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Influence of small-scale self-focusing on second harmonic generation in an intense laser field

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***Abstract*. Linearised equations for the amplitudes of harmonic perturbations of strong waves in quadratic and cubic nonlinear media are obtained within the model of plane monochromatic waves. The dependences of the gains of noise components of the first and second harmonic waves upon frequency doubling on the B integral are found. The maximally admissible noise level in the fundamental radiation beam is calculated by the example of a 0.5-mm-long KDP crystal at the peak intensity of 4.5 TW cm^{-2}.**

Keywords: *second harmonic generation, intense laser field, small-scale self-focusing.*

1. Introduction

The feasibility of obtaining coherent femtosecond radiation in the visible range as well as the necessity of increasing the temporal contrast of super-high-power laser pulses has aroused interest of the researchers in SHG since the discovery of this effect in 1961 [1] till now. The second harmonic generation of an intense laser field has a number of peculiarities. Because the intense laser fields, as a rule, are produced in pulses with duration of several tens of femtoseconds, the SHG efficiency depends on dispersion phenomena – the group velocity mismatch and dispersion spreading of pulses at the fundamental and doubled frequencies. However, the right choice of the crystal, its length, and the optical axis orientation makes it possible to minimise the effect of these phenomena [2 – 8].

At the same time, effects caused by the cubic nonlinearity of the medium become significant in intense laser fields. High-power waves propagating through a cubic nonlinear medium acquire an additional phase incursion leading to a breakdown of the phase matching conditin and a decrease in the conversion efficiency [2 – 8]. The instability of small-scale perturbations propagating in such a medium leads to small-scale self-focusing during which there appear filamentation [9 – 11], supercontinuum generation, and finally, nonlinear elements are damaged.

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This paper is devoted to investigation of the SHG peculiarities of a intense laser field. First, we consider the problems on conversion of high-power (noiseless) waves into second harmonic radiation in quadratic and cubic nonlinear media. Then, in the linear approximation we successively solve the problem of the small-scale self-focusing development for an arbitrary spatial perturbation (noise) spectrum. This problem is solved in analogy with the classical paper [9] but taking into account the quadratic nonlinearity: we found the gain of one perturbation harmonic, determined the gain integral in the entire harmonic spectrum, and estimated the maximum noise level at the crystal input, which does not lead to its optical breakdown and damage due to the small-scale self-focusing.

The linear stage of harmonic perturbation amplification is of great interest because it allows one to describe correctly the noise amplification (including the noise with a broad spatial spectrum) up to its power comparable with the fundamental wave power. The optical breakdown caused by the small-scale self-focusing limits the possibilities of frequency doubling at significantly lower noise powers; therefore, determination of its maximally admissible level (which does not lead to the optical breakdown or the damage of the nonlinear element) is an urgent problem in experiments on highly efficient SHG of an intense laser field. We have simulated the conversion of 1.5 – 4.5-TW cm^{-2} radiation from a petawatt femtosecond laser complex [12] into second harmonic radiation and presented the estimates and methods for determining the admissible noise level by the power in the first harmonic beam.

2. Peculiarities of SHG of an intense laser field

Before analysing the wave instability in quadratic and cubic nonlinear media, we will consider the conversion of intense (noiseless) laser fields into radiation at the second harmonic frequency. Despite their pulsed character, we will use the plane monochromatic wave approximation to describe the SHG. This approach is valid when such dispersion effects as the group velocity mismatch and dispersion spreading of the first and second harmonic pulses, is insignificant at distances in the order of the nonlinear element length of the frequency doubler. Fulfilment of the above requirements for the highly efficient SHG is possible if the nonlinear element length L meets the conditions:

$$L_{\rm nl} < L < L_{\rm gr} < L_{1,2},$$

where $L_{\rm nl}$ is the nonlinear length (characteristic scale of conversion into the second harmonic); $L_{\rm gr} = T_1|1/u_1-$

$1/u_2|^{-1}$ is the length of the group velocity mismatch of pulses; u_1 and u_2 are the group velocities of the waves at the first and second harmonic frequencies, respectively;

$$L_i = T_1^2 \left| 4 \ln 2 \frac{\partial^2 k_i}{\partial \omega^2} \right|_{\omega_i}^{-1} \quad (i = 1, 2)$$

are the lengths of dispersion spreading of the first and second harmonic pulses, respectively; T_1 is the pulse duration of fundamental radiation at the input to the nonlinear element. For the KDP crystal at $\lambda = 910$ nm and $T_1 = 50$ fs, $L_{\rm gr} = 0.13$ cm, $L_1 = 8$ cm, $L_2 = 1$ cm, which is significantly larger than the nonlinear length $L_{\rm nl} = 4 \times 10^{-2}$ cm calculated at the input intensity $I_0 \sim 1$ TW cm^{-2}.

