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Model of the radial gas-temperature distribution in a copper bromide vapour laser

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***Abstract*. An analytic model is proposed to calculate the buffer-gas temperature in the discharge-tube cross section of the copper bromide vapour laser. The model is the generalisation of the previous models developed by the authors. Assuming that the volume electric power is arbitrary distributed over the tube radius, the general solution of the quasi-stationary heat conduction equation with the boundary conditions of the first and second kinds is presented. Application of the model is considered by the example of a copper bromide vapour laser emitting at 510.6 and 578.2 nm at different specific radial distributions of the volume power. The obtained results are compared with the temperature profiles known to date. Application of this model to molecular lasers is also discussed.**

Keywords: *heat conduction equation, general solution, copper bromide vapour laser*.

1. Introduction

One of the main problems in designing new metal and metal compound vapour lasers consists in preliminary determination of the temperature regime of the active medium. The buffer-gas temperature distribution in the active volume is an important characteristic of a laser. The temperature influences the laser tube service life, the decrease in the laser output power in time, and the laser beam quality. It is well known that at a temperature exceeding the optimal one, there appear thermoionisation processes leading to a drastic increase in the current density at the tube centre. The gas discharge is compressed to a narrow filament, which results in a decrease in the laser generation power and deterioration of the mode composition of laser radiation. Often a black spot appears at the laser beam centre.

Until now the gas temperature in the active medium of the laser was determined analytically by using two expressions [1, 2]:

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$$T_{\mathrm{g}}(r)=\left[T_1^{m+1}+\frac{q_v(m+1)}{4\lambda_0}(R_1^2-r^2)\right]^{1/(m+1)}, \tag{1}$$

$$T_{\mathrm{g}}(r)=\left\{T_1^{m+1}+\frac{(m+1)Q_1}{\lambda_0}\left[\frac{a}{4}(R_1^2-r^2)+\frac{B}{9}(R_1^3-r^3)\right.\right.$$
$$\left.\left.+\frac{C}{16}(R_1^4-r^4)+\frac{D}{25}(R_1^5-r^5)\right]\right\}^{\frac{1}{m+1}}, \tag{2}$$

where R_1 is the tube radius; $0\leqslant r<R_1$; T_1 is the wall temperature of the quartz tube; q_v is the volume electric power released in the active medium (in W m^{-3}); λ_0 and m are the constants dependent on the gas; Q_1 is the effective deposited power (taking losses into account).

Expression (1) was derived in solving the heat conduction equation by assuming that the volume power q_v is constant, i.e., $q_v=\mathrm{const}$ in the entire active volume of the tube. This simplified expression was used in many papers to calculate the gas temperature in the kinetic models of different lasers when specifying the constant temperature at the tube wall [3 – 10]. The case of nonlinear boundary conditions is considered in [11].

Expression (2) was obtained in the case when the volume electric power in the tube was varied by the law

$$q_v(r)=Q_1\left[J_0\left(\frac{2.4r}{R_1}\right)\right]^2, \tag{3}$$

where $J_0(2.4r/R_1)$ is the zero-order Bessel function of the first kind. Because this special function is difficult to operate with, the volume power q_v was preliminary approximated by the third-order polynomial [2]:

$$q_v(r)=Q[a+b\beta r+c(\beta r)^2+d(\beta r)^3], \tag{4}$$

where $a=1.0044$, $b=-0.01768$, $c=-0.5657$, $d=0.1668$ and $\beta=2.4/R_1$.

Expression (2) uses the notations: $B=b\beta, C=c\beta^2$, $D=d\beta^3$ [2].

Comparison of the solutions of Eqns (1) and (2) showed that expression (2) reflects the temperature distribution more impartially [2]. This solution was obtained, by specifying $q_v(r)$ in the form (3), on the basis of most general theoretic assumptions and does not include all types of possible distributions. Depending on the specific pump-system parameters, active medium type, quality and technology of laser tube fabrication (absence of impurities) as well as on the materials used and the operating conditions, other distributions of q_v can also take place which are significantly different from (3). In this case, the heat conduction equation should be solved again.

The aim of this paper is to obtain the general solution of the heat conduction equation in the case of the arbitrary radial distribution of the volume pump power $q_v(r)$ and to study the solutions for some specific types of the dependence $q_v(r)$ by the example of the copper bromide vapour laser. Investigation of the effect of the buffer-gas temperature on the population inversion and the laser output power is a separate experimental and theoretical problem, which is beyond our consideration in this paper. The problem of this type is considered, for example, for the electric-discharge CO_2 lasers in [12].

