PROPAGATION OF LASER RADIATION IN ATMOSPHERE

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Probability density of strong intensity fluctuations of laser radiation in a weakly absorbing random medium

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Abstract. **Based on the quasi-optic parabolic equation, we derived analytically an expression for the probability density of strong intensity fluctuations of radiation propagating in a random attenuating medium. This probability density is compared with that obtained experimentally. It is shown that the agreement between the theory and the experiment in the entire range of variations in the radiation intensity is achieved by the combined account for the effect of small random attenuation on the radiation propagation and the action of noises on the radiation receiver.**

Keywords: *propagation of laser radiation, random attenuation, strong fluctuations, probability distribution.*

1. Introduction

The problem of the propagation of laser radiation in random media is one of the most important from the point of view of applications of optical systems, in which lasers are used as coherent radiation sources for operation in real atmosphere. The presence of random inhomogeneities in the medium leads to noticeable changes in the amplitude – phase distribution of the laser beam field, whose quantity and character significantly depend on the medium parameters and the path length. Already the first experiments on the propagation of laser beams along long paths demonstrated a nonmonotonic dependence of the relative variance σ_I^2 of the intensity fluctuations on the path length: first, σ_I^2 behaves naturally, i.e. increases, and then, after achieving the maximum, decreases monotonically and tends to a certain value equal to unity [1, 2]. The variation region of the problem parameters at which the relative variance of intensity fluctuations tends to the limiting value was called the saturation region or the region of strong intensity fluctuations.

Detailed theoretical investigations of the saturation effect [3 – 5] performed within the framework of approximation of the markovian random process with the use of the stochastic parabolic equation for the complex wave amplitude showed a rather good agreement with the experimental data (see, for example, [6, 7]). However, some discrepancy between the theory and the experiment [4] was found for the highest statistical moments and the probability density of the radiation intensity. Thus, it turned out that the theoretical function of the probability distribution of intensity fluctuations $W(I)$ does not coincide with that obtained experimentally. There appeared a paradox known in the literature as logarithmically normal [8, 9]. Indeed, the results of the experimental measurements yield a distribution $W(I)$ close to logarithmically normal [6, 10 – 12], while it follows from the theory that in the saturation region, the probability distribution should asymptotically tend to Rayleigh one [5]. The attempts to solve this paradox by approximating the distribution $W(I)$ by the piecewise function [13], K distribution [14], within the heuristic model [15], with the help of other representations (see, for example, [6]), and numerical simulations of propagation of a laser beam in turbulent atmosphere (see, for example, [16 – 20]) gave, finally, unsatisfactory results because all the above distributions rather rapidly tend to the Rayleigh distribution when the path length is increased.

The authors of papers [21 – 23] proposed a new mechanism for producing intensity fluctuations of laser radiation in the saturation region, based on the account for the influence of relatively small pulsations of the imaginary component of the permittivity of a random medium on the radiation statistics, which makes it possible to solve the logarithmically normal paradox.

In this paper, we derived expressions for the statistical moments and probability density of the laser radiation intensity propagating in a random medium with fluctuations of both real and relatively small imaginary part of the permittivity. The obtained probability distributions are compared with the experimental data in the entire range of variations in the radiation intensity.

2. Formulation of the problem

We will consider the propagation of a laser beam in a random medium with the fluctuations of the complex permittivity $\tilde{\varepsilon} = \tilde{\varepsilon}_R + i\tilde{\varepsilon}_I$ (where $\tilde{\varepsilon}_R$ and $\tilde{\varepsilon}_I$ are the real and imaginary parts of random variations in the permittivitys) at a noticeably long path for which the saturation condition of radiation intensity fluctuations

$$\beta_{RR}^2 = 0.31 C_{RR}^2 k^{7/6} z^{11/6} \gg 1 \quad (1)$$

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is fulfilled. Here, $\beta_{\rm RR}^2$ is the relative variance of intensity fluctuations of a plane wave in the approximation of the method of smooth perturbations for a transparent turbulent medium; z is the path length; k is the wave number; $C_{\rm RR}^2$ is the structural characteristic of fluctuations of the real component of the permittivity. In inequality (1), the expression for $\beta_{\rm RR}^2$ corresponds to the case of radiation propagation in a random medium with the Kolmogorov fluctuation spectrum of the permittivity (see, for example, [3]).

Let us obtain the expression for the probability distribution function of strong (saturated) intensity fluctuations of laser radiation during its propagation in a weakly absorbing random medium in which pulsation of the real part of the permittivity significantly exceeds its imaginary component: $\langle|\tilde{\varepsilon}_{\rm R}|\rangle \gg \langle|\tilde{\varepsilon}_{\rm I}|\rangle$. We will calculate the probability density of intensity fluctuations using the relation

$$W(I) = \frac{1}{2\pi}\int_{-\infty}^{\infty} {\rm d}\Omega F_I(\Omega)\exp(-{\rm i}\Omega I), \tag{2}$$

coupling $W(I)$ with the characteristic function $F_I(\Omega)$, which, in turn, is completely determined by the statistical moments $\langle I^N\rangle$ of the radiation intensity:

$$F_I(\Omega) = \sum_{N=0}^{\infty}\frac{({\rm i}\Omega)^N}{N!}\langle I^N\rangle. \tag{3}$$

Relations (2), (3) follow directly from the definition of the characteristic function (see, for example, [24]). According to these relations, the distribution of the intensity probabilities can be reconstructed if we know all the statistical radiation intensity moments.