Within the mentioned approximation frequency doubling (hereafter we consider the first type of wave interaction, oo – e) can be described with the help of a system of nonlinear differential equations for the coupled-wave amplitudes

$$\frac{d\varepsilon_1}{dz} = -i\beta\varepsilon_2\varepsilon_1^* e^{-i\Delta kz} - i\gamma_{11}|\varepsilon_1|^2\varepsilon_1 - i\gamma_{12}|\varepsilon_2|^2\varepsilon_1,$$
$$\frac{d\varepsilon_2}{dz} = -i\beta\varepsilon_1^2 e^{i\Delta kz} - i\gamma_{21}|\varepsilon_1|^2\varepsilon_2 - i\gamma_{22}|\varepsilon_2|^2\varepsilon_2 \tag{1}$$

with boundary conditions

$$\varepsilon_1(z = 0) = \varepsilon_{10}, \varepsilon_2(z = 0) = 0, \tag{2}$$

where $\varepsilon_1 = \rho_1 \exp(i\varphi_1)$ and $\varepsilon_2 = \rho_2 \exp(i\varphi_2)$ are the complex amplitudes of electric field strengths at the fundamental and second harmonic frequencies, respectively; z is the longitudinal coordinate of the wave propagation; $\Delta k = k_2 - 2k_1$ is the linear phase mismatch of wave vectors; β, γ_{ij} ($i,j = 1$, 2) are the nonlinear wave coupling coefficients [5]. The terms with the coefficients γ_{11} and γ_{22} are responsible for the self-action of the first and second harmonic waves, while the terms with γ_{12} and γ_{21} – for the cross-action.

The solutions of the system of equations (1) were analysed in detail in papers [5 – 8]. In converting an intense laser field into radiation at the second harmonic frequency, the terms in equations (1) responsible for self- and cross-action play a significant role. The matter is that the cubic nonlinearity leads to an additional phase accumulation in the first and second harmonic waves, which violates the phase-matching condition and reduces the conversion efficiency.

There exists a rather simple method to overcome this effect. The nonlinear phase incursion can be compensated for due to the change in the angle of radiation propagation to the crystal optical axis. The idea was put forward by the authors of papers [4, 5] and its experimental verification can be found in [8, 13]. As applied to the model of plane monochromatic waves, the optimal detuning of the wave vectors can be found as follows [8]:

$$\Delta k_{\rm opt} L_{\rm nl} = -(2\alpha + \Delta_0^{\rm nl}), \tag{3}$$

where

$$\alpha = \frac{2\gamma_{11} - 2\gamma_{12} - \gamma_{21} + \gamma_{22}}{4\beta^2 L_{nl}}; \quad \Delta_0^{\rm nl} = \frac{\gamma_{21} - 2\gamma_{11}}{\beta^2 L_{nl}}.$$

Using (3), it is easy to find the expression for the optimal angle of radiation propagation, $\theta_{\rm opt}$:

$$\sin^2\theta_{\rm opt} = \frac{(n_1 + \Delta n_1)^{-2} - n_{\rm o}^{-2}}{n_{\rm e}^{-2} - n_{\rm o}^{-2}}. \tag{4}$$

Here, n_1 is the refractive index of an ordinary wave of the first harmonic; $n_{\rm o}$, $n_{\rm e}$ are the main refractive indices for the second harmonic wave; $\Delta n_1 = \lambda A_{10}^2(2\gamma_{11} + 2\gamma_{12} - \gamma_{21} - \gamma_{22})/8\pi$ is an addition to the refractive index, caused by the cubic nonlinearity of the medium. At $I_0 \sim 4.5$ TW cm^{-2} and $\lambda = 910$ nm, we have $\Delta n_1 = 3.43 \times 10^{-4} \ll 1$; therefore, the detuning $\Delta\theta$ from the phase-matching angle can be found by using the Taylor series expansion of (4) over the parameter Δn_1:

$$\Delta\theta = \frac{\Delta n_1}{n_1^3(n_{\rm o}^{-2} - n_1^{-2})} \left(\frac{n_1^{-2} - n_{\rm o}^{-2}}{n_{\rm e}^{-2} - n_1^{-2}} \right)^{1/2}.$$

