronism of the laser frequency appears. In this case, equation (2) contains an addition term – the rate of spontaneous emission into the generated mode. It is convenient to rewrite this equation by introducing, instead of the dimensionless intensity I, the quantity n (the number of photons in the resonator), which is proportional to this intensity ($I = \beta n$, where β is the factor of spontaneous emission into the resonator mode [10]):

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{\omega}{Q}n + \frac{\beta N n}{T_1} + \frac{\beta N}{T_1}. \qquad (7)$$

The last term $\beta N/T_1$ in equation (7) is the average rate of spontaneous emission into the generated mode. Other equations do not change when spontaneous emission is taken into account. The stationary solution with the account for the spontaneous emission and at a large number of photons in the resonator ($n_0 \gg 1$) inherent in lasers, takes the form

$$n_0 = \frac{1}{2\beta}\left\{\eta + \left[\eta^2 + 4\beta(1+\eta)\right]^{1/2}\right\}, \qquad (8)$$

$$\frac{\beta N_0}{T_1} = \frac{\omega}{Q}\left(1 - \frac{1}{n_0}\right), \qquad (9)$$

$$\omega = \omega_n + \frac{\omega}{2Q}\alpha\left(1 - \frac{1}{n_0}\right). \qquad (10)$$

One can see from (10) that when spontaneous emission is taken into account, the non-isochronism appears, the laser frequency being dependent on the laser intensity (the number of photons inside the resonator n_0). This dependence proves the strongest near the lasing threshold. Figure 1 shows the dependence of the nonlinear laser frequency shift $\Delta\nu = [\omega_n + (\omega/2Q)\alpha - \omega]/2\pi$ on the pump excess over the threshold. This dependence is calculated for the 'thresholdless' solid-state laser studied in detail in [10–12]. Note that in these papers the solid-state chip laser was called 'thresholdless' because of the absence of a decrease in quantum fluctuations of the radiation intensity typical of conventional lasers in the region of the lasing threshold. Calculations were performed for the following values of the laser parameters: $\beta = 10^{-5}$, $\alpha = 0.2$, $\omega/Q = 10^{11}\ \mathrm{s}^{-1}$, $1/T_1 = 1.3 \times 10^4\ \mathrm{s}^{-1}$.

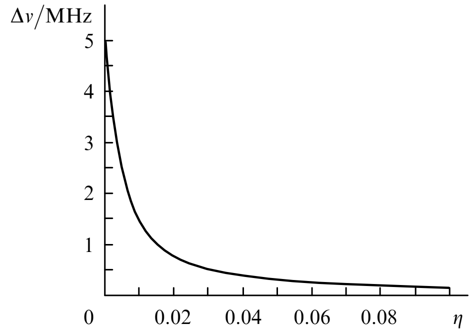

Figure 1. Nonlinear shift of the laser frequency $\Delta\nu = [\omega_n + (\omega/2Q)\alpha - \omega]/2\pi$ as a function of the pump excess over the threshold.

3. Thus, in this paper we have studied theoretically the dependence of the laser frequency of a solid-state laser on the laser radiation intensity (amplitude). We have shown that the non-isochronism of the laser frequency appears only when spontaneous emission is taken into account in a self-consistent semi-classical system of laser equations.

Of interest is the experimental investigation of the dependence of the laser frequency on the laser radiation amplitude. The results of such calculations would make it possible to determine the amplitude–phase coupling factor $\alpha = \alpha_1 + \alpha_2$. Note that in the case of a constant pump, the nonlinear shift of the laser frequency $\Delta\nu = [\omega_n + (\omega/2Q)\alpha - \omega]/2\pi$ is constant. In the case of quasi-static periodic modulation of the pump, this shift is periodically dependent of time, which can simplify its measurement.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant Nos 07-02-00204 and 08-02-00217).

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