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RESONATORS

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Selection of the higher transverse modes of a waveguide quasi-optical resonator

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***Abstract*. A method of spatial filtration for the selective excitation of individual higher-order transverse oscillation modes with a high degree of discrimination of undesirable modes in a waveguide quasi-optical resonator is described. The method is based on the use of a mirror with absorbing or scattering inhomogeneities discretely arranged on its surface. The efficient excitation of such modes in a waveguide dielectric resonator with a nonuniform amplitude-step mirror in the submillimetre wavelength range (at 0.4326 mm) is confirmed theoretically and experimentally.**

Keywords: *beam formation, laser resonators, transverse mode selection, amplitude filter, Fourier optics.*

1. Introduction

In recent years waveguide quasi-optical resonators (WQORs) are being widely used in lasers. The optical field in such resonators is formed not only by mirrors, but also by oversized waveguides placed between them. Such combined resonators are employed in capillary gas-discharge lasers [1 – 3], waveguide folding lasers [4], submillimetre free-electron lasers [5], etc. The accepted criteria for the choice of the resonator design are minimal diffraction and waveguide losses and the maximum separation between eigenfrequencies [6].

A substantial disadvantage of submillimetre hollow WQOR lasers is their multimode radiation because the losses of many transverse modes in them are very small. In this connection the selection of transverse modes and obtaining of single-mode lasing is an urgent problem for these lasers. The stability regions of a WQOR with respect to the fundamental mode are successfully produced by introducing selective losses due to diffraction from the open regions of the resonator [7 – 10].

In this paper, we present the results of theoretical and experimental studies of the selective excitation of an individual higher-order transverse mode in combined resonators by the example of a broadband small submillimetre resonator of an optically pumped laser with a nonuniform amplitude-step mirror [11].

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2. Theoretical relations

The scheme of a folding combined resonator is presented in Fig. 1. The resonator contains circular dielectric waveguides (*1*) and (*2*) of radius a_1 and length L sealed at one end by plane mirrors and optically coupled with a system of fold mirrors (SFM) consisting of two spherical mirrors (*3*) and (*4*) and one plane mirror (*5*). The use of the latter allows one to orient waveguides parallel to each other and to reduce the off-axial aberration due to small angles between the SFM optical axis and normals to spherical and plane mirrors. To simplify the problem, we will consider a resonator symmetrical with respect to its reflection in the plane mirror of the SFM. The transverse dimensions of the resonator elements are assumed to provide the fulfilment of the quasi-optical condition $(ka_i)^2 \gg 1$ ($i = 1-3$, $k = 2\pi/\lambda$, λ is the wavelength, and a_2 and a_3 are the radii of mirrors) and the paraxial condition $k_\parallel \gg k_\perp$ (the longitudinal wave number greatly exceeds the transverse wave number). We will treat spherical mirrors mounted at a distance of L_1 from waveguide ends as axially symmetric quadratic phase correctors with the focal distance F. A nonuniform plane mirror of diameter $2a_2$ with a spatial filter is located at a distance of L_2 from phase correctors. The other dimensions of the resonator are shown in Fig. 1.

Our theoretical consideration is based on the methods of Fourier optics and the study of eigenmodes [12, 13]. The formation of the resonator oscillation modes is described as the interference of wave beams reflected from mirrors and counterpropagating in the waveguide and in open regions.

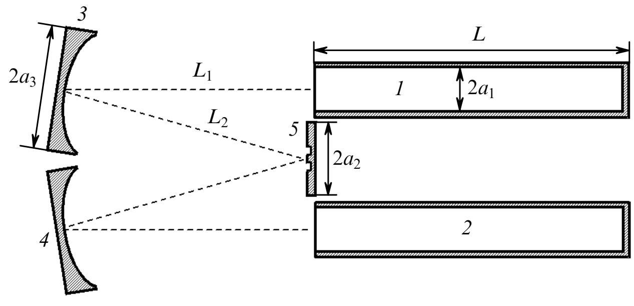

Figure 1. Scheme of a waveguide quasi-optical resonator: (*1*, *2*) waveguides; (*3*, *4*) spherical mirrors; (*5*) plane mirror.

