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# Collision-induced amplification of radiation in inversionless two-level systems

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***Abstract*. Amplification of probe radiation by an inversionless two-level system in the 'red' wing of its spectral line is studied theoretically during the resonance absorption of intense cw laser radiation. The effect is caused by the inequality of the spectral densities of the Einstein coefficients for absorption and stimulated emission outside the absorption line under conditions when homogeneous broadening, due to the interaction of particles with the buffer gas, significantly exceeds the natural broadening (at high pressures of the buffer gas). It is found that the larger the inversionless gain, the higher the buffer gas pressure and the pump radiation intensity. It is established that at high enough pump intensity, the probe field is amplified along the entire region of the 'red' wing of the line (at any negative detunings of the probe field frequency).**

***Keywords***: *inversionless amplification of radiation, probe field, collisions, Einstein coefficients, level populations, spectral line wing.*

## 1. Introduction

The authors of papers [1 – 4] found theoretically that the equality of spectral probability densities of radiation absorption and stimulated emission is violated when particles interact with a thermostat (for example, during frequent collisions in a gas). In this case, the spectral densities of the Einstein coefficients for absorption [$b_{nm}(\Omega)$] and stimulated emission [$b_{mn}(\Omega)$] are related with each other by the expression

$$b_{mn}(\Omega) = b_{nm}(\Omega)\exp[-\hbar\Omega/(k_{\rm B}T)], \tag{1}$$

where $\Omega = \omega - \omega_{mn}$ is the detuning of the laser frequency $\omega$ from the frequency $\omega_{mn}$ of the $m - n$; transition; $\hbar$ is Planck's constant; $k_{\rm B}$ is the Boltzmann constant; and $T$ is the temperature. At $\hbar|\Omega| \ll k_{\rm B}T$, the canonical equality for the absorption and stimulated emission probabilities follows from (1). The physical interpretation of relation (1) between the spectral densities of the Einstein coefficients is presented in [3 – 6].

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According to (1), the population inversion can be established in a two-level system during the nonresonance absorption of cw radiation, and, as a result, lasing is possible at the resonance frequency. In recent papers [3, 5, 6], this effect is explained theoretically and registered experimentally. In the experiment, lasing appeared in the regime of super-radiance on the resonance transitions of sodium atoms upon pump absorption in the 'blue' wing of the spectral line.

Another interesting effect, caused by the inequality of the spectral densities of the Einstein coefficients, is the amplification of radiation by an inversionless two-level system. If, due to some reasons, the populations of the excited and ground levels prove close, the amplification regime is realised for the probe radiation when the probability of the stimulated emission exceeds that of the absorption in the 'red' wing of the spectral line. Paper [4] reports the possible existence of this effect*; however, this effect has not been studied so far. In this paper, we study in detail this effect.

## 2. Balance equations

Consider the following formulation of the problem. Let two-level absorbing particles (with the ground level $n$ and excited level $m$) reside in the atmosphere of a buffer gas. We assume that the populations of the absorbing particles are levelled by the intense laser radiation with a frequency resonant with the frequency of the $m - n$ transition. Inversionless amplification of radiation can be detected by comparing the intensities of probe radiation propagating through a medium in the absence and presence of intense pump radiation: in the first case, probe radiation is absorbed, while in the second case, probe radiation is amplified.

In the case of significant collision line broadening (compared to the natural and Doppler line broadening), the balance equation for the $m$ level population has the form:

$$\frac{{\rm d}\rho_m}{{\rm d}t} + \Gamma_m\rho_m = I\,[b_{nm}(\Omega)\rho_n - b_{mn}(\Omega)\rho_m]$$

$$+ I_\mu[b_{nm}(\Omega_\mu)\rho_n - b_{mn}(\Omega_\mu)\rho_m], \tag{2}$$

*S.A. Babin was first to pay attention to the principal possibility of this effect existence.