The model proposed here is a self-consistent closed thermotechnical problem and can be used for a comparative analysis at the stage of experimental planning. In the course of computer simulation, by varying the geometrical parameters and materials of the tube, thermal insulation, electric power, and operating conditions, it is possible to determine the main tendencies for changes in the temperature profile and, hence, the most suitable parameters in order to decrease costs of the experiments.

2. Mathematical model and finding of a general solution of the problem

The distribution of the buffer-gas temperature T_g in the laser-tube cross section is determined by solving the stationary heat conduction equation

$$\operatorname{div}(\lambda_g \operatorname{grad} T_g) + q_v = 0, \tag{5}$$

where λ_g is the heat conductivity of a buffer gas (W m^{-1} K^{-1}). We assume that in the general case the volume density of the released power q_v is the function of the radius r, i.e., $q_v = q_v(r)$.

For simplicity, we will use the boundary conditions of the first and second kinds in the form

$$T_g(R_1) = T_1, \tag{6}$$

$$\left.\frac{dT_g}{dr}\right|_{r=0} = 0. \tag{7}$$

The boundary condition (6) means that the temperature T_1 of the internal laser-tube wall is known. The boundary condition (7) points at the presence of the axial symmetry of the temperature distribution in the active medium.

In solving (5)–(7), we proceed from the ordinary representation of the heat conductivity:

$$\lambda_g = \lambda_0 T_g^m, \tag{8}$$

where λ_0 and m are the constants depending on the type of the buffer gas (for a copper bromide vapour laser is a gas mixture of neon and hydrogen, see Table 1).

Let us introduce the function $U(R)$:

$$U(R) = T_g^{m+1}. \tag{9}$$

In this case, Eqns (5)–(7) are reduced to the form

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} + q_v(r)\frac{(m+1)}{\lambda_0} = 0, \tag{10}$$

$$U(R_1) = T_1^{m+1}, \quad \left.\frac{dU}{dr}\right|_{r=0} = 0. \tag{11}$$

Multiplying both sides of equation (10) by r^2, we obtain the Euler-type equation:

$$\frac{r^2 d^2U}{dr^2} + r\frac{dU}{dr} = -r^2 q_v(r)\frac{(m+1)}{\lambda_0}.$$

By introducing an independent variable τ, with the help of substitution $r = e^{\tau}$, we transform (10), (11) to the form

$$\frac{d^2U}{d\tau^2} = -e^{2\tau} q_v(e^{\tau})\frac{(m+1)}{\lambda_0}, \tag{12}$$

$$U|_{\tau=\ln R_1} = T_1^{m+1}, \quad \frac{dU}{dr} = o[e^{\tau}], \tau \to -\infty, \tag{13}$$

where o is the Landau symbol.

Double integration of equation (12), taking into account the boundary conditions (13) and variable r, yields the expression

$$U(r) = T_1^{m+1} - \frac{(m+1)}{\lambda_0}\int_{\ln R_1}^{\ln r} dy \int_{-\infty}^{y} e^{2t} q_v(e^t) dt. \tag{14}$$

Using (9), we find the searched-for general solution of the problem under study:

$$T_g(r) = \left[T_1^{m+1} - \frac{(m+1)}{\lambda_0}\int_{\ln R_1}^{\ln r} dy \int_{-\infty}^{y} e^{2t} q_v(e^t) dt\right]^{\frac{1}{m+1}},$$

$$0 \leqslant r < R_1. \tag{15}$$

When the function $q_v(r)$ has an arbitrary form, it is preferable to perform symbol and numerical calculations with the help of specialised mathematical software, for example Mathematica, Mathlab, etc. [13, 14].

3. Application of the obtained solution by the example of a copper bromide laser

Because we lack reliable experimental data for describing concrete distributions $q_v(r)$, we will consider application of solution (15) by the example of some model distributions.

3.1 Determining the gas temperature for specific volume-power distributions

In this section we present the calculations of the gas temperature at different radial distributions of the volume density of the released power $q_v(r)$. We considered two types of qualitatively different discharges (Fig. 1). Curve (*2*) corresponds to the case of a smooth discharge, while curve (*3*) – to a discharge concentrated at the laser-tube centre and having an inflection point. In some cases, in the presence of distribution of type (*3*), strong deformation of the discharge and its compression to a filament as well as the appearance of thermoionisation instability and other negative phenomena are possible. For comparison, Fig. 1 shows also the distribution at $q_v(r) = q_0 = \text{const}$, where it is arbitrarily assumed that $q_0 = 1$. All the curves are plotted for the same power deposited into the tube (the areas limited by each of the curves and the x axis are equal).