Spherical mirrors and inhomogeneities on mirrors are described with the help of amplitude–phase correction functions [14]. We will restrict the numerical study of the characteristics of the modes of a dielectric WQOR to the practically important case of generation of linearly polarised radiation in the form of LP_{nm} waves (where n and m are the integer azimuthal and radial wave indices, respectively) [15]. For $n = 0$, the notation LP_{nm} corresponds to symmetric EH_{1m} waves, for $n = 1$ – to combined $TE_{0m} + EH_{2m}$ waves, and for $n \geqslant 2$ – to combined $EH_{-|n-1|m} + EH_{|n+1|m}$ waves. The amplitudes of the transverse components of the electric field for LP_{nm} waves, forming the compete system of orthonormalised functions, have the form

$$V_{nm}(\rho_1, \varphi_1) = f_{nm} J_n(U_{nm}\rho_1)\cos n\varphi_1, \qquad (1)$$

where $\rho_1 = r_1/a_1$; r_1 and φ_1 are cylindrical coordinates in the waveguide; J_n are the n-order Bessel functions of first kind; U_{nm} are the roots of the equation $J_n(U_{nm}) = 0$; and

$$f_{nm} = \begin{cases} \left[\sqrt{\pi}J_1(U_{0m})\right]^{-1} \text{ for } n = 0, \\ \sqrt{2}\left[\sqrt{\pi}J_{n+1}(U_{nm})\right]^{-1} \text{ for } n > 0 \end{cases}$$

are the normalisation factors. The propagation constants of the LP_{nm} waves are [16]

$$\gamma_{nm} \approx k\left[1 - \frac{1}{2}\left(\frac{U_{nm}\lambda}{2\pi a_1}\right)^2\left(1 - \frac{\mathrm{i}\nu_0\lambda}{\pi a_1}\right)\right], \qquad (2)$$

where $\nu_0 = 0.5(\nu^2 + 1)/(\nu^2 - 1)^{1/2}$ and ν is the refractive index of the waveguide walls.

The method of numerical calculation of a waveguide quasi-optical resonator in the general form is described in [10]. The field in waveguides is represented as a superposition of natural waves LP_{nm}, while in the open regions of the resonator the field is described by the diffraction integral in the Fresnel approximation. This eigenmode problem in the resonator is reduced to the solution of the system of linear algebraic equations

$$\mu C_k = \exp(\mathrm{i}\gamma_{nk}L)\sum_{m=1}^{M}\exp(\mathrm{i}\gamma_{nm}L)\int_0^1 V_{nm}(\rho_1)Q_k(\rho_1)\rho_1\mathrm{d}\rho_1,$$
$$k = 1, \ldots, M, \qquad (3)$$

where

$$Q_k(\rho_1) = \int_0^1 Q_0(\rho_1, \rho_1')V_{nk}(\rho_1')\rho_1'\mathrm{d}\rho_1';$$
$$Q_0(\rho_1, \rho_1') = \int_0^1 Q(\rho_1, \rho_2)Q(\rho_2, \rho_1')T(\rho_2)\mathrm{d}\rho_2; \qquad (4)$$