The dependences $\eta = |A_2(z)|^2/|A_{10}|^2$ of the conversion efficiency in a 0.5-mm-long KDP crystal on the detuning from the phase-matching angle for the specified radiation parameters are presented in Fig. 1. We used expressions for the coefficients β and γ_{ij} from paper [5]. At the first harmonic wavelength $\lambda_1 = 910$ nm, the nonlinear wave coupling coefficients in the KDP crystal are: $\gamma_{11} = 2.302 \times 10^{-9}$, $\gamma_{12} = 1.711 \times 10^{-9}$, $\gamma_{21} = 1.398 \times 10^{-9}$ and $\gamma_{22} = 2.872 \times 10^{-9}$ CGS units, $\beta = 3.329 \times 10^{-4}$ CGS units. Unlike the case $\gamma_{ij} = 0$, the cubic nonlinearity leads to the fact that the maximum efficiency is achieved at $\Delta\theta = -0.45°$. Note that the quantity $\Delta\theta$ is independent of the quadratic nonlinearity and the nonlinear element length and directly proportional to the input signal intensity. The characteristic distance between the zeroes of the main maximum is 20 mrad.

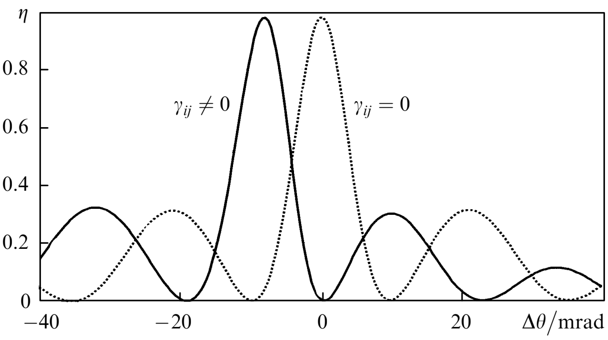

Figure 1. The conversion efficiency η into the second harmonic versus the detuning $\Delta\theta$ from the phase-matching angle taking into account and neglecting the cubic nonlinearity of the medium at $I_0 \sim 4.5$ TW cm^{-2} and the KDP crystal length $L = 0.5$ mm.

Let us compare the conversion efficiency of 910-nm, 4.5-TW cm^{-2} radiation into the second harmonic for the wave vector detunings $\Delta k L_{\rm nl} = 0$ and $\Delta k L_{\rm nl} = \Delta_{\rm opt}$ (Fig. 2).

One can see from Fig. 2 that at $\Delta k L_{\rm nl} = 0$, the effect of the cubic nonlinearity leads to the phase-matching violation and inverse energy transfer, while at $\Delta k L_{\rm nl} = \Delta_{\rm opt}$, the conversion efficiency into the second harmonic radiation increases. According to the model of the plane monochro-

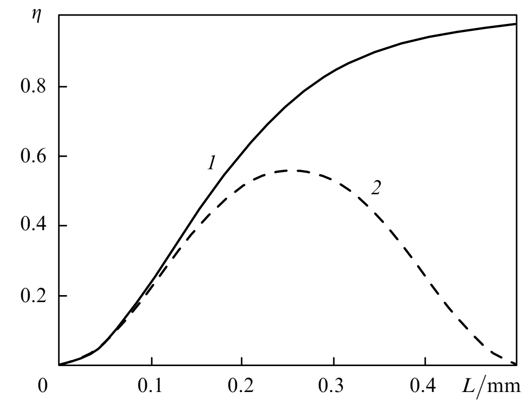

Figure 2. Dependences of the efficiency η on the crystal length L at $I_0 \sim 4.5$ TW cm^{-2} for $\Delta k_{\rm opt} L_{\rm nl} = \varDelta_{\rm opt}$ (*1*) and $\Delta k L_{\rm nl} = 0$ (*2*).

