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Error statistics in a high-speed fibreoptic communication line with a phase shift of odd bits

E.G. Shapiro

***Abstract*. The propagation of optical pulses through a fibreoptic communication line with a phase shift of odd bits is directly numerically simulated. It is shown that simple analytic expressions approximate well the error probability. The phase shift of odd bits in the initial sequence is statistically shown to decrease significantly the error probability in the communication line.**

Keywords: *fibreoptic communication lines, dynamics of optical pulses, error statistics.*

Optimisation of optical communication line parameters is a key problem in designing a communication system. The quality of a communication line is estimated by the bit error rate (BER), which is the ratio of the number of error bits to the total number of transmitted bits. Because the direct measurement of small BERs is often technically complicated, statistical and numerical methods for estimating the error probability in a communication system play an important role. The knowledge of the error statistics plays a decisive role in the development and application of efficient forward error correction (FEC) methods, which can significantly improve the efficiency of a communication line.

In the literature and for practical calculations, the 'tails' of the probability density distributions of unit and zero bits are often approximated by Gaussians. In this case, the calculation of the BER requires the knowledge of the Q factor, which is determined by the expression $Q = (\mu_1 - \mu_0) \times(\sigma_1 + \sigma_0)^{-1}$, where μ_1 and μ_0 are the average values of unit and zero bits, respectively, and σ_1 and σ_0 are their average root-mean-square deviations. The BER is calculated by the expression

$$\mathrm{BER}(Q) = \frac{1}{2}\,\mathrm{erfc}\,\frac{Q}{\sqrt{2}} \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}. \qquad (1)$$

The Gaussian approximation is simple but the accuracy of predicting the error probability for this model is low.

In this paper, we present the results of the direct numerical simulation of the error statistics in a fibreoptic communication line based on a standard single-mode SMF fibre with a 40-Gbit s^{-1} data transmission rate. The initial signal was produced in such a way that odd bits have a phase shift $\Delta\varphi$, which is equivalent to multiplication of pulses located on the odd time intervals by the constant $\exp(\mathrm{i}\Delta\varphi)$. It is shown statistically that the use of the format with the phase shift of each second bit significantly reduces the error probability in the communication line when the phase modulation $\Delta\varphi$ is properly selected. In addition, simple analytic expressions are shown to approximate well the error probability in the case of the data transfer by using the phase shift of odd bits.

We considered an optical communication line whose periodic section has the following configuration and dimensions:

$$\mathrm{SMF}(85\ \mathrm{km}) + \mathrm{EDFA} + \mathrm{DCF}(14.85\ \mathrm{km}) + \mathrm{EDFA},$$

where DCF is the dispersion-compensating fibre; EDFA is an erbium-doped fibre amplifier. The parameters of the optical fibres are presented below.

	SMF	DCF
Attenuation at 1550 nm/dB km^{-1}	0.2	0.65
Effective mode area/μm^2	80	19
Dispersion/ps nm^{-1} km^{-1}	17	−100
Dispersion slope/ps nm^{-2} km^{-1}	0.07	−0.41
Nonlinear refractive index/m^2 W^{-1}	2.7×10^{-20}	2.7×10^{-20}

Erbium amplifiers had a noise factor of 4.5 dB and a gain of 13.4 dB necessary to compensate completely for the optical signal attenuation over the length of the periodic section. The average dispersion of the periodic section was -0.4 ps nm^{-1} km^{-1}. The communication line had 31 sections and an additional segment of a standard single mode fibre compensating for the accumulated dispersion. The length of the latter (72.6 km) was selected so that the Q factor at the end of the communication line be maximal. The dispersion accumulated in 31 periodic sections was -1238.1 ps nm^{-1}, while the total dispersion of the additional segment was 1234.2 ps nm^{-1}. Thus, the average dispersion of the entire communication line was almost zero.

As unit bits, we used 7.5-ps Gaussian pulses with a 5-mW peak power. We considered the data transfer in one frequency channel at a rate of 40-Gbit s^{-1}.