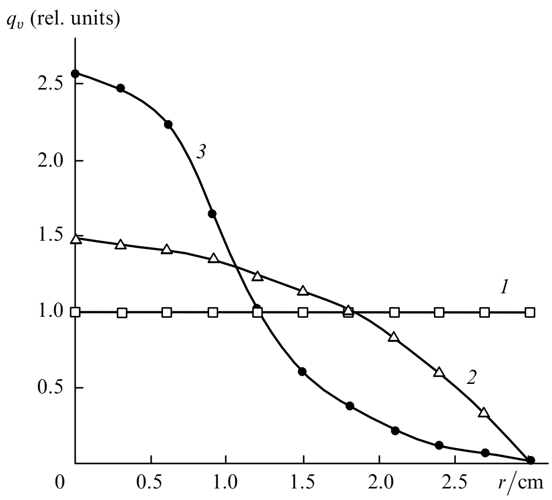

Figure 1. Distribution of the volume density of the released power in the laser-tube cross section: □ is curve (*1*), $q_v(r) = q_0 = \text{const}$; △ is curve (*2*), $q_v(r) = q_{v,2}(r) = K_1 q_0(a + br^2)$ [expression (16)]; ● is curve (*3*), $q_v(r) = q_{v,3}(r) = K_2 q_0(n + pr^2 + sr^3)$ [expression (17)].

Dependences (*2*) and (*3*) can be represented in the form of second- and third-order polynomials, respectively:

$$q_{v,2}(r) = K_1 q_0(a + br^2), \tag{16}$$

$$q_{v,3}(r) = K_2 q_0(n + pr^2 + sr^3), \tag{17}$$

where

$$K_1 = 1.4383, a = 1.0183471, b = -0.001077; \tag{18}$$

$$K_2 = 2.57365,\ n = 0.966892,\quad p = -0.47399,\ s = 0.1249822. \tag{19}$$

Coefficients (18), (19) are obtained by the method of least squares. By substituting (16) and (17) into the general solution (15) we obtain after integration the expressions of the gas temperature distribution:

$$T_{g,2}(r) = \left[T_1^{m+1} - \frac{(m+1)K_1 q_0(r^2 - R_1^2)(4a + br^2 + bR_1^2)}{16\lambda_0}\right]^{\frac{1}{m+1}}, \tag{20}$$

$$T_{g,3}(r) = \left[T_1^{m+1} - \frac{(m+1)K_2 q_0[100n(r^2 - R_1^2) + 25(r^4 - R_1^4) + 16s(r^5 - R_1^5)]}{400\lambda_0}\right\}^{\frac{1}{m+1}}. \tag{21}$$

3.2 Investigation of the gas temperature in the active medium of the copper bromide vapour laser

Consider a copper bromide vapour laser [15] for which we have been already derived and studied expressions (1), (2) [2, 11].

The total consumed electric power of the laser is 5 kW. Taking into account the losses in the feed system, power $Q = 4080$ W is supplied to the active volume. In this case, the output laser power is 120 W. The geometrical dimensions of the laser are presented in Fig. 2. The laser tube is made of quartz and is covered at the top by an insulating material from glass wool, mineral wool, or zirconium oxide in the active volume region. The volume density of the supplied power is $q_v = 0.7219$ W cm^{-3}. The inert gas is the mixture of Ne (15 Torr) and H_2 (0.3 Torr). In this case, the heat conductivity is represented in the form (8), where $\lambda_0 = 5.8935 \times 10^{-5}$, $m = 1.091$.

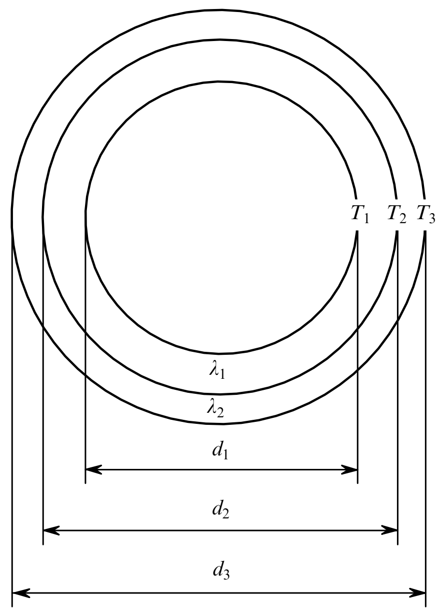

Figure 2. Laser-tube cross section in the active region of the copper bromide vapour laser. The internal diameter of the quartz tube is $d_1 = 60$ mm, the external diameter is $d_2 = 64$ mm, the external diameter of the heat-insulating coat is $d_3 = 74$ mm.