$$\Omega_\mu \equiv \omega_\mu - \omega_{mn}, \quad \rho_n + \rho_m = N.$$

Here, $\rho_n$ and $\rho_m$ are populations of the $n$ and $m$ levels; $N$ is the concentration of absorbing particles; $\Gamma_m$ is the rate of spontaneous decay of the $m$ level; $\omega_\mu$ is the probe radiation frequency; $I$ and $I_\mu$ are the pump and probe radiation intensities. In equation (2), the spectral densities of the Einstein coefficients $b_{nm}(\Omega)$ and $b_{mn}(\Omega)$ are related with each other by (1). The coefficients $b_{nm}(\Omega_\mu)$ and $b_{mn}(\Omega_\mu)$ are related by a similar expression (with a substitution $\Omega \to \Omega_\mu$).

In the theory based on widely used quantum kinetic equations for the density matrix (see, for example, [7, 8]), relations

$$Ib_{nm}(\Omega) = \frac{2|G|^2\Gamma}{\Gamma^2 + \Omega^2}, \; b_{nm}(\Omega) = b_{mn}(\Omega), \; G = \frac{d_{mn}E}{2\hbar}, \quad (3)$$

$$I_\mu b_{nm}(\Omega_\mu) = \frac{2|G_\mu|^2\Gamma}{\Gamma^2 + \Omega_\mu^2}, \; b_{nm}(\Omega_\mu) = b_{mn}(\Omega_\mu), \; G_\mu = \frac{d_{mn}E_\mu}{2\hbar} \quad (4)$$

are fulfilled, where $E$ and $E_\mu$ are the electric field amplitudes of pump and probe radiation; $d_{mn}$ is the matrix element of the dipole moment of the transition; $\Gamma$ is the collision half-width of the absorption line. The relaxation constant $\Gamma$ is independent of the radiation frequencies and intensities. The above kinetic equations have been derived by assuming that all the radiation processes proceed within the mean free time, while in the collisions they can be neglected. This assumption is valid in the region of frequency detunings which are not too far from the collision linewidth. At large frequency detunings ($|\Omega|, |\Omega_\mu| \gg \Gamma$, the wing of the line absorption), the situation changes drastically: radiation processes proceed mainly in collisions [9, 10] (the so-called optical collisions [10]). Under these conditions the collision half-width $\Gamma$ in relations (3) and (4) should be replaced by the quantities $\Gamma_{\rm oc}(\Omega)$ and $\Gamma_{\rm oc}(\Omega_\mu)$, respectively, which mean the phase relaxation rates in optical collisions [10]. These quantities depend on the frequency detunings; they, as the collision half-width $\Gamma$, are proportional to the buffer gas pressure but can be significantly higher than this pressure.

At $|\Omega|, |\Omega_\mu| \gg \Gamma$ two first formulae in (3) and (4) change because in relation (1) for the spectral densities of the Einstein coefficients, the exponential multiplier becomes substantial. It is obvious that the right-hand sides of these expressions [of course, with the necessary substitution of $\Gamma$ by $\Gamma_{\rm oc}(\Omega)$ and $\Gamma_{\rm oc}(\Omega_\mu)$] are responsible for the radiation processes in which all the particles participate, independently of the their kinetic energy before the collision. If the frequency detunings are positive ($\Omega > 0, \Omega_\mu > 0$), this is the process of radiation absorption [$Ib_{nm}(\Omega)$, $I_\mu b_{nm}(\Omega_\mu)$], while if the detunings are negative ($\Omega < 0$, $\Omega_\mu < 0$), this is the process of stimulated emission [$Ib_{mn}(\Omega)$, $I_\mu b_{mn}(\Omega_\mu)$] [4, 5]. Thus, for the quantities $Ib_{nm}(\Omega)$ and $Ib_{mn}(\Omega)$ under conditions $|\Omega| \gg \Gamma$, the expressions

$$\begin{aligned} Ib_{nm}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Omega^2}, \\ Ib_{mn}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Omega^2}\exp\left(-\frac{\hbar|\Omega|}{k_{\rm B}T}\right), \; \Omega > 0; \\ Ib_{nm}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Omega^2}\exp\left(-\frac{\hbar|\Omega|}{k_{\rm B}T}\right), \\ Ib_{mn}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Omega^2}, \; \Omega < 0 \end{aligned} \quad (5)$$