The dynamics of optical pulses was described by the generalised Schrödinger equation for the complex envelope A of the electromagnetic field [1]:

E.G. Shapiro Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Koptyuga 1, 630090 Novosibirsk, Russia; e-mail: e.shapiro@iae.nsk.su

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$$\mathrm{i}\,\frac{\partial A}{\partial z}-\frac{\beta_2(z)}{2}\frac{\partial^2 A}{\partial t^2}+\sigma(z)|A|^2A = \mathrm{i}\Big[-\gamma(z)+\sum_{k=1}^{N}r_k\delta(z-z_k)\Big]A. \quad (2)$$

Here, z is the distance along the line; t is the time; $|A|^2$ is the pulse power; β_2 is the group velocity dispersion parameter; $\sigma = 2\pi n_2/(\lambda_0 A_{\mathrm{eff}})$ is the Kerr nonlinearity coefficient; n_2 is the nonlinear refractive index; λ_0 is the carrier wavelength; A_{eff} is the effective area of the fibre eigenmode; $\gamma(z)$ is the signal attenuation coefficient; r_k is the gain; location points of amplifiers are denoted by z_k; the quantities σ and β_2 are presented as functions of z to take into account the changes in these parameters when passing from one fibre type to another.

In dispersion-controlled systems, fibres with the chromatic dispersion of opposite signs are used to ensure the control of the dispersion pulse broadening. If the average dispersion of the communication line was equal to zero, in the linear case in the absence of attenuation and noise, the signal shape was reconstructed at the end of the line [1]. Within the framework of the generalised nonlinear Schrödinger equation describing the propagation of optical pulses, we take into account the following effects responsible for the signal distortion: Kerr nonlinearity, dispersion broadening, amplified spontaneous emission as well as fluctuations of the positions of individual bits (the so-called Gordon – Haus effect [2]). Apart from this effect, there are a number of physical reasons resulting in the jitter of the pulses. They are electrostriction [3] and polarisation mode dispersion; however, they are beyond the scope of the model under study and this paper.

The data were statistically processed after the propagation of optical signals over a distance of 3000 km. A rectangular optical filter with the transmission bandwidth $B_{\mathrm{op}} = 100$ GHz and an electric third-order Butterworth filter with the transmission bandwidth $B_{\mathrm{el}} = 40$ GHz are used to receive signals. Depending on the specified threshold decision level (DL) of the electric current, either a zero bit or a unit bit is detected. If the current is smaller than the DL, the bit is recognised as a zero one, and if the current is higher, – as a unit bit. The number of errors depends on the choice of the DL value, and there exists an optimal level of division of zeros and units for which the number of errors is minimal.

The basic effects causing signal degradation are the amplified spontaneous emission and Kerr nonlinearity. Agrawal [1] describes the physical mechanism of the nonlinearity suppression with the help of the phase shift of each second bit of the initial sequence in a fibreoptic line without the amplified spontaneous emission.

In this paper, to demonstrate the effect of the phase shift on the nonlinearity, we considered two models of propagation of optical pulses: with ideal noiseless amplifiers and with the amplified spontaneous emission of erbium amplifiers.

The generalised nonlinear Schrödinger equation (2) was solved numerically by using the method of splitting into physical processes [1].

Let us denote by $w_0(y)$ the probability density function of the zero bit distribution constructed using the sampling of current values y on the detector. Figure 1 shows the dependences of $F_0(y) = \ln(w_0(y))$, calculated by neglecting the amplifier noise, for a standard format without a phase shift and for a format with the phase shift of each second bit by $\Delta\varphi = \pi/2$. The sampling size of the zero bits was 99948 values of the standard format and 100460 values of the format with the phase shift. It is obvious that the application of the $\Delta\varphi = \pi/2$ phase shift for each second bit significantly decreases the error probability in detecting the zero.

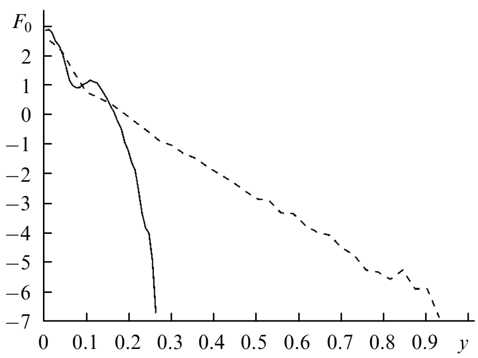

Figure 1. Dependence of the logarithm $F_0(y) = \ln(w_0(y))$ of the probability density distribution of zero bits on the current y (in relative units) on the detector for $\Delta\varphi = \pi/2$ (solid curve) and $\Delta\varphi = 0$ (dashed curve) in a model with ideal amplifiers.

