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On the transmission of gamma images in the guided modes of a crystal

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***Abstract*. It is shown that the propagation of gamma rays in crystals is accompanied by phenomena similar to those appearing in low-frequency optical and microwave waveguides, including effects of the image transmission and transformation.**

Keywords: *quantum nucleonics, waveguides, Bragg diffraction, Mössbauer radiation, Borrmann effect, resonance fluorescence, image transmission.*

1. Introduction

Although the wave properties of gamma rays have long been known, the specific radiophysical concepts and methods have been developed for this spectral range considerably less than for microwave and optical ranges. This is mainly explained by fundamental difficulties encountered in the development of coherent gamma radiation sources of the laser type (see, for example, [1]).

However, it is known that the absence of coherent gamma radiation sources cannot be an insurmountable obstacle, and the success can be achieved by using stable narrowband spontaneous radiation sources such as Mössbauer isomeric nuclei emitting zero-phonon lines with the natural linewidth. The radiation coherence length of some Mössbauer isomers (for example, $^{67\mathrm{m}}$Zn, $^{73\mathrm{m}}$Ge, $^{181\mathrm{m}}$Ta) [2] is ~ 1 km, which is quite acceptable for solving many purely radiophysical problems, for example, for manufacturing highly reflecting crystal gamma reflectors for interferometry [3], high-Q monoblock crystal resonators [4], etc.

The consideration presented below, which is based on the Bragg diffraction in crystals and the theory and applications of electromagnetic waveguides, is motivated, along with general interest, by a pragmatic quest to extend the radiophysical ideology to X-ray and gamma-ray spectral regions. The approach used in the paper can be applied not only to the propagation of gamma rays in crystals but also to any material fields with the wave properties inherent in them.

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The main attention is paid to the reproduction of the known phenomenon of electromagnetic image transmission in regular smooth waveguides [5] in the gamma-ray spectral range. This phenomenon involves the expansion of the input image, i.e. a complex electromagnetic field exciting a waveguide into a series in eigenfunctions (eigenmodes) of a waveguide problem, the propagation of separate mode waves (terms of the series) in the waveguide, and the phased summation of them in a cross section away from the input. This summation leads to the synthesis of the terms of the series and the reproduction of the input image. In this case, both the expansion in the series and synthesis occur automatically, without any additional operations (see also review [6]). The possibility of similar processes for the wave functions of cold neutral atoms in an extended quantum channel is pointed out in [7].

Before solving the problem, it is necessary to consider the peculiarities of the regular propagation of waves in crystal structures playing the role of gamma waveguides. Because the general analysis for an arbitrary Bravais lattice is rather complicated, we will demonstrate the main rules by the example of a simplest cubic lattice projected on a plane for further simplification, i.e. of a plane square lattice with the period a_0 and Bragg resonance condition

$$2a_i \sin\vartheta_i = n_i\lambda \quad (i = 0, 1, 2, \ldots, \quad n_i = 1, 2, \ldots), \tag{1}$$

where a_i is the distance between planes of the ith system and ϑ_i is the beam grazing angle. The systems of lattice planes differ in the value $a_i \leqslant a_0$ and the number of scattering nodes per unit area of the plane (for the plane lattice under study – per unit length), which decreases with increasing i and decreasing the ratio a_i/a_0 .

2. Guided gamma-wave modes propagating in a crystal

Guided gamma-waves in a crystal are travelling gamma waves with the stationary transverse structure and high longitudinal directivity. The multiple reflections of waves at angles $\pm\vartheta_i$ from the ith system of planes in a plane square lattice produce a standing wave in the transverse direction y and select the propagation direction of a group of waves parallel to atomic planes along the longitudinal axis z. This picture of the common wave field exactly reproduces the propagation of microwave radiation in metal waveguides, where multiple reflections from the walls are known to produce the longitudinal travelling and transverse standing waves, the period of the latter being determined by the distance between the walls with metal boundary conditions corresponding to the metal. The difference consists only in the form of boundary conditions, which are periodic in a single crystal and zero for the electric component on the wall of a metal waveguide.