are valid. Similar expression at $|\Omega_\mu| \gg \Gamma$ are valid for the quantities $I_\mu b_{nm}(\Omega_\mu)$ and $I_\mu b_{mn}(\Omega_\mu)$ [in formulae (5), the following substitutions should be made: $\Omega \to \Omega_\mu$, $I \to I_\mu$, $|G| \to |G_\mu|$).

Relations (3)–(5) can be combined by rewriting them in the form

$$\begin{aligned} Ib_{nm}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Gamma^2 + \Omega^2}, \\ Ib_{mn}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Gamma^2 + \Omega^2}\exp\left(-\frac{\hbar|\Omega|}{k_{\rm B}T}\right), \; \Omega > 0; \\ Ib_{nm}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Gamma^2 + \Omega^2}\exp\left(-\frac{\hbar|\Omega|}{k_{\rm B}T}\right), \\ Ib_{mn}(\Omega) &= \frac{2|G|^2\Gamma_{\rm oc}(\Omega)}{\Gamma^2 + \Omega^2}, \; \Omega < 0. \end{aligned} \quad (6)$$

The expressions for the quantities $I_\mu b_{nm}(\Omega_\mu)$ and $I_\mu b_{mn}(\Omega_\mu)$ are derived from (6) using the substitution $\Omega \to \Omega_\mu$, $I \to I_\mu$, $|G| \to |G_\mu|$. These formulae can be used at small frequency detunings ($|\Omega|, |\Omega_\mu| \lesssim \Gamma$). Taking into account that in this case the quantities $\Gamma_{\rm oc}(\Omega)$ and $\Gamma_{\rm oc}(\Omega_\mu)$ are equal to the collision half-width of the absorption line $\Gamma$ [10].

The expression similar to (6) but without the exponential multiplier was presented by Yakovlenko in paper [10] and called the modified Lorentz expression. In the case of the weak radiation intensity, it approximates the entire contour of the spectral line including far wings. In the collision region ($|\Omega|, |\Omega_\mu| \ll \Omega_{\rm B}$, where $\Omega_{\rm B}$ is the Weisskopf frequency [10]), the relation $\Gamma_{\rm oc}(\Omega) \simeq \Gamma_{\rm oc}(\Omega_\mu) \simeq \Gamma$ is fulfilled, and in the absence of the exponential multiplier, expression (6) is transformed, as should be, to the Lorentz formula.

Balance equation (2) taking into account (6) takes the form

$$\frac{{\rm d}\rho_m}{{\rm d}t} + \Gamma_m\rho_m = NP + NP_\mu, \quad (7)$$

where

$$\begin{aligned} P &= \begin{cases} \varkappa\Gamma_m(\rho_n - \xi\rho_m)/N, & \Omega > 0, \\ \varkappa\Gamma_m(\xi\rho_n - \rho_m)/N, & \Omega < 0, \end{cases} \\ P_\mu &= \begin{cases} \varkappa_\mu\Gamma_m(\rho_n - \xi_\mu\rho_m)/N, & \Omega_\mu > 0, \\ \varkappa_\mu\Gamma_m(\xi_\mu\rho_n - \rho_m)/N, & \Omega_\mu < 0 \end{cases} \end{aligned} \quad (8)$$

are the probabilities of radiation absorption by the atoms at the frequencies $\omega$ and $\omega_\mu$, respectively (the number of absorption events per unit time per one absorbing atom);

$$\xi \equiv \exp\left(-\frac{\hbar|\Omega|}{k_{\rm B}T}\right); \quad \xi_\mu \equiv \exp\left(-\frac{\hbar|\Omega_\mu|}{k_{\rm B}T}\right);$$

$$\varkappa = \frac{2|G|^2 \Gamma_{\rm oc}(\Omega)}{\Gamma_m(\Gamma^2 + \Omega^2)}; \quad \varkappa_\mu = \frac{2|G_\mu|^2 \Gamma_{\rm oc}(\Omega_\mu)}{\Gamma_m(\Gamma^2 + \Omega_\mu^2)}. \tag{9}$$

The quantities $\varkappa$, $\varkappa_\mu$ have the meaning of the saturation parameters for the $m - n$ transition. For simplicity, we will assume below that the probe field intensity is small and the condition

$$\varkappa_\mu \ll 1 \tag{10}$$

is fulfilled.