3. Statistical intensity moments

In the general form, the expression for the Nth intensity moment within the framework of approximations of quasi-optics and the markovian random process (see, for example, [3]) for the fluctuations of the complex permittivity with the use of the complex wave amplitude in the form of the Huygens–Kirchhoff integral, as well as with the use of the Feynman integral over the trajectories for the Green function of the parabolic equation [5, 23] during the radiation propagation in a weakly absorbing random medium can be written in the form:

$$\langle I^N\rangle(\boldsymbol{R})\rangle = \langle I(\boldsymbol{R})\rangle^N \exp\left[\frac{N(N-1)}{2}\,\sigma_\tau{}^2(z)\right]$$
$$\times\hat{L}^{(N)}\exp\left[-\frac{k^2}{8}\int_0^z {\rm d}\xi\psi'_{NN}(\xi)\right], \tag{4}$$

where

$$\hat{L}^{(N)} = \left(\frac{c}{8\pi}\right)^N \frac{\exp\left[N\sigma_\tau{}^2(z)/2\right]}{\langle I(\boldsymbol{R})\rangle^N}\prod_{j=1}^N\iint {\rm d}^2\rho'_{2j-1}{\rm d}^2\rho'_{2j}$$
$$\times U_0(\boldsymbol{R}'_{2j-1})U_0^*(\boldsymbol{R}'_{2j})G_0(\boldsymbol{R},\boldsymbol{R}'_{2j-1})G_0^*(\boldsymbol{R},\boldsymbol{R}'_{2j})$$
$$\times|C|^{2N}\prod_{j=1}^N\iint D^2v_{2j-1}(\xi)D^2v_{2j}(\xi)\times$$
$$\times\exp\left\{\frac{{\rm i}k}{2}\int_0^z{\rm d}\xi\left[\dot{v}^2_{2j-1}(\xi)-\dot{v}^2_{2j}(\xi)\right]-\frac{k^2}{4}\int_0^z{\rm d}\xi\tilde{\psi}_{NN}(\xi)\right\} \tag{5}$$

is the integral operator at $\hat{L}^{(N)}\cdot 1$; $U_0(\boldsymbol{R}'_j)$ is the complex wave amplitude at the medium input (at $z=0$); $\boldsymbol{R}'_j = \{\boldsymbol{\rho}'_j, 0\}$ is the radius vector in the source plane; $\boldsymbol{R} = \{\boldsymbol{\rho}, z\}$ is the radius vector of the observation point; C is the normalisation constant; $\int D^2v(\xi)$ means the functional integration over all the trajectories $\boldsymbol{v}(\xi)$ starting at point $(\boldsymbol{\rho}',0)$ and finishing at point $(\boldsymbol{\rho},z)$; $\dot{\boldsymbol{v}} = {\rm d}\boldsymbol{v}/{\rm d}\xi$; $G_0(\boldsymbol{R},\boldsymbol{R}') = [k/(2\pi{\rm i}z)]\exp({\rm i}k|\boldsymbol{\rho}-\boldsymbol{\rho}'|^2/z)$ is the Green function of the parabolic equation for the complex wave amplitude in a medium without fluctuations of the permittivity;

$$\tilde{\psi}_{NN}(\xi) = \sum_{j=1}^N D_+\big(\boldsymbol{\rho}^{(0)}_{2j-1}(\xi)-\boldsymbol{\rho}^{(0)}_{2j}(\xi)+\boldsymbol{v}_{2j-1}(\xi)-\boldsymbol{v}_{2j}(\xi)\big);$$
$$\psi'_{NN}(\xi) = \sum_{j=1}^N\sum_{l=1}^N\big[2(1-\delta_{jl})D_-\big(\boldsymbol{\rho}^{(0)}_{2j-1}(\xi)-\boldsymbol{\rho}^{(0)}_{2l}(\xi)$$
$$+\boldsymbol{v}_{2j-1}(\xi)-\boldsymbol{v}_{2l}(\xi)\big)-D_-\big(\boldsymbol{\rho}^{(0)}_{2j-1}(\xi)-\boldsymbol{\rho}^{(0)}_{2l-1}(\xi)+\boldsymbol{v}_{2j-1}(\xi)$$
$$-\boldsymbol{v}_{2l-1}(\xi)\big)-D_-\big(\boldsymbol{\rho}^{(0)}_{2j}(\xi)-\boldsymbol{\rho}^{(0)}_{2l}(\xi)+\boldsymbol{v}_{2j}(\xi)-\boldsymbol{v}_{2l}(\xi)\big)\big]$$
$$+2{\rm i}\sum_{j=1}^N\sum_{l=1}^N\big[D_{\rm RI}\big(\boldsymbol{\rho}^{(0)}_{2j}(\xi)-\boldsymbol{\rho}^{(0)}_{2l}(\xi)+\boldsymbol{v}_{2j}(\xi)-\boldsymbol{v}_{2l}(\xi)\big)$$
$$-D_{\rm RI}\big(\boldsymbol{\rho}^{(0)}_{2j-1}(\xi)-\boldsymbol{\rho}^{(0)}_{2l-1}(\xi)+\boldsymbol{v}_{2j-1}(\xi)-\boldsymbol{v}_{2l-1}(\xi)\big)\big]$$
$$+4\sum_{j\neq1}^N\sum_{l=1}^N D_{\rm II}\big(\boldsymbol{\rho}^{(0)}_{2j-1}(\xi)-\boldsymbol{\rho}^{(0)}_{2l}(\xi)+\boldsymbol{v}_{2j-1}(\xi)-\boldsymbol{v}_{2l}(\xi)\big); \tag{6}$$