Thus, let $w_1(y)$ be the probability density function of the unit bit distribution constructed using the sampling of current values y on the detector. Figure 1 shows the function of $F_1(y) = \ln(w_1(y))$ for the propagation model with ideal amplifiers. The curves were plotted using 99732 values of the current on the detector for the standard formation of the initial bit sequence and using 100424 values for the format with a phase shift. It is obvious that when the initial bit sequence is produced using the $\Delta\varphi = \pi/2$ phase shift, the error probability in detecting the unity on the detector is smaller than in the standard case, when $\Delta\varphi = 0$.

We will present below the results of the numerical simulation of propagation of optical signals with the phase shift of each second bit $\Delta\varphi = 0$ (standard format), $\Delta\varphi = \pi/2$, and $\Delta\varphi = \pi$ taking into account the amplified spontaneous emission and nonlinearity effect.

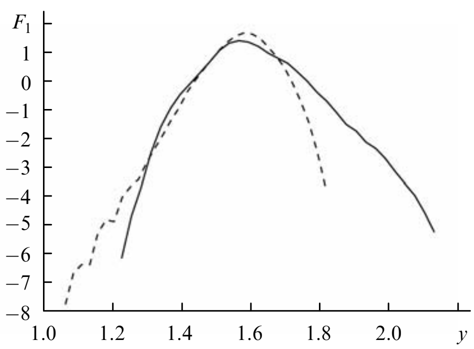

Figure 2. Dependence of the logarithm $F_1(y) = \ln(w_1(y))$ of the probability density distribution of unit bits on the current y (in relative units) on the detector for $\Delta\varphi = \pi/2$ (solid curve) and $\Delta\varphi = 0$ (dashed curve) in a model with ideal amplifiers.

The dependences of the probability density distribution of zero bits in this case, $F_0(y) = \ln(w_0(y))$, are shown in Fig. 3. The statistical sampling consisted of 128024 current values on the detector for $\Delta\varphi = 0$, of 89164 values for $\Delta\varphi = \pi/2$, and of 83490 values for $\Delta\varphi = \pi$. One can see that the tails of the probability density distribution decrease exponentially [4]. The same figure presents analytic approximations of the probability density tails by the linear functions: $-7.5y + 2$ for $\Delta\varphi = 0$, $-15.6y + 3.8$ for $\Delta\varphi = \pi/2$, and $-8.5y + 2$ for $\Delta\varphi = \pi$.

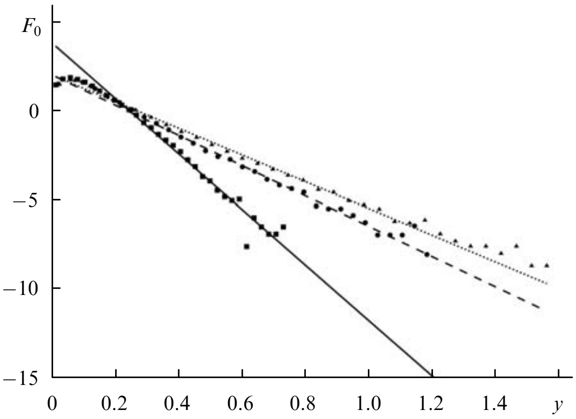

Figure 3. Dependence of the logarithm $F_0(y) = \ln(w_0(y))$ of the probability density distribution of zero bits on the current y (in relative units) on the detector for $\Delta\varphi = \pi/2$ (calculation, ■) and analytic approximation ($-15.6y + 3.8$, solid line), for $\Delta\varphi = \pi$ (●) and analytic approximation ($-8.5y + 2$, dashed line), as well as for $\Delta\varphi = 0$ (▲) and analytic approximation ($-7.5y + 2$, dotted line).

Figure 4 illustrates the probability density distributions of zero bits $F_1(y) = \ln(w_1(y))$ for the above formats and analytic approximation (solid curve) for $\Delta\varphi = 0$. The statistical sampling consisted of 127464 current values on the detector for $\Delta\varphi = 0$, of 89012 values for $\Delta\varphi = \pi/2$, and of 83422 values for $\Delta\varphi = \pi$. The analytic approximation $P_1(y)$ of the probability density distribution of the unit bits $w_1(y)$ is expressed as [5, 6]

$$P_1(y) = \frac{1}{2}\sqrt{\frac{\bar{M}}{\pi I_0}}\left[\frac{I_1}{(y + d - I_0)^3}\right]^{1/4} \times \exp\left[-\frac{\bar{M}}{I_0}\left(\sqrt{y + d - I_0} - \sqrt{I_1}\right)^2\right], \quad (3)$$

where I_0 and I_1 are the average values of the electric current on the detector. The parameters $\bar{M}$ and d were selected from the sampling of zero and unit bits on the detector so that the difference between the analytic probability density distribution and the results of the numerical experiment be minimised. In this paper, we used the least-squares method. The comparison of the curves shows that the main reason of signal distortion for unit bits is the amplified spontaneous emission.