Under stationary conditions, which we will consider below, we obtain from (7) under condition (10) the expressions for the difference in the populations of the $m$ and $n$ levels and the probability $P_\mu$ of the probe radiation absorption:

$$\rho_m - \rho_n = N \frac{\varkappa(1-\xi)\,\mathrm{sign}\,\Omega - 1}{1 + (1+\xi)\varkappa}, \tag{11}$$

$$P_\mu = \varkappa_\mu \Gamma_m \frac{\varphi_\mu + [(1+\xi)\varphi_\mu - (1+\xi_\mu)\varphi]\varkappa}{1 + (1+\xi)\varkappa}, \tag{12}$$

where

$$\varphi = \begin{cases} 1, & \Omega \geqslant 0, \\ \xi, & \Omega < 0, \end{cases} \quad \varphi_\mu = \begin{cases} 1, & \Omega_\mu \geqslant 0, \\ \xi_\mu, & \Omega_\mu < 0. \end{cases} \tag{13}$$

It follows from (11), (12) that inversionless amplification of probe radiation ($P_\mu < 0$) is possible at $\rho_m < \rho_n$. Indeed, according to (11) the population inversion on the $m - n$ transition appears if the condition

$$\varkappa(1-\xi)\,\mathrm{sign}\,\Omega > 1 \tag{14}$$

is fulfilled. This condition can be fulfilled only when the frequency detuning of pump radiation ($\Omega > 0$) is positive. According to (12), the probability of probe radiation absorption becomes negative (amplification appears) if

$$[(1+\xi_\mu)\varphi - (1+\xi)\varphi_\mu]\varkappa > \varphi_\mu. \tag{15}$$

This condition can be fulfilled only if the detuning of the probe field frequency is smaller than the detuning of the pump field frequency:

$$\Omega_\mu < \Omega. \tag{16}$$

At $\Omega \leqslant 0$, the population inversion does not appear; however, for probe radiation in the case of the 'red' frequency detuning ($\Omega_\mu < \Omega$), the amplification regime is realised (if the intensity of exciting radiation is high enough for the saturation parameter $\varkappa$ to be greater than unity).

The gain $g_\mu$ of probe radiation is related to the probability $P_\mu$ of probe radiation absorption by the expression

$$g_\mu = -\frac{N\hbar\omega_\mu P_\mu}{I_\mu}. \tag{17}$$

Taking into account the expression

$$|G_\mu|^2 = \frac{\lambda_{mn}^3 \Gamma_m I_\mu}{16\pi^2 \hbar c} \tag{18}$$

($\lambda_{mn}$ is the wavelength of the $m - n$ transition), formula (17) for the gain is reduced to the form

$$g_\mu = \alpha_\mu^0 \frac{[(1+\xi_\mu)\varphi - (1+\xi)\varphi_\mu]\varkappa - \varphi_\mu}{1 + (1+\xi)\varkappa},$$
$$\alpha_\mu^0 = \frac{N\lambda_{mn}^3 \Gamma_m \Gamma_{\rm oc}(\Omega_\mu)}{4\pi\lambda_\mu(\Gamma^2 + \Omega_\mu^2)}, \tag{19}$$

where $\lambda_\mu$ is the probe radiation wavelength; $\alpha_\mu^0$ is the absorption coefficient of probe radiation under conditions when pump radiation is absent ($\varkappa = 0$) and all the particles reside on the lower level $n$. Expression (19) is valid at any values of the the frequency detunings $\Omega$ and $\Omega_\mu$. One can see from it that the probe radiation gain in the spectral line wing ($|\Omega_\mu| \gg \Gamma$) is directly proportional to the phase relaxation rate $\Gamma_{\rm oc}(\Omega_\mu)$ in collisions. Because the quantity $\Gamma_{\rm oc}(\Omega_\mu)$ is proportional to the buffer gas pressure [10], one should expect an enhancement in the effect with increasing the buffer gas pressure.