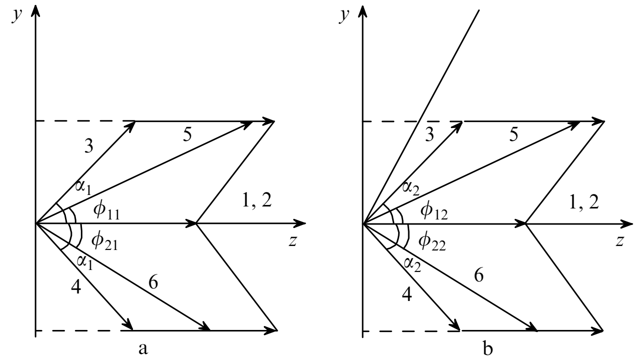

Figure 3. Scheme of propagation of the intense waves of the first (wave 1) and second (wave 2) harmonics as well as their harmonic perturbations (waves 3, 4 and 5, 6, respectively) in the planes noncritical (a) and critical (b) with respect to the phase-matching angle.

matic waves, the conversion efficiency of 4.5-TW cm^{-2} radiation into the second harmonic in a nonlinear 0.5-mm-long KDP element exceeds 90 %. A detailed analysis of 50-fs frequency-doubled Gaussian pulses with the same parameters of laser radiation and the nonlinear element performed in paper [8] showed that the conversion efficiency is 83 %.

Therefore, the cubic nonlinearity can significantly affect the conversion efficiency into the second harmonic. For each value of the intensity at the nonlinear element input, it is possible to select an optimal propagation angle ensuring the highest conversion.

3. Instability of plane monochromatic waves in media with quadratic and cubic nonlinearities

Consider the influence of the small-scale self-focusing effects on the SHG in the KDP crystal at the first harmonic radiation intensities of 1.5–4.5 TW cm^{-2}. To this end, we will calculate the gains of harmonic perturbations in the first and second harmonic radiation and determine the critical noise level in the radiation of the fundamental frequency.

3.1 Linear equations for amplitudes of harmonic perturbations

Let us obtain the linear equations for the electric field strength amplitudes of spatial harmonic perturbations of the first and second harmonic waves. As in the first paper on the small-scale self-focusing [9], we assume that the noise component amplitudes are substantially smaller than the strong wave amplitudes at the fundamental and doubled frequencies (waves 1 and 2), i.e., the conditions

$$|\varepsilon_i| \ll |\varepsilon_1|, |\varepsilon_1|,\ i = 3 - 6 \qquad (5)$$

are fulfilled. Here, $i = 3$, 4 corresponds to the harmonic perturbation strengths of radiation at the fundamental frequency, and $i = 5$, 6 – at the second harmonic frequency (Fig. 3).

By applying the standard linearisation procedure [because conditions (5) are fulfilled] to quasi-optic equations describing the dynamics of variation of each frequency component and by grouping the terms with the identical transverse wave vectors, it is easy to obtain equations for the amplitudes of harmonic perturbations:

$$\begin{aligned}
\frac{d\varepsilon_3}{dz} &= \frac{-i}{\cos\alpha_1\cos\alpha_2}[\beta(E_1^*E_5 + E_4^*E_2) + \gamma_{11}(E_1^2E_4^* \\
&\quad + 2|E_1|^2E_3) + \gamma_{12}(|E_2|^2E_3 + E_1E_2E_6^* + E_1E_5E_2^*)] \\
&\quad \times \exp(ik_1z\cos\alpha_1\cos\alpha_2), \\
\frac{d\varepsilon_4}{dz} &= \frac{-i}{\cos\alpha_1\cos\alpha_2}[\beta(E_1^*E_6 + E_3^*E_2) + \gamma_{11}(E_1^2E_3^* \\
&\quad + 2|E_1|^2E_4) + \gamma_{12}(|E_2|^2E_4 + E_1E_2E_5^* + E_1E_6E_2^*)] \\
&\quad \times \exp(ik_1z\cos\alpha_1\cos\alpha_2), \\
\frac{d\varepsilon_5}{dz} &= \frac{-i}{\cos\phi_{11}\cos\phi_{12}}[2\beta E_3E_1 + \gamma_{21}(|E_1|^2E_5 \\
&\quad + E_1E_2E_4^* + E_1^*E_2E_3) + \gamma_{22}(|E_2|^2E_6^* + 2|E_2|^2E_5)] \\
&\quad \times \exp(ik_5z\cos\phi_{11}\cos\phi_{12}), \\
\frac{d\varepsilon_6}{dz} &= \frac{-i}{\cos\phi_{21}\cos\phi_{22}}[2\beta E_4E_1 + \gamma_{21}(|E_1|^2E_6 \\
&\quad + E_1E_2E_3^* + E_1^*E_2E_4) + \gamma_{22}(|E_2|^2E_5^* + 2|E_2|^2E_6)] \\
&\quad \times \exp(ik_6z\cos\phi_{21}\cos\phi_{22}).
\end{aligned} \qquad (6)$$