As in a multimode microwave waveguide, Bragg condition (1) in the general case is valid for many groups of waves propagating in different directions with different grazing angles $\pm\vartheta_i$ and indices n_i, which form a set of allowed modes for a selected system of crystal planes. If the problem consists in the preservation of a small number of 'waveguide' groups (ideally, of the one lowest type with $n_0 = 1$ coupled to the ith system of planes) and in the exclusion of propagation of the rest of the set, then the wavelength range in the square lattice under study is determined by the relation

$$2a_i < \lambda < 2a_0 \quad (i = 1, 2, \ldots), \tag{2}$$

when condition (1) is fulfilled only for $n_0 = 1$ in the system of principal crystallographic planes with $i = 0$. Because in the nearest system of planes with $i = 1$ the distance a_1 is $a_0/\sqrt{2}$, inequality (2) is reduced to the form

$$\sqrt{2}a_0 < \lambda < 2a_0. \tag{3}$$

The right inequalities in (2) and (3) show that λ is smaller than the waveguide critical value for fundamental waves, while the left inequality in (3) shows that λ exceeds the critical value for any other type of the waves with $i > 0$, i.e. such a 'waveguide' is a single-mode one for radiation at the wavelength λ.

Although the waveguide should be single-mode not always, these inequalities restrict the spectral interval of radiation capable of propagating in a guided mode to the subnanometre range (thus, 0.14 nm $< \lambda <$ 0.2 nm for $a_0 =$ 0.1 nm).

The condition $n_0 = 1$ (which is not necessary but is sometimes desirable) means that only the lowest type wave with $\lambda > a_0$ propagates in the guided mode in the system of principal planes with $i = 0$. This means that λ exceeds the critical value for the $n_0 = 2$ mode. As a result, the fulfilment of double inequality (2) is favourable for the guided directional single-mode propagation of the lowest-type modes $(i = 0, n_0 = 1)$.

Along the longitudinal axis z in a crystal a set of travelling waves with phase velocities

$$v_i(n_i) = c/\cos\vartheta_i = c/[1 - (n_i\lambda/2a_i)^2]^{1/2} > c \tag{4}$$

and group velocities

$$u_i(n_i) = c\cos\vartheta_i = c[1 - (n_i\lambda/2a_i)^2]^{1/2} < c \tag{5}$$

propagates. These expressions are analogous to the corresponding formulas for usual electromagnetic waveguides, in particular, to relation $v_i u_i = c^2$. In the transverse direction along the y axis, standing waves with a period of a_i/n_i are established, which propagate without a change in the stationary transverse structure, together with the travelling wave, along the z axis. As a whole, the field of each guided mode in the crystal is a superposition of two plane waves propagating at angles $\pm\vartheta_i$ to the z axis.

Borrmann effect in a waveguide crystal. If the stationary field structure of a guided mode provides the coincidence of the nodes of the electric component of a transverse standing wave with crystal lattice sites, the Borrmann effect [8] can appear and photon losses for the total field of the guided mode produced by the standing and travelling components considerably decrease. This does not prevent the interaction of the guided mode radiation with nuclear radiative transitions if the latter have a high enough multipolarity [9, 10]. The Borrmann effect favours cleaning of the guided-mode field from photons, which can be emitted by an isotropic (for example, Mössbauer) source at grazing angles ϑ_i not equal to Bragg angle (1), and from higher-mode photons with $i \gg 1$ because their mean free paths are considerably smaller than those of 'Borrmann' photons of the lower guided modes. As a result, the photon flux, which is not captured in guided modes, decays with distance from the entrance to the crystal by the order of the mean free path of 'non-Borrmann' photons, and only the gamma-ray flux in the selected guided mode propagates in the crystal.