$T(\rho_2)$ is the amplitude correction function of a nonuniform mirror; $\rho_2 = r_2/a_2$ and $\rho_1' = r_1/a_1$ are the dimensionless radial coordinates on the nonuniform mirror and the open end of waveguide (*2*), respectively. Here, the kernels of integral transformations are described, accurate to a constant phase factor, by the expression

$$Q(\rho_p, \rho_{3-p}) = -4\pi^2 N_3\frac{N_p}{1 - \gamma_{3-p}}\exp\left[\mathrm{i}\pi\left(N_p\rho_p^2 + N_{3-p}\rho_{3-p}^2\right)\right]$$
$$\times\int_0^1 \exp\left[\mathrm{i}\pi\left(N_3 Z_p\rho_3^2\right)J_n(2\pi N_p\xi_p\rho_p\rho_3)\times\right.$$
$$\times J_n(2\pi N_{3-p}\xi_{3-p}\rho_{3-p}\rho_3)\rho_3\mathrm{d}\rho_3, \qquad (5)$$

where

$$N_p = \frac{a_p^2}{\lambda L_p}; \quad Z_p = \frac{1 - \gamma_p\gamma_{3-p}}{(1 - \gamma_p)(1 - \gamma_{3-p})};$$
$$\gamma_p = 1 - \frac{L_p}{F}; \quad \xi_p = \frac{a_3}{a_p}; \quad p = 1, 2.$$

The solution of system of equations (3) gives M eigenvalues μ and M eigenvectors, whose components are the expansion coefficients of resonator modes in waveguide waves. The fraction of the resonator mode energy transferred by waveguide LP_{nm} waves are determined by the value of $|C_m|^2$. The relative energy losses and the additional phase incursion of the mode after the round-trip transit of radiation in the resonator are determined by the expressions

$$\delta = 1 - |\mu|^4, \quad \Phi = 2\arg\mu, \qquad (6)$$

respectively.

To perform the spatial filtration of higher-order modes in the WQOR, it is necessary to study the field structure of its waveguide LP_{nm} waves on plane mirror (*5*) (in the far-field zone). The Fourier–Bessel transform of these waves in the case of an infinite phase corrector, for example, mirror (*3*) in Fig. 1 has, accurate to a constant factor, the form [17]

$$G(\rho_2) = \frac{U_{nm}J_{n+1}(U_{nm})J_n(2\pi N_{12}\rho_2)}{U_{nm}^2 - 4\pi^2 N_{12}^2},$$

where $N_{12} = a_1 a_2/(\lambda F)$ is the Fresnel number for the waveguide–mirror free space region. Taking this relation into account, we arrange absorbing or scattering elements on the plane SFM mirror of the resonator in the radial node lines of the Fourier spectrum of selected modes so that the radius is

$$\rho_{2g} = \frac{\chi_{ng}}{2\pi N_{12}}, \qquad (7)$$

where χ_{ng} are the roots of the corresponding functions J_n for the WQOR modes; $g = 1, 2, 3, \ldots$. Taking into account the possibility of selecting waveguide modes with the help of these elements [18, 19], we can expect that the solution of system (3) will be functions close to analytic forms (1). In this case, the transverse dimensions of regions of different types at which boundaries a jump of material constants exists should exceed the wavelength.

To solve numerically system (3) by the matrix method [20], we prepared a program using the quadrature Simpson formula (101 × 101 matrix). The dependences of the characteristics of the lowest-loss transverse modes of the dielectric resonator of the submillimetre laser on the mirror size and parameters of a spatial filter on a nonuniform mirror were studied in [21].

3. Experimental setup. Comparison of experimental results with numerical calculations

The scheme of the experimental setup for studying the mode spectrum of the WQOR and the transverse distribution of mode intensities is presented in Fig. 2. To obtain symmetric resonance curves and to study the intensity distribution at the resonator output, the resonator was switched on 'in transit' [22]. The resonator was formed by hollow glass waveguides of diameter 25.2 mm and length 445 mm. As semitransparent reflectors, the grids made of nickel strips of width 25 μm and thickness 17 μm arranged with a period of 100 μm were used. The transmission coefficient of the grids was 6 % at a wavelength of 0.4326 mm at which measurements were performed. Two spherical mirrors of diameter 42 mm with a focal distance of 80 mm were used as phase correctors in the SFM playing the role of elements of the Fourier transform in the laser resonator model.