The temperature profile is simulated under the following assumptions: (i) the temperature profile of the discharge is determined in the quasi-stationary operation regime of the laser; (ii) the gas temperature between the pulses varies insignificantly; (iii) all the electric power (4080 W) supplied to the active volume is converted to the thermal energy; (iv) the power imparted to the tube walls during the discharge emission and deactivation of excited and charged particles on the walls is not taken into account.

The third assumption is introduced due to the specific character of the laser under study. It is known that atomic and ion lasers are characterised by a relatively low efficiency because laser transitions take place between the highly excited levels of atoms and ions. For the input electric power of 4080 W supplied to the active volume of the discharge, the corresponding output laser power is 120 W, which yields an efficiency of about 3 %. Upper laser levels with the energy of 3.8 eV can be excited by a direct electron impact, in accordance with the scheme $\mathrm{Cu}(^2\mathrm{S}_{1/2}) + e \rightarrow \mathrm{Cu}^*(^2\mathrm{P}_{1/2}, ^2\mathrm{P}_{3/2}) + e$ [16]. In this case, their population due to the thermal buffer-gas energy is impossible.

The last assumption is based on the properties and energy diagram of the buffer single-atom gas (neon). Because the energies of its long-lived excited s-levels ($3\,s_2$ and $2\,s_2$) are equal to ~ 20 eV, they are weakly populated in the plasma of this discharge with the electron energy of 3–4 eV [16]. Therefore, in this case only lower metastable laser levels of copper atoms are significantly populated, these levels relaxing mainly in collisions with the electrons: $Cu(^2D_{3/2}, ^2D_{5/2}) + e \rightarrow Cu(^2S_{1/2}) + e$ [16]. Thus, the energy remains in the discharge volume. In this case, the spontaneous emission intensity of the discharge plasma is small and its influence can be neglected.

We will solve equation (5), apart from boundary conditions (6) and (7), inside the quartz tube by using the mixed boundary conditions of the third and fourth kinds for the radial heat flow through the tube wall, which in cylindrical coordinates have the form [17]:

$$T_1 = T_2 + \frac{q_l \ln(d_2/d_1)}{2\pi\lambda_1}, T_2 = T_3 + \frac{q_l \ln(d_3/d_2)}{2\pi\lambda_2}, \quad (22)$$

$$Q_1 = \alpha F_3(T_3 - T_0) + F_3\varepsilon c\left[\left(\frac{T_3}{100}\right)^4 - \left(\frac{T_0}{100}\right)^4\right]. \quad (23)$$

Here, T_1, T_2, T_3 are the temperatures of the tube walls and heat-insulating coat (see Fig. 2); $q_l = Q_1/l_a$ is the released thermal power per unit length; $l_a = 2$ m is the active length of the laser; λ_1, λ_2 are the heat conductivities of the quartz tube and heat insulation, respectively; $d_{1,2,3}$ are the tube diameters; $Q_1 = 4080$ W is the thermal flow equal to the consumed electric power (according to the third assumption); α is the heat transfer coefficient of the external heat-insulating surface to the surroundings; F_3 is the area of the external surface of the heat-insulating coat of the tube in the region of the active zone; $c = 5.67$ W m^{-2}K^{-4} is the emissivity of an ideal black body; $T_0 = 300$ K is the air temperature; ε is the integral radiation capacity of the external surface of the tube (heat-insulating coat). The values of the parameters used in calculations are listed in Table 1.

To use the obtained solutions (20) and (21), it is necessary to specify the temperature T_1. Two cases are possible:

(i) Temperature T_2 of the external wall of the quartz tube (under the heat insulation) is known. For real lasers it can be measured, for example, with a thermocouple. Then, the first equality (22) can be used to calculate T_1.

(ii) Temperatures T_2 and T_3 are not known. This situation takes place in designing new lasers of this type. In this case, we can use the boundary condition (23), where the surrounding temperature ($T_0 = 300$ K is most often used) is specified. It is also possible to specify the geometrical and other parameters. The unknown quantity is T_3. It is determined from the nonlinear equation (23). After it, T_2 and T_1 are found successively from (22). In [2] we considered in detail the second case and obtained $T_1 = 1020$ K for the natural convection. This value is used in our calculations.