## 3. Inversionless amplification

The case of exact resonance for the pump field ($\Omega = 0$) is of main interest in realising the inversionless amplification of radiation. To this end, probe radiation is amplified in the region of 'red' frequency detunings ($\Omega_\mu < 0$). In this case ($\Omega = 0$, $\Omega_\mu < 0$), formula (19) for the gain takes the form

$$g_\mu = \alpha_\mu^0 \frac{(1-\xi_\mu)\varkappa - \xi_\mu}{1 + 2\varkappa}. \tag{20}$$

From this it follows that the inversionless gain ($g_\mu > 0$) appears when the condition

$$\varkappa > \frac{1}{\exp[\hbar|\Omega_\mu|/(k_{\rm B}T)] - 1} \tag{21}$$

is fulfilled. According to (21), the larger the detuning of the probe field frequency, the lower the pump radiation intensity required for the inversionless amplification to appear. In the 'red' wing of the line, the inversionless amplification is possible at large detunings even at the saturation parameter $\varkappa < 1$, if $\Omega_\mu < -k_{\rm B}T(\ln 2)/\hbar$ (the gain in this case will be very small due to a decrease in $\alpha_\mu^0$ with increasing $|\Omega_\mu|$).

At $\varkappa \gg 1, \xi_\mu/(1-\xi_\mu)$, we obtain from (20) for $g_\mu$ a simple expression [4]:

$$g_\mu = \frac{\alpha_\mu^0}{2}\left[1 - \exp\left(-\frac{\hbar|\Omega_\mu|}{k_{\rm B}T}\right)\right]. \tag{22}$$

From this it follows that the gain can be comparable with the maximum possible absorption coefficient $\alpha_\mu^0$ at the same frequency detuning. In the case of not too large detuning of the probe field frequency ($|\Omega_\mu| \ll k_{\rm B}T/\hbar$, $\lambda_\mu \simeq \lambda_{mn}$), expression (22) takes the simplest form (recall that it is valid at $\Omega = 0$, $\Omega_\mu < 0$):

$$g_\mu = \frac{N\lambda_{mn}^2}{8\pi} \frac{\Gamma_m \Gamma_{\rm oc}(\Omega_\mu)}{(\Gamma^2 + \Omega_\mu^2)} \frac{\hbar|\Omega_\mu|}{k_{\rm B}T}. \tag{23}$$

The gain achieves its maximum value at the frequency detuning of the probe field $\Omega_\mu = -\Gamma$. Then, as the detuning increases (at $|\Omega_\mu| \gg \Gamma$), the gain decreases proportionally to the quantity $\Gamma_{oc}(\Omega_\mu)/|\Omega_\mu|$. Theoretically, in optical collisions, the dependence $\Gamma(\Omega_\mu) \propto |\Omega_\mu|$ is possible (see, for example, [10]). In this case, the gain will not decrease with increasing $|\Omega_\mu|$ in a rather large region of the frequency detunings ($\Gamma \ll |\Omega_\mu| \ll k_B T/\hbar$).