Mirror (*10*) in experiments was a plane aluminium-coated glass mirror (diameter 10 mm, $N_{12} = 1.82$) or an aluminium nonuniform amplitude-step mirror of the same diameter. This mirror was simultaneously a part of the SFM. The parameters of the nonuniform mirror were preliminarily calculated by using equation (3). After manufacturing the mirror, the measured widths of its reflecting and absorbing parts were again substituted into (3) and mode characteristics were calculated for the real resonator model.

The amplitude-step mirror was fabricated mechanically with the help of a special cutter. On the plane mirror surface one groove of width 0.475 mm (1.1λ) and depth ~ 0.2 mm was stitched. The inner surface of the groove was oriented at an angle of $\sim 30^\circ$ to the reflecting surface of the mirror. This provided the escape of the beams reflected from the groove surface from the laser resonator, which is similar to almost complete absorption of radiation in this region of the mirror. The measured replacement of the groove centre from the mirror centre was 1.25 mm, which corresponded to relation (7) for the selective excitation of the LP_{02} mode.

All the elements of the resonator were mounted on an IZA-2 measurement line, which provided the precise movement (with skewing of no more than $1''$) of either semitransparent sealing reflectors or the SFM along the optical axis with the help of electrical drive (*11*). The resonator was excited through one of the semitransparent reflectors by radiation from an optically pumped submillimetre laser consisting of CO_2 pump laser (*1*) and submillimetre cell (*2*). The submillimetre laser operated at the transition in formic acid molecules (HCOOH) at a wavelength of 0.4326 mm. The laser radiation was modulated with chopper (*3*) and was matched with the resonator with the help of mirrors (*4–6*) and Teflon lens (*7*) with a focal distance of 30 cm. The radiation transmitted through the resonator was detected with instruments (*13–16*).

The measurement method is similar to that described in [22]. The eigenmode spectrum of the resonator was recorded by changing the resonator length with the help of electric drive (*11*). The transverse modes were identified by intermode intervals, which were calculated by the phase shifts of the modes during the round-trip transit of radiation in the resonator from relation (6) and by transverse distributions of the intensity and degree of polarisation of waveguide modes, which are known from the theory. The transfer coefficient of the resonator was optimised for a specified mode by the displacement of lens (*7*) along and perpendicular to the excitation beam. The total energy losses during the round-trip transit of radiation in the resonator were determined by measuring the resonance curve width. The transverse intensity distributions near the output reflector of the resonator were measured by scanning pyroelectric detector (*13*) with a spatial resolution of 1 mm in a plane perpendicular to the radiation beam. The detector was fixed on azimuthal displacement mechanism (*12*). Recorder (*16*) was synchronised in time with displacement mechanisms of the detector or resonator mirror.

Figure 3 presents the spectra of the excited resonator modes obtained by using the uniform or amplitude-step mirror as mirror (*10*). As the length of the resonator with a