$\boldsymbol{\rho}^{(0)}_j = [\boldsymbol{\rho}\xi+\boldsymbol{\rho}'_j(z-\xi)]/z$; δ_{ij} is the Kronecker delta; $D_{\alpha\alpha'}(\boldsymbol{\rho}) = A_{\alpha\alpha'}(0)-A_{\alpha\alpha'}(\boldsymbol{\rho})$ are the structural fluctuation functions of the real ($\alpha=\alpha'={\rm R}$) and imaginary ($\alpha=\alpha'={\rm I}$) parts of the permittivity and their correlations ($\alpha={\rm R}$, $\alpha'={\rm I}$); $\Phi_{\alpha\alpha'}(\boldsymbol{q})$ are their corresponding spectra; $A_{\alpha\alpha'}(\boldsymbol{\rho}) = 2\pi\int{\rm d}^2q\Phi_{\alpha\alpha'}(\boldsymbol{q})\cos(\boldsymbol{q\rho})$; $\sigma_\tau{}^2(z) = k^2\times\ A_{\rm II}(0)z$ is the mean square of the fluctuations of the optical path thickness τ of length z; $D_\pm(\boldsymbol{\rho}) = D_{\rm RR}(\boldsymbol{\rho})\pm D_{\rm II}(\boldsymbol{\rho})$.

By generalising the results [23] to the case of calculations of the highest statistical moments of the radiation intensity and using the asymptotic method [5, 6] for the analysis of strong intensity fluctuations and the 'cumulant' method for calculating the integrals [25], we obtain from (4) the expression

$$\langle I^N(\boldsymbol{R})\rangle = N!\langle I(\boldsymbol{R})\rangle^N\exp\left[\frac{N(N-1)}{2}\,\gamma(\boldsymbol{R})\right] \tag{7}$$

for the Nth intensity moment, where

$$\gamma(\boldsymbol{R}) = k^2A_{\rm II}(0)z+K_1^{(2,2)}(\boldsymbol{R}) = \sigma_\tau^2(z)+K_1^{(2,2)}(\boldsymbol{R}); \tag{8}$$
$$K_1^{(2,2)}(\boldsymbol{R}) = -\frac{k^2}{8}\,\hat{L}^{(2)}\int_0^z{\rm d}\xi\psi'_{2,2}(\xi)$$

is the first order 'cumulant' (see, for example, [23]). One can see from the definition (5) of the integral operator $\hat{L}^{(N)}$ that the 'cumulant' $K_1^{(2,2)}$ depends on the distribution $U_0(\boldsymbol{R}')$ of the complex amplitude of the radiation field at the medium input. Because in most experimental papers studying the statistics of strong fluctuations of the laser radiation intensity there were fulfilled the conditions for the propagation of a laser beam in the plane wave regime, we present the expression for $K_1^{(2,2)}$, corresponding to this case:

$$K_1^{(2,2)}(\boldsymbol{R}) = \pi k^2 z \int_0^1 \mathrm{d}\xi \iint \mathrm{d}^2 q \left\{ \Phi_{-}(\boldsymbol{q}) \left[1 - \cos\left(\frac{q^2 z}{k} \xi \right) \right] - 2\Phi_{\mathrm{RI}}(\boldsymbol{q}) \sin\left(\frac{q^2 z}{k} \xi \right) \right\} \exp\left[-\frac{k^2 z}{2} D_{+}\left(\frac{\boldsymbol{q} z}{k} \xi \right) \right] \quad (9)$$

$$- 2\pi k^2 z \int_0^1 \mathrm{d}\xi \iint \mathrm{d}^2 q \Phi_{\mathrm{II}}(\boldsymbol{q}) \left\{ 1 - \exp\left[-\frac{k^2 z}{2} D_{+}\left(\frac{\boldsymbol{q} z}{k} \xi \right) \right] \right\},$$

where $\Phi_{-}(\boldsymbol{q}) = \Phi_{\mathrm{RR}}(\boldsymbol{q}) - \Phi_{\mathrm{II}}(\boldsymbol{q})$.