Splitting and combining of photon beams. The radiation field at the crystal output, which corresponds to the guided-mode field, is represented by the standard interference pattern of two plane mutually coherent waves propagating at an angle of $2\vartheta_i$ to each other. In this field the splitting of the wave occurs, which is necessary, in particular, for various interferometric experiments. Similarly, when two external waves are incident at angles $\pm\vartheta_i$ on the end of a waveguide single crystal, a pair of waves is emitted from its opposite end, each of them being a superposition of the two incident waves. This produces the mixing of both waves, which is also required for quadratic detection in interferometric experiments.

3. Resonance narrowing of a gamma line

Although the coherence length of many sources (for example, Mössbauer sources) is usually quite large, a gamma line can be considerably narrowed down to the width smaller than the natural linewidth by the nuclear resonance fluorescence (resonance scattering) during constructive gamma interference [11–16]. To realise this possibility in the case of a Mössbauer source, a waveguide crystal should consist of unexcited nuclei that are identical to emitting Mössbauer nuclei of the source or at least are enriched with them.

The rigorous quantum theory of fluorescence [17] gives substantially different results for two limiting cases of radiation linewidths, which are clearly described in classical wave language.

If the incident radiation linewidth noticeably exceeds the natural linewidth of a transition in a scattering atom, the atom is excited by a nearly delta-shaped electromagnetic pulse of duration smaller than the spontaneous decay time τ of the upper state. This state emits spontaneous radiation after the end of the exciting pulse and is free of its influence, decaying exponentially with the decay time τ and the linewidth $\Delta\omega_0 \approx 2\pi/\tau$.

If the exciting radiation linewidth is much narrower than the transition linewidth, then the primary radiation is described by a nearly monochromatic sinusoid of duration considerably exceeding the spontaneous decay time τ. Correspondingly, fluorescence proceeds under the continuous synchronising influence of the field. In this case, the secondary wave is coherent with the primary wave and their linewidths are the same.

The evolution of the fluorescence line during the resonance scattering of primary radiation with the linewidth, which is close to or even coincides with the atomic linewidth, noticeably differs from the two limiting cases. During multiple successive resonance scattering events, both the shape and width of the fluorescence line change. Thus, after N successive events of scattering of primary radiation with the Lorentzian line $f_0(\omega)$ normalised to unity with the central frequency ω_0 and width $\Delta\omega_0$ by atoms with the same transition line, the fluorescence line shape $f_N(\omega)$ and its width $\Delta\omega_N$ are described by the expressions

$$f_N(\omega) \approx \frac{(\Delta\omega_0/2)^{2(N+1)}}{[(\omega-\omega_0)^2 + (\Delta\omega_0/2)^2]^{N+1}}, \tag{6}$$

$$\Delta\omega_N \approx \Delta\omega_0 (2^{1/(N+1)} - 1)^{1/2}. \tag{7}$$

Examples $\Delta\omega_1/\Delta\omega_0 \approx 0.64$ for $N = 1$ and $\Delta\omega_3/\Delta\omega_0 \approx 0.43$ for $N = 3$ demonstrate the fluorescence line narrowing even after a small number N of scattering events.

Of course, a resonance-fluorescence gamma-ray source emitting the line of width much narrower than the natural linewidth can be only partially used instead of a nuclear laser because stimulated emission in the inverted medium is accompanied by the multiplication of photons, whereas during resonance fluorescence the primary photons are only reproduced without increasing their number. Therefore, the number of photons in a laser can be maintained or even increased, whereas the initial photon flux in the case of resonance fluorescence only decreases due to inevitable losses. Of course, both these processes require the external energy input (sometimes, with not very high efficiencies and quantum yields), either upon pumping in one case or excitation of the primary wave in the other.

The narrowing of the resonance fluorescence line can lead to the positive result when the central radiation and scattering frequencies of nuclei coincide, but it is not always applicable in the case of noticeable different frequencies. Thus, even small difference between central frequencies of radiation and scatterers leads to the shift of the central frequencies of scattered waves to the centre of the scatterer line, which sometimes can introduce the undesirable error (in particular, in interferometric problems).