Here, $E_i = \varepsilon_i \exp(-ik_{iz}z)$; k_1, k_5, k_6 are the wave-vector moduli of radiation at the fundamental frequency and harmonic perturbations of the second harmonic. The angles α_1 and α_2 determine the propagation directions of harmonic perturbations of the first harmonic in the planes [noncritical (α_1) and critical (α_2) with respect to the phase-matching angle] of the nonlinear element of the frequency doubler (see Fig. 3). The propagation direction of noise at the second harmonic frequency in the nonlinear element planes noncritical (ϕ_{11}, ϕ_{21}) and critical (ϕ_{12}, ϕ_{22}) with respect to the phase-matching angle are determined from the boundary conditions (7) responsible for the equality of the corresponding transverse components of the wave vectors:

$$k_1 \sin\alpha_2 = k_5 \sin\phi_{12}, \quad k_1 \sin\alpha_2 = k_6 \sin\phi_{22},$$

$$k_1 \cos\alpha_2 \sin\alpha_1 = k_5 \cos\phi_{12} \sin\phi_{11}, \qquad (7)$$

$$k_1 \cos\alpha_2 \sin\alpha_1 = k_6 \cos\phi_{22} \sin\phi_{21}.$$

Let us assume that the boundary conditions

$$\varepsilon_3(z=0) = \varepsilon_4(z=0) = \varepsilon_{30}\mathrm{e}^{\mathrm{i}\varphi}, \quad \varepsilon_{5,6}(z=0) = 0 \qquad (8)$$

are fulfilled at the nonlinear element input, and boundary condition (2) are valid for the strong-wave amplitudes. Here, φ is the initial phase of the harmonic perturbation of the first harmonic wave. The noise in the beam at the second harmonic frequency appears due to the interaction of the harmonic perturbations of the signal at the fundamental frequency with a strong wave at the same frequency. The second harmonic beam modulation acquired in this way increases due to the influence of the cubic nonlinearity, which may lead to the small-scale self-focusing development and to destruction of the nonlinear element of the frequency doubler.

3.2 Gains of harmonic perturbations

The gain dynamics of harmonic perturbations depends in a complicated way on the high-power wave intensity at the nonlinear element input, the quadratic and cubic nonlinearities of the frequency doubler medium, the linear wave-vector detuning as well as on the initial phase φ of the field perturbations at the fundamental frequency at the nonlinear element input.

Let us determine the gains G_i of harmonic perturbations of the first and second harmonic waves ($i = 3 - 6$):

$$G_i(z, \alpha_1, \alpha_2, \varphi) = \frac{|\varepsilon_i(z, \alpha_1, \alpha_2, \varphi)|^2}{|\varepsilon_{30}|^2}.$$

The initial phase φ is, as a rule, a random quantity; therefore, of greatest interest is the averaged gain

$$G_{\mathrm{av}\,i}(z, \alpha_1, \alpha_2) = \frac{1}{2\pi}\int_0^{2\pi} G_i(z, \alpha_1, \alpha_2, \varphi)d\varphi.$$

For a medium with the cubic nonlinearity only, paper [14] showed that in the case of an ideal (nonideal) phase, the gain can be twice lower (greater) than the average gain G_{av}.

The dependences of $G_{\mathrm{av}\,1,2}$ on the propagation directions of harmonic perturbations of the first harmonic radiation (α_1, α_2) are presented in Fig. 4 for a 0.5-mm-long crystal at $I_0 \sim 4.5$ TW cm^{-2} and $\Delta = \Delta_{\mathrm{opt}}$.

According to Fig. 4 the maximal initial-phase-averaged gains of the harmonic perturbations of the first and second harmonic waves at the nonlinear element output are as follows: $G_{\mathrm{av}\,1} = 14$, $G_{\mathrm{av}\,2} = 270$.