Expression (7) together with the expression for the 'cumulant' $K_1^{(2,2)}$ reflects the main features in the behaviour of the statistical intensity moments of laser radiation propagating in a weakly absorbing medium along a noticeably long path. Its derivation is based on the fact that in the region of strong intensity fluctuations $[\beta_{\mathrm{RR}}(z) \gg 1]$, where the effects of multiple radiation scattering on the inhomogeneities of the permittivity of the medium are significant, the correlation function of the intensity fluctuations is characterised by two length scales. The first – the radius of the wave coherence ρ_{c} – allocates the region in which the largest correlation of the intensity fluctuations is achieved. The second – $r_{\mathrm{c}}/(k\rho_{\mathrm{c}})$ (in the case under study, $r_{\mathrm{c}}/\rho_{\mathrm{c}} = z/(k\rho_{\mathrm{c}}^2) \sim \beta_{\mathrm{RR}}^{6/5} \gg 1$) – determines the behaviour of the correlation function in the region of large scales (the 'tail' of the correlation function). As a result, the complex wave amplitude represents a superposition of the fields obeying the Rayleigh and logarithmically normal statistics. The Rayleigh component is caused by the multiple scattering of the waves on the turbulent vortices whose scales do not exceed r_{c}. The logarithmically normal component appears due to scattering of the Rayleigh component on the vortices of the atmospheric turbulence whose scales lie in the region $r_{\mathrm{c}} \ll r \ll L_0$, where L_0 is the external scale of the turbulence. Fluctuations of the imaginary part of the permittivity, as follows from (7) and (9), produce a dual influence on the intensity moments. On the one hand, the Bouguer factor $\exp\sigma_\tau^2$ is determined by the contribution of the entire fluctuation spectrum of the imaginary part of the permittivity. In this case, the main contribution is made by large-scale, of the order of the external scale L_0, turbulent vortices. On the other hand, the attenuation fluctuations corresponding to the inertial and viscous intervals of the turbulence also affect the quantity $\langle I^N \rangle$ through the structural functions D_{II} and D_{RI}. All this leads to the fact that the change in the wave amplitude due to the multiple scattering of radiation on the turbulent vortices whose scales lie in the range $l_0 \ll r \ll r_{\mathrm{c}}$ (where l_0 is the internal scale of the turbulence) does not obey the Bouguer law. In addition, the correlation effects determining D_{RI} make a contribution to the instant $\langle I^N \rangle$. Therefore, due to the presence of random attenuation, the dependence of the radiation intensity moments in the saturation region (due to competition of two mechanisms) on the problem parameters will have an essentially different character for different $\tilde{q}_0 = r_{\mathrm{c}}/L_0$. This circumstance, in particular, explains the difference in the applicability conditions of expressions (7), which have the form

$$|\gamma(R) - k^2 A_{\mathrm{II}}(0) z| \ll 1 \text{ at } \tilde{q}_0 < 1,$$

$$|\gamma(R)| \ll 1 \text{ at } \tilde{q}_0 \gg 1.$$

Note that the dependence of type (7) for the statistic moments of the radiation intensity on the moment order N in the case of the multiple scattering of radiation in a transparent random medium was first established in paper [26] while elucidating the applicability conditions of the K distribution for the description of the distribution function of strong intensity fluctuations. Independently of paper [26], the expression of type (7) was derived based on the variational principle in [21] while studying the probability distributions of strong intensity fluctuations in a weakly absorbing random medium. However, only the use of the 'cumulant' method developed in [25] makes it possible to calculate successively the statistical moments with the required accuracy. This was demonstrated in paper [23] analysing the variance of strong intensity fluctuations as well as in this paper while deriving relation (7).

To compare the results obtained with the help of expression (7) with the measurements of the statistical intensity moments of laser radiation propagating in the turbulent atmosphere, Fig. 1 shows the dependences $J_N(J_2)$ of the normalised moments $J_N = \langle I^N \rangle / \langle I \rangle^N$ calculated by the expression

$$J_N = N!(J_2/2)^{N(N-1)/2}, \quad (10)$$

which follows from (7), as well as the similar dependences corresponding to the experimental data from papers [27–29]. First of all, Fig. 1 demonstrates good agreement of the calculated and empirical data for the third moment of the radiation intensity. One can also see that as the order of the statistical moment is increased, the discrepancy between the theory and the experiment, and between the experimental data of different authors, increases. The authors of papers [30, 31] (see also [6]) showed that the reason for this discrepancy is the limited region of the experimentally

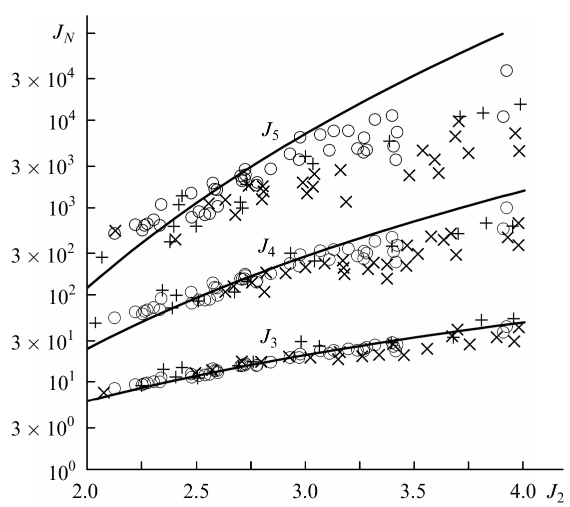

Figure 1. Dependences of the Nth moment of the laser radiation intensity on the second moment. The points are the experiment [27] (+), [28] (×), and [29] (○). The solid curves are calculated by using expression (10).

detected values of the radiation intensity, which leads to an increase in the errors of the moment measurements with increasing N. As a result, we observe, on the one hand, a systematic understatement of the measurement results with increasing J_2 compared to the probability moments, and, on the other hand, their overstatement at $J_2 \leqslant 2$. The authors of the same papers established that to compare correctly the theoretical and experimental results, it is needed to use truncated probability moments calculated by integrating over the finite intervals in the range from the minimal to the maximal intensity values in the experimental realisation under study.