The obtained profiles of temperature distributions are presented in Table 2. One can see that the temperature distributions calculated with expressions (20) and (21) are close. Their maximum relative difference in the central part of the tube does not exceed 2.4 %.

Table 2. Temperatures calculated with the help of expressions (1), (20), and (21).

r/cm	T_g/K calculated by (1)	T_g/K calculated by (20)	T_g/K calculated by (21)
0.0	1967	2047	2059
0.6	1939	2009	1994
1.2	1851	1889	1841
1.8	1694	1689	1568
2.4	1442	1403	1265
3.0	1020	1020	1020

Table 3 lists the temperature distributions with respect to the radius r obtained from expressions (20) and (2). It follows from Table 3 that at the tube centre the relative temperature difference is no more than 2 %.

Table 3. Temperatures calculated with the help of expressions (20) and (2).

r/cm	T_g/K calculated by (20)	T_g/K calculated by (2)
0	2047	2070
0.5	2019	2031
1.0	1937	1919
1.5	1799	1746
2.0	1603	1528
2.5	1346	1283
3.0	1020	1020

The performed calculations show that for different radial distributions $q_v(r)$, the corresponding expressions for $T_g(r)$ at the tube centre yield different results within 3 %. For $q_v(r) \neq$ const, the simplest representation is obtained with the help of second-order polynomial from expression (16). Solution (20) corresponding to it is the simplest and at the same time the most convenient for computations.

Therefore, we can draw the main conclusion that expression (20) is a kind of universal. It can be successfully used for temperature determination with a sufficient accuracy for all types of distributions $q_v(r)$ in similar lasers.

3.3 Application of the developed model for determining the gas temperature in the case of molecular lasers

The main task in determining the gas temperature in molecular lasers (including CO_2 lasers) is to find the relative portion of the electric power Q_2 supplied to the discharge. This power is used to heat directly the neutral gas: $Q_2 = \eta Q_1$. In a molecular gas, the main portion of the energy from electrons is used to excite vibrational molecular levels, which after vibrational relaxations is released as heat. A part of the energy can be imparted from the discharge volume to the walls due to diffusion of vibrationally excited molecules as well as can be transformed into laser radiation, whose contribution cannot be always neglected because for some CO_2 lasers the efficiency achieves 30 %.

Table 1. Data used to calculate the temperature profile [11].

Q_1/W	l_a/m	q_v/W cm^{-3}	q_l/W m^{-1} K^{-1}	$\lambda_g = \lambda_0 T_g^m$/W m^{-1} K^{-1}	λ_1/W m^{-1} K^{-1}	λ_2/W m^{-1} K^{-1}	ε
4080	2	0.7219	2040	$\lambda_0 = 5.8935 \times 10^{-5}$ ($m = 1.091$, $p_{Ne} = 15$ Torr, $p_{H_2} = 0.3$ Torr)	1.96 ($T = 800 - 1100$ K)	0.12 ($T = 800 - 1100$ K, mineral wool)	0.72

Determination of reliable coefficients η for some types of discharges can be a challenging problem. In particular, for nitrogen η can achieve 0.5 [18].

The model proposed in this paper can be applied to molecular laser after determining η.

4. Conclusions

We have obtained the general solution of the heat conduction equation in the case of an arbitrary radial distribution of the volume density of the power deposited in the discharge, $q_v = q_v(r)$. For the cases when the reliable data on the volume electric power distribution are absent, we have considered two specific types of distributions $q_v = q_v(r)$. Using the general solution of the heat conduction equation for these distributions at different nonlinear boundary conditions of the third and fourth kinds, we have calculated the corresponding temperature profiles for a copper bromide vapour laser. It has been established that although these profiles differ, the gas temperatures at the tube centre are identical with an accuracy up to 2 % – 3 %. Therefore, we recommend to use the volume power distribution in the form of the second-order polynomial and the solution of the boundary problem corresponding to this polynomial for the heat conduction equation.

An important advantage of the proposed general solution of the problem on calculating the gas temperature is the possibility to specify arbitrary distributions of the volume density of the power supplied to the discharge and to derive equations for the gas temperature distributions. Another advantage is the possibility of specifying different geometrical parameters, surrounding temperature, the laser-tube wall temperature as well as other parameters, which is convenient in computer computations of the gas temperature that is an element of more general kinetic and other models used to assess the parameters of the existing laser sources and new laser sources being developed.

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