The angular detuning in the plane critical with respect to the phase-matching angle from the optimal direction of wave propagation in the crystal imposes restrictions on generation and amplification of parasitic waves. As a result, the angular diagrams of the harmonic perturbation gains are symmetric in the plane noncritical with respect to the phase-matching angle, i.e., $G_{\mathrm{av}\,i}(z, -\alpha_1, \alpha_2) = G_{\mathrm{av}\,i}(z, \alpha_1, \alpha_2)$,, and asymmetric in the critical plane: $G_{\mathrm{av}\,i}(z, \alpha_1, -\alpha_2) \neq G_{\mathrm{av}\,i}(z, \alpha_1, \alpha_2)$. According to Fig. 4, the harmonic perturba-

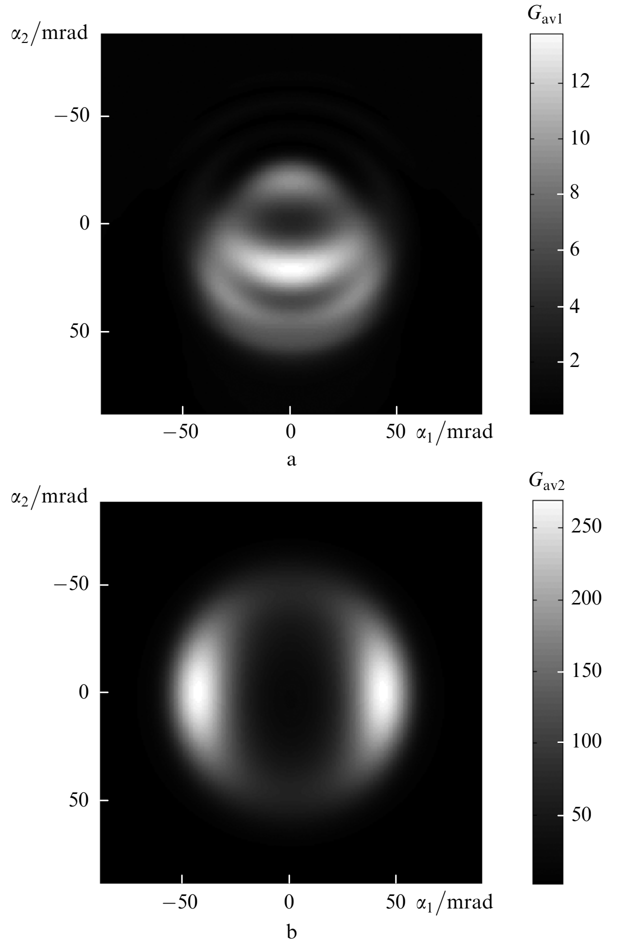

Figure 4. Angular dependences of the harmonic perturbation gains $G_{\mathrm{av}1,2}$ of the first (a) and second (b) harmonic waves at the input of the 0.5-mm-long KDP crystal on the propagation direction of harmonic perturbations of first harmonic radiation (α_1, α_2) at $\Delta = \Delta_{\mathrm{opt}}$ and $I_0 =$ 4.5 TW cm^{-2}. The dependences were obtained by solving numerically the system of equations (6) with boundary conditions (8).

tion of the second harmonic are maximally amplified for the given parameters in the direction corresponding to the angles $\alpha_1 = 42$ mrad and $\alpha_2 = 0$.

To check the results of the numerical calculations, we will consider a medium without a quadratic nonlinearity, i.e., we set $\beta = 0$. In this case, the distribution of $G_{\mathrm{av}\,1}$ can be found analytically [10]:

$$G_{\mathrm{theor}} = \frac{1}{4}\left[2\cosh^2(B_{11}x) + \left(\frac{2B_{11}x}{\kappa_1^2 - 4B_{11}}\right)^2 \sinh^2(B_{11}x) + \left(\frac{\kappa_1^2 - 4B_{11}}{2B_{11}x}\right)^2 \sinh^2(B_{11}x)\right],$$

where $B_{11} = \gamma_{11}A_{10}^2L$ is the B-integral; $x^2 = \kappa_1^2/B_{11} - \kappa_1^4/4B_{11}^2$; $\kappa_1 = k_{1\perp}(L/k_1)^{1/2}$ is the normalised transverse wave vector.