4. Probability distribution function

Let us find now the density of probability distributions of the radiation intensity fluctuations. By using expression (7) for $\langle I^N \rangle$ and the relation of characteristic function (2) with distribution function (1), we obtain

$$W(I) = \frac{\exp\gamma(\boldsymbol{R})}{\sqrt{2\pi\gamma(\boldsymbol{R})}\langle I(\boldsymbol{R})\rangle} \times \int \mathrm{d}\tau \exp\left\{-\frac{\tau^2}{2\gamma(\boldsymbol{R})} - \frac{I}{\langle I(\boldsymbol{R})\rangle}\exp\left[\frac{3}{2}\gamma(\boldsymbol{R}) + \tau\right]\right\} \quad (11)$$

for the case $\gamma > 0$. Note that the expression similar to (1) was derived in paper [32] within the framework of the heuristic model by neglecting the fluctuations of the radiation attenuation and in paper [21] by taking into account the random attenuation of radiation to solve the logarithmically normal paradox. The numerical simulation [17, 20] of the laser beam propagation in a transparent turbulent medium gives a probability distribution of strong intensity fluctuations of radiation, similar to (11).

One can easily see that distribution (11) (at $\gamma > 0$) satisfies the necessary requirements imposed on the probability density. The obtained distribution at any nonnegative I is positive, represents a real function, and is normalised to unity.

However, when $\gamma < 0$, the requirement to the nonnegativity for $W(I)$ is violated. The authors of paper [23] showed that at some values of the problem parameters, it is possible to realise the conditions under which the relative variance σ_I^2 of strong intensity fluctuations of radiation in a random absorbing medium can take values smaller than unity: $\sigma_I^{2<1}$. In this case, due to a substantial manifestation of compensation effects caused by the correlation $\tilde{\varepsilon}_{\mathrm{R}}$ and $\tilde{\varepsilon}_{\mathrm{I}}$ the function $\gamma(\boldsymbol{R})$ takes negative values. The reconstruction, according to (1), (2), and (7), of the distribution function of strong fluctuations at $\gamma = -\Gamma < 0$ yields for it the following integral representation:

$$W(I) = \frac{\exp(\Gamma/2)}{\sqrt{2\pi\Gamma}\langle I\rangle}\int \mathrm{d}\tau \exp\left(-\frac{\tau^2}{2\Gamma} - \frac{I}{\langle I\rangle}\,\mathrm{e}^{\Gamma/2}\cos\tau\right) \times \cos\left(\tau - \frac{I}{\langle I\rangle}\,\mathrm{e}^{\Gamma/2}\sin\tau\right). \quad (12)$$

The impracticability of the condition $W(I) \geqslant 0$ at some values of I can be found in this case from expression (7) for $\langle I^N \rangle$. Indeed, apart from the nonnegativity of σ_I^2, a more general nonnegativity condition for the quantity $S_I^{2N} = \langle (I^N/\langle I^N\rangle - 1)^2\rangle$ should be fulfilled, which, at $N = 1$, coincides with the relative fluctuation variance of the random quantity ($S_I^2 = \sigma_I^2$). As follows from relations (7), at any negative γ starting with some $N = N^{(-)}$, the condition $S_I^{(2N)} \geqslant 0$ is not fulfilled. The violation of this condition means that at the given values of the argument (the smaller the larger $|\gamma|$, and vice versa), the distribution function can take any negative values. Therefore, at $\gamma < 0$, the approximation of the first 'cumulants' used in deriving expression (7) for the statistical moments of the radiation intensity is not sufficient during the reconstruction of the distribution function. (Note that this problem also takes place in the probability theory when analysing non-Gaussian random processes with the help of a limited set of true (probable) cumulants [24].) Finishing the discussion of the case $\gamma < 0$, note that distribution function (12) takes negative values (has the oscillating character near zero) in the region of the intensity spikes whose probability is extremely small, while in the significant domain of definition it is positive in this case as well.

Figure 2 demonstrates the behaviour of the distrubution function of the intensity probabilities in the case of strong fluctuations at different parameters γ. One can see that as the parameter γ is increased, the probability of the so-called signal fading and its outliers also increases. At the same time, in some range (0.5, 3) of the values of the normalised radiation intensity, the distribution function decreases with increasing γ.