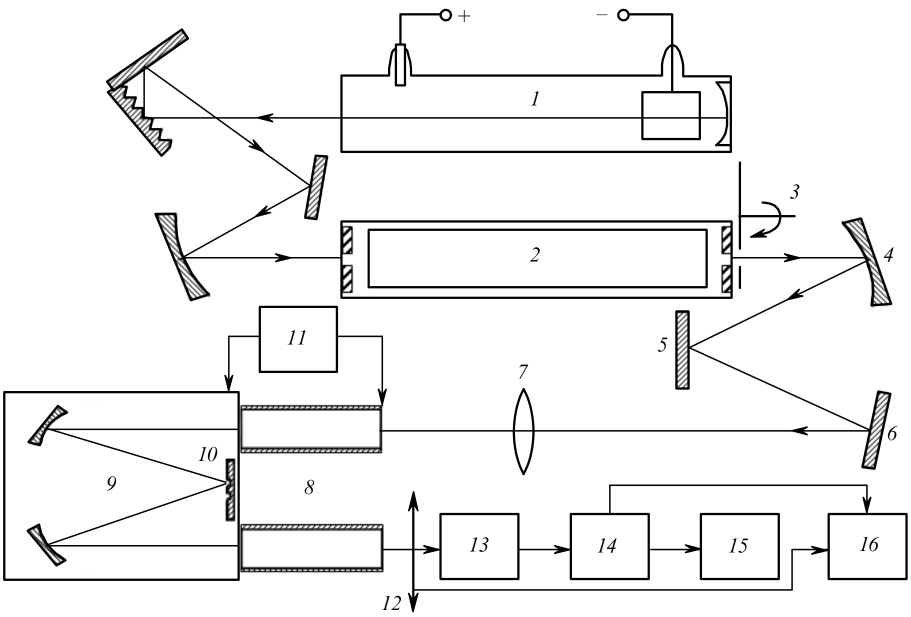

Figure 2. Scheme of the experimental setup: (*1*) CO_2 laser; (*2*) HCOOH laser; (*3*) chopper; (*4*–6) mirrors; (*7*) lens; (*8*) combined resonator; (*9*) SFM; (*10*) amplitude-step mirror; (*11*) electrical drive for the displacement of reflectors or the SFM; (*12*) device for the azimuthal displacement of a detector; (*13*) pyroelectric detector; (*14*) amplifier; (*15*) oscilloscope; (*16*) recorder.

uniform mirror was changed, three resonator modes LP_{01}, LP_{11}, and LP_{21} with the lowest losses were observed (Fig. 3a). When the uniform mirror was replaced by a mirror with an absorbing groove, these modes were considerably suppressed and the required higher LP_{02} mode was predominantly excited (Fig. 3b). The total energy losses measured after the round-trip transit of radiation in the resonator with the uniform mirror were 31.7 % and 54.6 % for the first highest-Q factor modes LP_{01} and LP_{11}, respectively; in the case of the nonuniform reflector, the measured energy losses were 61.8 % and 84.7 % for the first highest-Q factor modes LP_{02} and LP_{03}, respectively. The energy losses after the round-trip transit of radiation in the resonator calculated from expression (6) for the first highest-Q factor modes were 7.8% and 34.8%, respectively, when the uniform mirror was used, and 42.4% and 67.3%, respectively, with the amplitude-step mirror.

We neglected in calculations thermal losses and mirror coupling losses. The mirror coupling losses were measured to be 12 %, and the thermal losses on five mirrors were 10 %. Some discrepancy between theoretical and experimental results is related to the imperfection of waveguides and the measurement error of total losses, which was ± 5 %. The experimental intermode distances for the two highest-Q factor modes correspond to calculations and are equal to 102 and 85 MHz for resonators with the uniform and amplitude-step mirrors, respectively.

Figure 4 presents experimental and calculated transverse radiation distributions on the output resonator mirrors for modes with highest losses. The transverse intensity distribution for the resonator with uniform mirror (*10*) corresponds to the intensity distribution for the fundamental waveguide LP_{01} mode. When the amplitude-step mirror is used, the experimental radiation intensity distribution coincides qualitatively with that calculated for the LP_{02} waveguide mode [16]. Some difference between calculated and experimental intensity distributions is caused by the misalignment of resonator mirrors and variations in the dimensions of the glass waveguide (because of ellipticity, conicity and surface roughness), which were neglected in calculations.

4. Conclusions

We have described the method of spatial filtration for selective excitation of individual higher-order transverse modes with a high degree of discrimination of undesirable modes in a waveguide quasi-optical resonator. The method is based on the use of a mirror with absorbing or scattering inhomogeneities discretely arranged on its surface along the