Figure 5a presents the calculated angular dependence of the initial-phase-averaged gain of harmonic perturbations in a medium without a quadratic nonlinearity at $B_{11} = 2.89$ calculated for 910-nm, 4.5-TW cm^{-2} radiation propagating in a 0.5-mm-long KDP crystal. The dependence of the harmonic perturbation gain on α_1, α_2 represents a ring

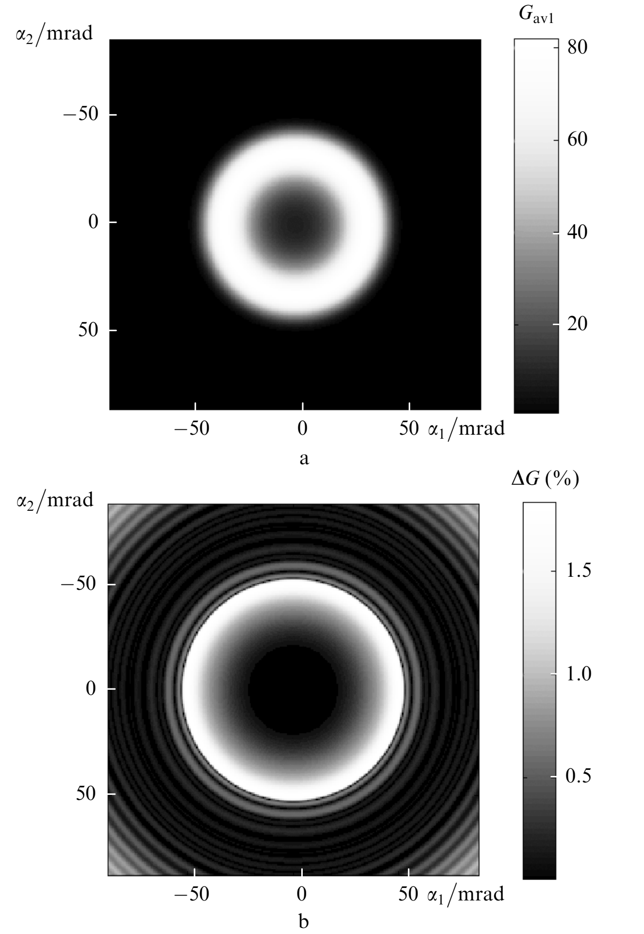

Figure 5. Angular dependence of the harmonic perturbation gain $G_{av1,2}$ of the first harmonic wave (a) and the relative error of the numerical calculation $\Delta G = |G_{theor} - G_{av1}|/G_{theor}$ (b) at the input of the 0.5-mm-long KDP crystal on the propagation direction of harmonic perturbations of first harmonic radiation (α_1, α_2) at $\varDelta = 0$ and $I_0 = 4.5$ TW cm^{-2}. The dependences were obtained by solving numerically the system of equations (6) with boundary conditions (8) at $\beta = 0$.

structure. In this case, according to Fig. 5b, the relative error of numerical calculations does not exceed 2 %.

Therefore, in the limiting case of passage to a medium without a quadratic nonlinearity the results obtained with the help of the considered model well agree with the theory [10].

Another important parameter characterising the small-scale self-focusing is the harmonic perturbation gain integral over the angular spectrum. Let us determine this gain for the problem under study as:

$$G_{\mathrm{int}\,j} = \frac{1}{\pi\alpha_{cr}^2}\int\!\!\int_{\varOmega} G_{\mathrm{av}\,j}\mathrm{d}\alpha_{j1}\mathrm{d}\alpha_{j2}.$$

Here, $j = 1, 2$ corresponds to the gains of noise in the first and second harmonic beams; α_{cr} is the angle in the noncritical plane at which the gain of the second harmonic noise becomes e times smaller than its maximal value (or, in the case of the medium with $\beta = 0$, the first harmonic gain); $\varOmega$ is the circle of radius α_{cr} with the centre at the coordinate origin. Despite the awkwardness of the integral gain determination, it is rather physical because it takes into account the non-isotropic gain structure (see Fig. 4a).