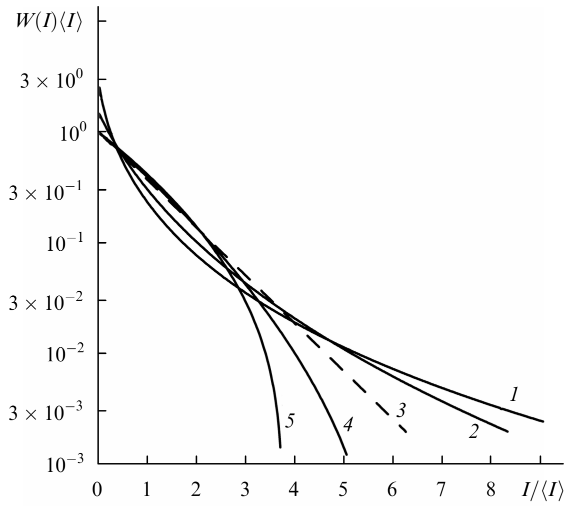

Figure 2. The behaviour of the normalised density of the probability distributions of the strong intensity fluctuations of radiation at $\gamma = 1.0$ (*1*), 0.4 (*2*), $\gamma \to 0$ (*3*), $\gamma = -0.1$ (*4*), and -0.2 (*5*).

Let us analyse now distribution (12) corresponding to the case $\gamma(\boldsymbol{R}) > 0$. First of all, we will mention some features of the asymtotic behaviour of the function $W(I)$ at different γ. Thus, at $\gamma \to 0$, the function $W(I)$ tends to the Rayleigh exponential distribution. In the other case, when $\gamma \gg 1$ (formally, at $\gamma \to \infty$ and as show the results of the numerical experiment, already at $\gamma \geqslant 3$), distribution (12) at $I\exp(\gamma/2)/\langle I\rangle > 1$ is close to logarithmically normal. The description of distribution (12) in a broad range of variations in the intensity and the parameter γ is given with a good accuracy by the expression:

$$W(I) \approx \frac{e^{\gamma}}{\sqrt{1+T(J)}} \exp\left[-\frac{T^2(J)+2T(J)}{2\gamma}\right], \quad (13)$$

where $T(J)$ is the root of the equation $T = \gamma J \exp(-T)$; $J = I\exp(^3/_2\gamma)/\langle I\rangle$. Expression (13) is obtained calculating the integral in distribution (11) by the saddle point method.

We will compare distribution (11) with some experimental distribution functions obtained while investigatng the propagation of a laser beam in the turbulent atmosphere [10, 11]. In the experiments under study, propagation of the laser beam was studied in the plane-wave regime under the saturation conditions of the intensity fluctuations. In this case, the dependence of the parameter γ on the problem conditions is determined by the expression [23]:

$$\gamma(z) = k^2 A_{\rm II}(0)z + 0.43\beta_{\rm RR}^{-4/5}(z). \quad (14)$$

Figure 3 shows the calculation results of the function $W(I)$ obtained using expression (11) and processed experimental data from papers [10, 11].

Figure 3 illustrates the consistency of the theoretical values of distribution function (11) with the experiemntal data [10].

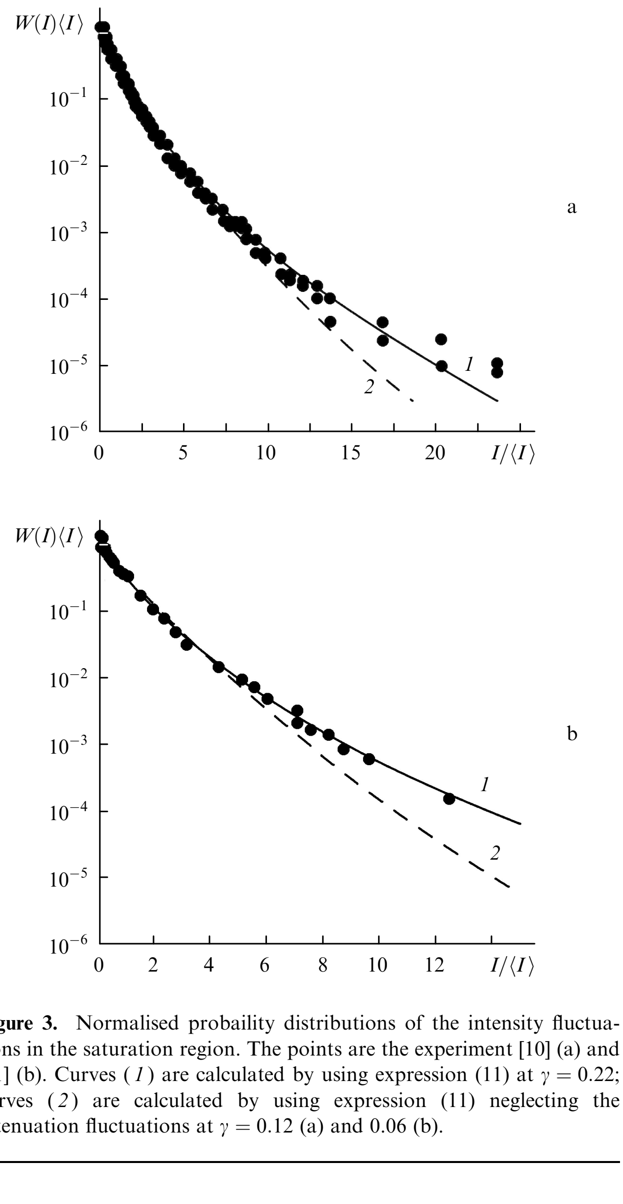

Figure 3. Normalised probaility distributions of the intensity fluctuations in the saturation region. The points are the experiment [10] (a) and [11] (b). Curves (*1*) are calculated by using expression (11) at $\gamma = 0.22$; curves (*2*) are calculated by using expression (11) neglecting the attenuation fluctuations at $\gamma = 0.12$ (a) and 0.06 (b).