Figure 5 shows the numerically calculated BER probability as a function of the DL for zero and unit bits for the above formats of the initial bit sequence as well as the analytic approximation. The error appears if the electric current on the detector for the unit of the initial bit sequence is smaller than the DL and for the zero – higher than the

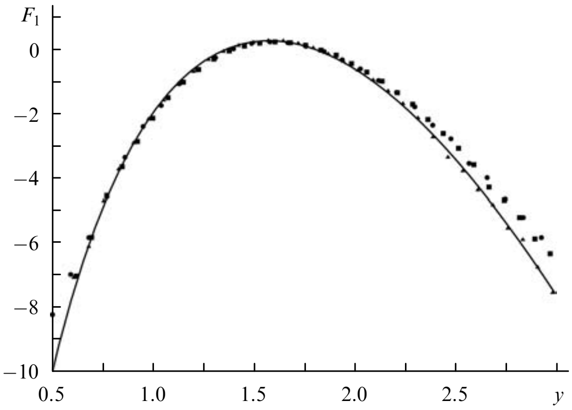

Figure 4. Dependence of the logarithm $F_1(y) = \ln(w_1(y))$ of the probability density distribution of unit bits on the current y (in relative units) on the detector for $\Delta\varphi = \pi/2$ (■), $\Delta\varphi = \pi$ (●), and $\Delta\varphi = 0$ (▲) and analytic approximation (solid curve) given by expression (3) at $M = 4.83$, $d = 0.144$, $I_0 = 0.137$, $I_1 = 1.62$.

DL. It follows from the curves that the phase shift of each second bit can significantly improve the signal quality on the detector. Note that the best format from the point of view of minimising the probability error is the format $\Delta\varphi = \pi/2$. The BER for this format, obtained by the direct numerical simulation, is equal to 7×10^{-5} for the optimal DL, while for the format without a phase shift, the minimal error probability is 1.7×10^{-3}. Thus, the error probability optimal with respect to DL in the case of formation of an initial bit sequence with a phase shift by $\Delta\varphi = \pi/2$ is 20 times lower than the minimal error probability in the absence of the phase shift of odd bits.

It is shown statistically that the use of the phase shift of each second bit interval significantly reduces the effect of the Kerr nonlinearity, when optical pulses propagate in a fibreoptic communication line, and improves the signal quality on the detector.

The simple analytic expressions are also shown to approximate well the tails of the probability density distribution of zero and unit bits for the formats with a phase shift of each second bit, which makes it possible to

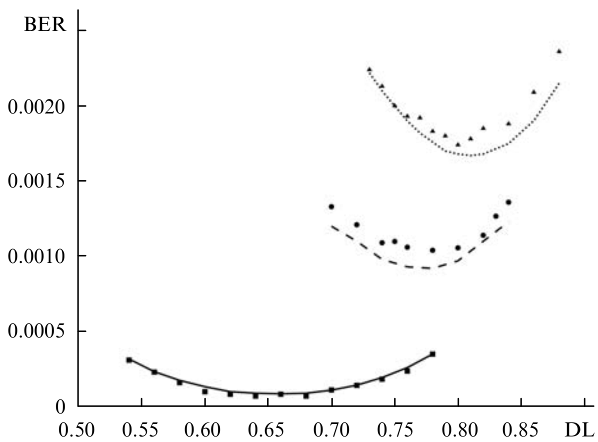

Figure 5. BER coefficient obtained in a numerical experiment for $\Delta\varphi = \pi/2$ (■) and analytic approximation (solid curve), for $\Delta\varphi = \pi$ (●) and analytic approximation (dashed curve), and for $\Delta\varphi = 0$ (▲) and analytic approximation (dotted curve).

reduce the sampling size required to estimate adequately the error rate.

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