The greater the gain of probe radiation, the larger the concentration of the active particles ($g_\mu \propto N$). However, the concentration should not be too large; otherwise, pump radiation will be completely absorbed at the active medium input. The effective usage of the radiation energy requires the pump radiation to be rather strongly absorbed by the active medium and, at the same time, to have the intensity at the medium output sufficient to maintain the maximum gain of probe radiation (the condition $\varkappa \gg 1$) should be fulfilled at the medium output). This requirement can be satisfied when the condition

$$\alpha L \gtrsim 1 \tag{24}$$

is fulfilled, where $\alpha$ is the absorption coefficient of pump radiation; $L$ is the active medium length. We will find the absorption coefficient of pump radiation taking into account the fact that radiation can bleach the medium. The absorption coefficient is related to the absorption probability $P$ by the expression

$$\alpha = \frac{N\hbar\omega P}{I}. \tag{25}$$

Under stationary conditions, from equation (7) with condition (10) we obtain, for the probability of pump radiation absorption, the expression

$$P = \frac{\varkappa \Gamma_m \varphi}{1 + (1 + \xi)\varkappa}. \tag{26}$$

It follows from (25), (26) that in the case of exact resonance for pump radiation ($\Omega = 0$) and its rather high intensity ($\varkappa \gg 1$), the absorption coefficient of pump radiation is given by the expression

$$\alpha = \frac{N}{I}\frac{\pi\hbar c \Gamma_m}{\lambda_{mn}}. \tag{27}$$

One can see that the higher the intensity $I$, the larger the admissible concentration of the active particles and, hence the larger the gain of probe radiation.

We will calculate, using expression (20), the probe radiation gain in the 'red' wing of the $D_1$ line of sodium atoms (the $3P_{1/2} - 3S_{1/2}$ transition). For this transition, we have $\lambda_{mn} = 0.59$ μm, $\Gamma_m = 6.15 \times 10^7$ s$^{-1}$ [11]. We take $\alpha L \simeq 3$ (when pump radiation propagates through the medium, its intensity decreases by 20 times). Then, at the active medium length $L = 10$ cm, the coefficient is $\alpha \simeq 0.3$. Let the pump radiation intensity be $I = 10^6$ W cm$^{-2}$. In this case, it follows from (27) that $\alpha \simeq 0.3$ corresponds to the concentration of active particles $N \simeq 2.9 \times 10^{16}$ cm$^{-3}$ (this vapour concentration of sodium atoms is achieved at temperature $T \simeq 747$ K [12]). The phase relaxation rate $\Gamma_{oc}(\Omega_\mu)$ in collisions will be considered equal to the collision half-width $\Gamma$ of the line, assuming that the buffer gas is xenon at a pressure of 10 atm: $\Gamma_{oc}(\Omega_\mu) \simeq \Gamma = 1.55$ cm$^{-1}$ [13].

Figure 1 shows the calculated gain $g_\mu$ of probe radiation as a function of the frequency detuning $\Omega_\mu$. One can see that for the above parameters the results of calculations obtained using approximate expression (23) and exact formula (20) coincide satisfactorily. For the parameters corresponding to Fig. 1, the maximum gain is achieved for the frequency detuning $\Omega_\mu = -1.75$ cm$^{-1}$ and is equal to 1.12 cm$^{-1}$. This means that the probe radiation intensity increases by $7.3 \times 10^4$ times per one pass through the active medium of length $L = 10$ cm. This increase does not provide the super-radiance regime per single pass through the medium. In this sense, the effect discussed here is weaker than the amplification effect when the population inversion is produced [3–6]. However, if a resonator providing at least 10 passes through the active medium is used, lasing from spontaneous noises appears. The laser radiation frequency can be tuned in a rather broad range.