The experiment [10] studied the propagation of the laser beam in the plane-wave regime at a path of length $z =$ 1.75 km at $\beta_{\rm RR} = 5$, $k \approx 10^5$ cm^{-1} and the square of the relative variance of the intensity fluctuations $\sigma_I^2 = 1.46$. One can see from this figure that the theoretical distribution with the parameter $\gamma = 0.22$ [curve (*1*)] well describes the experimental data at the entire measurement domain, while curve (*2*) plotted by neglecting the influence on propagation of random attenuation radiation describes them much worse. For the conditions of the experiment under study, the contribution to the statistics of the laser radiation intensity of the refractive index fluctuations and the absorption coefficient is approximately the same: $0.43\beta_{\rm RR}^{-4/5}(z) = 0.12$ and, in accordance with expression (14), $k^2A_{\rm II}(0)z = 0.10$. Using the latter relation, we obtain that the parameter $A_{\rm II}(0) = 5.7 \times 10^{-17}$ cm corresponds to the experimental conditions of paper [10].

The agreement of distribution (11) with the experimental data borrowed from paper [11] is shown in Fig. 3b. The experiment [11] studied the propagation of a laser beam in the plane-wave regime at a natural path of length $z =$ 2.5 km at $\beta_{\rm RR} = 11.5$ and $k = 10^5$ cm^{-1}. The best fit of the expreimental and theoretical curves is achieved as in the experiment [10] at $\gamma = 0.22$, to which $A_{\rm II}(0) = 6.4 \times 10^{-17}$ cm corresponds. Unlike paper [10], in the conditions of the experiment [11], the dominating contribution to the radiation intensity fluctuations gave randon changes in the attenuation because in this case, the contribution $0.43\beta_{\rm RR}^{-4/5}(z) = 0.06$ approximately three times smaller than the contribution $k^2A_{\rm II}(0)z = 0.16$. One can see from Fig. 3b that curve (*1*) plotted taking into account the attenuation fluctuations describes the entire set of the expreimental data [11], while curve (*2*) plotted by neglecting the attenuation fluctuations is in agreement with the measurement results only at a limited interval of the intensities.

As we have already mentioned, in theoretical investigations of the laser beam propagation in the turbulent atmosphere, the influence on the radiation propagation of attenuation fluctuations was traditionally neglected due to their relative smallness. Therefore, no special investigations of either the mechanism of their appearance or their quantities have been performed. At the same time, the performed analysis showed that the probability distribution of strong intenisty fluctuations is very sensitive to the radiation stochastization caused by the relatively weak pulsations of the imaginary part of the permittivity. The measurement of the function $W(I)$ allows one to obtain only the integral characteristic of random variations in the imaginary part of the permittivity – the parameter $A_{\rm II}(0)$. The characteristics of the random field $\tilde{\varepsilon}_I$ can be studied in detail by using the following combined approach. First of all, studying the phase fluctuations of the wave reflected by the phase-conjugate mirror (whose properties are considered in [25]), we can establish the principles of the behaviour of the fluctuations in the imaginary part of the permittivity in the region of high spatial frequencies. The agreement of these principles with the behaviour of the attenuation fluctuations in the low-frequency region can be achieved by the following investigation of the radiation statistics in the region of strong intensity fluctuations. Based on the results of these measurements, we can make certain conclusions both about the quantity of the parameters characterising randon attenuation variations in the turbulent atmosphere as a whole and about the most probable reason for their appearance.

5. Probability distribution function in the region of the signal fading

In conclusion, we will consider one more important issue concerning directly the topic of the article. Different models of the distribution function of the intensity probabilities in the theory of the wave propagation in the turbulent atmosphere were constructed based on the requirements for the best description of the experimental data in the region of large values of the radiation intensity ($I \geqslant \langle I \rangle$). The authors of paper [11] paid attention to the fact that the significant discrepancy between the experimental and theoretical distribution functions is also observed in the region of the signal fading at the arguments of the function $W(I)$ much smaller that the average intensity. For the correct interpretation of the measurment results at $I \ll \langle I \rangle$, it is necessary to take into account that in the general case, the signal registered by the detecor contains noises whose equivalent average intensity is significantly lower than the average intensity of the laser beam under study. Therefore, the model distributions [in particular, distribution (11) as well] obtained using the assumption about the fact that the solely reason of the detected radiation stochastization is its interaction with the atmospheric turbulence, cannot correctly describe the measured probabilty distributions in the entire region of the intensities. The significant influence of the noises on the behaviour of the experimental distribution function was demonstrated in paper [29].

To describe adequately the distribution function of the intensity fluctuations of the measured signal in the fading region, we assume that the radiation intensity I_s detected by the receiver is a sum of uncorrelated intenisties of the studied wave I and noise I_n ($I_s = I + I_n$), the average noise intensity being significantly smaller than the average signal intensity: $\delta_n = \langle I_n \rangle / \langle I \rangle \ll 1$. The distribution function of the probabilities of the detected radiation $W_s(I_s)$ will represent a convolution (see, for example, [33])

$$W_s(I_s) = \int_0^{I_s} dI' W_n(I_s - I') W(I') \tag{15}$$

of probability distributions $W_n(I_n)$ and $W(I)$ of the noise and the wave intensities, respectively.