4. Gamma-image transmission and transformation in a waveguide crystal

The presence of guided modes in a crystal facilitates the gamma-image transmission in a crystal, which is similar to such processes in the low-frequency range mentioned in the introduction.

Let the fields of a guided mode be excited by a gamma image projected on the input end of a single crystal. Such an image can be produced, for example, by irradiating the end surface by a monochromatic gamma-ray source through a contrast mask with a picture inscribed on it. The amplitudes and phases of partial waves excited in this case correspond to the terms of the image-field expansion as a series in the eigenmodes of the waveguide. This expansion occurs automatically, being the more complete, the greater is the number of paraxial modes propagating in the crystal ($n_0 \gg 1$), i.e. under the condition

$$\lambda/a_0 \ll 1, \tag{8}$$

which for the values of a_0 in real crystals of the order of fractions of nanometre can be fulfilled only for very short wavelengths (the latter almost completely excludes Mössbaur isotopes as possible radiation sources).

Thus, the gamma-ray field excited at the crystal input is a superposition of the fields of individual modes. Then, the fields of individual modes propagate with different phase velocities $v_i(n_i)$ along the crystal (4). For paraxial modes of a multimode waveguide [i.e. when inequality (8) is fulfilled] and for $a_i = a_0$, we have

$$v_0(n_0) \approx c[1 + (n_0\lambda/2a_0)^2/2] \quad (n_0 = 1, 2, \ldots). \tag{9}$$

In this case, the phase difference between any pairs (A and B) of such paraxial mode waves with indices $n_{0\mathrm{A}}$ and $n_{0\mathrm{B}}$ at a point z on the longitudinal axis of the waveguide crystal is

$$\Delta\varphi_{\mathrm{AB}} \approx 2\pi \frac{\lambda z}{8a_0^2}(n_{0\mathrm{B}}^2 - n_{0\mathrm{A}}^2). \tag{10}$$

Because the difference of the squares of two numbers in parentheses in (10) is an integer, the requirement that the phase difference $\Delta\varphi_{\mathrm{AB}}$ (10) would be a multiple of 2π, i.e. that $|\Delta\varphi_{\mathrm{AB}}| = 2\pi(n_{0\mathrm{B}}^2 - n_{0\mathrm{A}}^2)s$ $(s = 0, 1, 2, \ldots)$, means that the so-called in-phase cross sections with coordinates

$$z_s = 8\frac{a_0^2}{\lambda}s \quad (s = 0, 1, 2, \ldots) \tag{11}$$

exist in the crystal, where, as in usual smooth waveguides [5], all the partial mode waves have the same phases as for $z = 0$. Thus, in the cross section z_s the terms of the series are added with initial phase differences and amplitudes (the decay for different modes being approximately the same) and therefore the input gamma-image is synthesised and reproduced.

It can be shown that for special types of excitation of a waveguide crystal, except in-phase cross sections (11), there also exist additional cross sections in which, as in [5], input images are transformed (for example, multiplicated).

5. Conclusions

The sketch consideration presented in the paper has shown that gamma rays can be subjected in single crystals to various manipulations, similarly to lower-frequency microwaves and optical waves, such as propagation in guided modes, splitting and mixing of gamma-ray beams, gamma-image transmission and transformation, etc. The analysis has been performed for the simplest cubic crystal lattice projected to a plane, but simple rules established for realisation of operations are clear enough (also possibly are more complicated) for other Bravais lattices. It is also clear that similar phenomena are possible both in crystal structures of different types (photonic and ultrasonic crystals, optical potential-well gratings, etc.) and for various material waves (cooled ensembles of neutral atoms, ultracold neutrons, etc.). There is reason to hope that the development of this direction in quantum nucleonics will improve the understanding of various wave phenomena and open up new experimental possibilities.

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