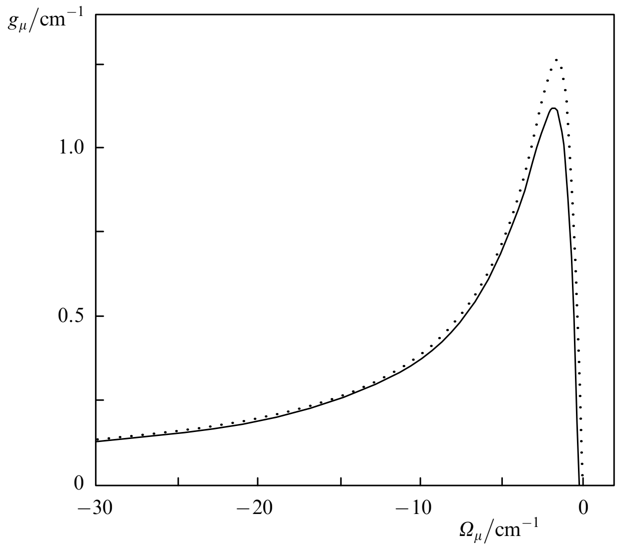


**Figure 1.** Gain $g_\mu$ of probe radiation in the 'red' wing of the $D_1$ line of sodium atoms (the $3P_{1/2} - 3S_{1/2}$ transition) as a function of the frequency detuning $\Omega_\mu$ of probe radiation from the central transition frequency at a xenon (buffer gas) pressure of 10 atm, $\lambda_{mn} = 0.59$ μm, $\Gamma_m = 6.15 \times 10^7$s$^{-1}$, $\Omega = 0$, $I = 1$ MW cm$^{-2}$ ($|G| = 0.84$ cm$^{-1}$), $T =$ 747 K ($N = 2.9 \times 10^{16}$ cm$^{-3}$), $\Gamma(\Omega_\mu) = \Gamma = 1.55$ cm$^{-1}$. The solid curve is calculation with exact formula (20), the dashed curve – with approximate formula (23).

## 4. Conclusions

We have studied inversionless amplification of probe radiation in the 'red' wing of the spectral line during the resonance absorption of intense cw laser radiation by active particles residing in the buffer gas atmosphere at its high pressure. The reason for the appearance of this effect is the inequality of spectral densities of the Einstein coefficients for absorption and stimulated emission under conditions when the homogeneous broadening caused by the interaction of particles with the buffer gas significantly exceeds the natural broadening (at large pressures of the buffer gas). Using balance equations, we have derived a formula for the probe field gain at any frequency detunings $\Omega$ and $\Omega_\mu$ of the pump field and probe field, respectively.

We have shown that amplification can appear when the frequency detuning of the probe field is smaller than the frequency detuning of the pump field ($\Omega_\mu < \Omega$), the probe field amplification is caused by the appearance of the population inversion upon absorption of cw laser radiation. In this case, the amplification is maximal, which was earlier studied theoretically and experimentally in papers [3, 5, 6]. At $\Omega \leqslant 0$, the population inversion does not appear; however, for probe radiation, the amplification regime is realised in the case of the 'red' frequency detuning ($\Omega_\mu < \Omega$).

The case of exact resonance for the pump field ($\Omega = 0$) is of main interest in realising the regime of inversionless amplification of radiation, this case being analysed in this paper. We have found that the gain of probe radiation is proportional to the phase relaxation rate $\Gamma_{\rm oc}(\Omega_\mu)$ in collisions. In this case, the higher the buffer gas pressure, the stronger the effect. The gain is proportional to the concentration $N$ of absorbing particles. The concentration cannot be too large; otherwise, pump radiation will be completely absorbed at the input to the active medium. The higher the pump radiation intensity, the higher (due to the medium bleaching) the maximum admissible concentration of the active particles and the gain of probe radiation. At a high enough pump radiation intensity, the gain achieved its maximum value for the quantity of the 'red' frequency detuning of the probe field equal to the collision half-width of the line. When the frequency detuning increases, the gain decreases proportionally to the quantity $\Gamma_{\rm oc}(\Omega_\mu)/|\Omega_\mu|$ [see formula (23)] and, hence, is strongly sensitive to the dependence $\Gamma_{\rm oc}(\Omega_\mu)$.

At high enough pump radiation intensity, the probe field is amplified along the entire region of the 'red' wing of the line (at any detuning $\Omega_\mu < 0$). The larger the detuning $|\Omega_\mu|$, the smaller the pump radiation intensity required for the amplification to appear. In the 'red' wing of the line, at larger detunings $|\Omega_\mu|$, inversionless amplification is possible even for the saturation parameter $\varkappa < 1$. When a resonator is used, one can obtain tunable lasing.

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