Consider, as an example, the case when the noise intensity obeys the exponential Rayleigh distribution law:

$$W_n(I_n) = \frac{1}{\langle I_n \rangle} \exp\left(-\frac{I_n}{\langle I_n \rangle}\right). \tag{16}$$

Figure 4 demonstrates the results of calculations of integral (15) with the distributions of intensity probabilities (11) and (16) of the studied radiation and noise, respectively, for different parameters δ_n and γ. One can see that the distribution $W_s(I_s)$ is nonmonotonic: increasing at the initial interval of I_s variations, at $I_s \approx \langle I_n \rangle$ it achieves a maximum after which it decreases and at $I_s \gg \langle I_n \rangle$ tends to the wave intensity distribution under study.

The comparison of the results of the experiment [11] and calculations by expressions (15), (11), (16) in the entire range of variations is shown in Fig. 5. One can see that the theoretical curve describes to the best advantage the experimental curves at $0.05 < \delta_n \leqslant 0.1$. For more exact consistency of the theoretical and experimental distribution functions of the intensity probabilites of the detected signal in the region of the signal fading, it is necessary to use the noise distribution corrresponding to the conditions of the specific measurements. The example under consideration shows that the nonmonotonic dependence of the function $W_s(I_s)$ observed in the experimetns is caused by the action of radiation noises on the detector and in no way is related to the character of the radiation intensity fluctuations resulting from the medium turbulence.

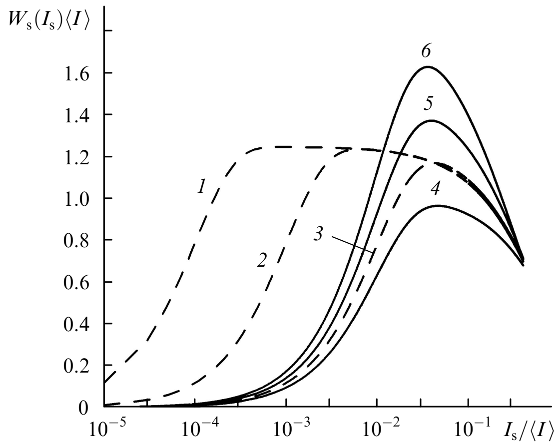

Figure 4. Normalised probaility distributions of the detectected signal intensity at $\delta_n = 10^{-4}$ (*1*), 10^{-3} (*2*), 10^{-2} (*3–6*), and $\gamma = 0.22$ (*1–3*), 0.01 (*4*), 0.4 (*5*), and 0.6 (*6*).

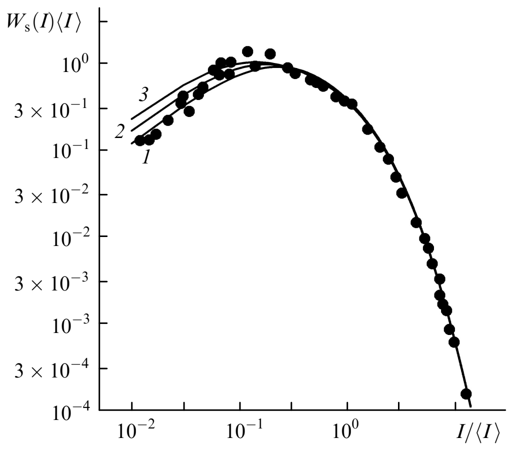

Figure 5. Comparison of the probability distribution function of radiation intensity fluctuations calculated by expression (11) at $\gamma = 0.22$ and obtained experimentally in the entire range of measurements for $\delta_n = 0.1$ (*1*), 0.07 (*2*), and 0.05 (*3*). The points are the experiment [11].

6. Conclusions

We have studied theoretically the probability distribution of strong intensity fluctuations of laser radiation propagating in a random attenuating medium. The use of the 'cumulant' method makes it possible to take into account multiple scattering effects of the Rayleigh radiation component and to obtain analytically, based on the quasi-potic parabolic equation, the distribution function of intensity probabilities allowing for the absorption fluctuations. The comparison of this distribution fucntion with that obtained in the experiment has shown that the consistency of the theory and the experiment is achieved by taking into account the influence of the small randon attenuation on the radiation propagation. The neglect of this influence leads to a noticeable difference in the theoretical results and the experimental data. We have shown in addition that the reason for the nonmonotonic distribution dependence of the strong intensity fluctuations observed experimentally is the action of the radiation noises on the detector. The total effect of two factors on the singal being measured – the small random attenuation and noises – yields an experimental distribution close to the logarithmically normal, while the theoretical simulation of the laser beam propagation in the turbulent medium by neglecting these factors leads to noticeably different distributions which tend asymptotically to the Rayleigh distribution. We have also shown that the combined allowance for the fluctuation influence of the imaginary part of the permittivity of the turbulent medium on the radiation propagation and the action of noises on the radiation detector during its registration makes it possible to give a reasonable explanation of the logarithmically normal paradox.

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