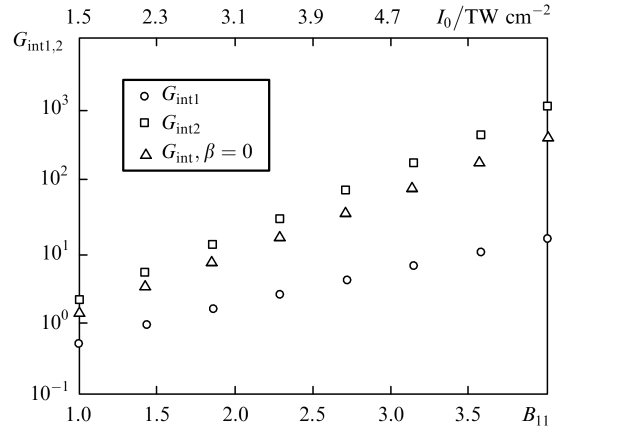

Figure 6. Dependences of the integral harmonic perturbation gains $G_{int1,2}$ of the first and second harmonics and G_{int} (at $\beta = 0$) on the parameter B_{11} and the intensity for the 0.5-mm-long KDP crystal.

The integral gains G_{int1} and G_{int2} calculated in this way at $I_0 = 4.5$ TW cm^{-2}, $\varDelta = \varDelta_{opt}$ and $L = 0.5$ mm are equal to 5 and 107, respectively. The dependences of the integral gains on B_{11} are shown in Fig. 6.

One can see from Figs 4 and 6 that during the SHG of an intense laser field harmonic perturbations in the second harmonic experience a noticeable amplification (the initial-phase-averaged gain is $G_{av2} = 270$ and the integral gain is $G_{int2} = 107$), which can lead to a significant decrease in the conversion efficiency and the small-scale self-focusing development accompanied by the destruction of the nonlinear element of the frequency doubler. In this connection it is necessary to determine the requirements to the critical noise level in the beam at the first harmonic frequency.

3.3 Estimates of the critical noise levels in a wave at the first harmonic frequency

Let us determine the critical noise level in the beam at the first harmonic frequency. The peak intensity I_{peak} and its roof-mean-square deviation I_{rms} in the beam profile from the average value I_{av} are related to the relative noise power P_n/P by the empirical expressions [10]:

$$I_{peak}/I_{av} = (1 + 5\sqrt{P_n/P})^2,$$
$$I_{rms}/I_{av} = (1 + \sqrt{P_n/P})^2 - 1. \qquad (9)$$

According to paper [15], the KDP crystal can endure the peak intensity of 18.5 TW cm^{-2} at the laser pulse duration of 100 fs and central wavelength of 795 nm. Suppose that from the point of view of the crystal breakdown this peak intensity is the threshold one. Then its ratio to the average intensity at $I_{av} = 4.5$ TW cm^{-2} is $K_{th} = I_{peak}/I_{av} = 4.1$. Using (9) and taking into account the fact that the noise power at the nonlinear element output is $P_{n\,out} = GP_n$, it is easy to find the critical noise level with respect to power $K_n = P_n/P$ in the input beam at the first harmonic frequency:

$$K_n = \frac{1}{G}\left[\frac{1}{5}(\sqrt{K_{th}} - 1)\right]^2.$$

For the gain $G = 107$, we obtain $K_n = 4 \times 10^{-4}$ and $I_{rms}/I_{av} = 4 \times 10^{-2}$ taking into account (9).

Thus, as applied to the frequency doubling in a 0.5-mm-long nonlinear KDP element by 4.5-TW cm^{-2} radiation, the root-mean-square intensity deviation should not exceed 4 %. Minimisation of the small-self-focusing effects is possible due to a decrease in the power noise level in the input beam.

4. Conclusions

We have considered the peculiarities of the second harmonic generation by highly intense laser pulses by the example of the 0.5-mm-long KDP crystal at $I_0 = 4.5$ TW cm^{-2} and analysed the influence of the small-scale self-focusing effects on this process. The numerical solution of linearised equations has allowed us to estimate the maximal gains of harmonic perturbations of the waves at first and second harmonic frequencies. We have found that for the SHG the noise level with respect to the power should not exceed $10^{-4} - 10^{-5}$ of the peak input power. When this condition is fulfilled, the SHG efficiency can exceed 80 %. We plan to perform experiments on frequency doubling of radiation with the peak power of 0.56 PW and the beam 10 cm in diameter in the 0.5-mm-long KDP crystal.

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