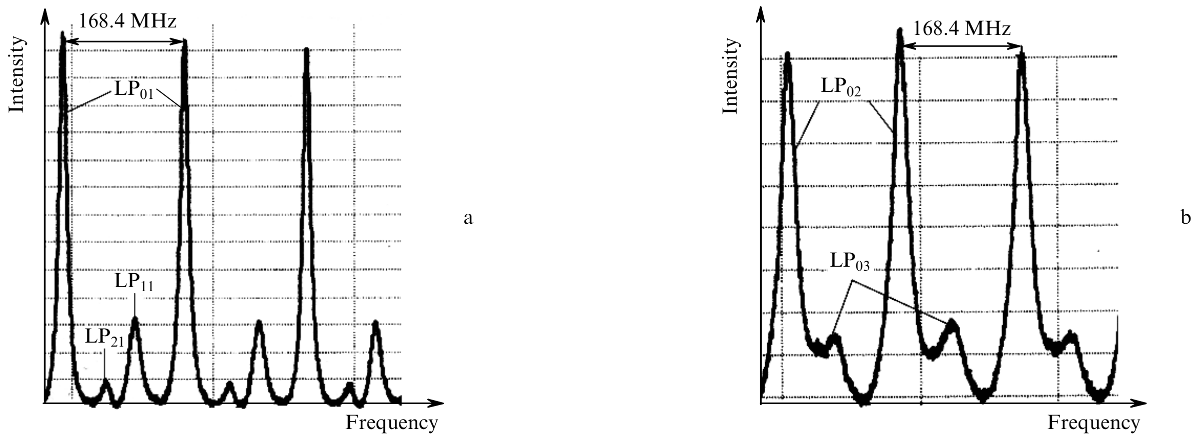

Figure 3. Mode spectra of waveguide resonators with the uniform (a) and amplitude-step (b) mirrors.

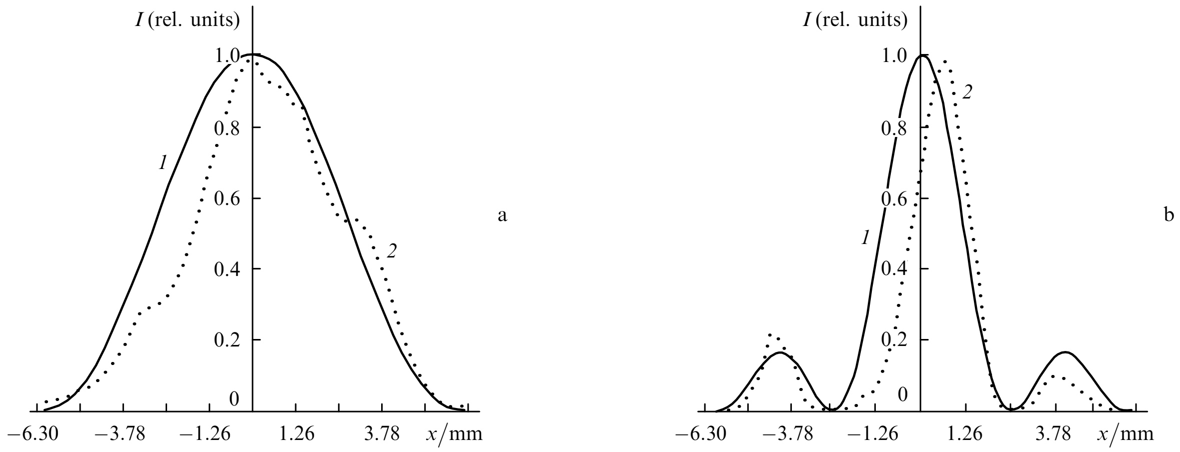

Figure 4. Calculated (*1*) and experimental (*2*) intensity distributions along the output mirror diameter for the fundamental modes of waveguide resonators with the uniform (a) and amplitude-step (b) mirrors.

node lines of the Fourier spectrum of the selected mode. The experimental studies confirmed the results of numerical calculations and showed the principal possibility of using the proposed intracavity method to generate the higher-order transverse modes in waveguide lasers. The principles behind this selection method (filtration of the spatial Fourier spectrum of waveguide modes and the use of their interference) can be used in similar laser resonance systems containing waveguide and open regions.

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