Quantum Electronics **40** (4) 363 – 367 (2010)

RESONATORS AND MODES

PACS numbers: 42.60.Da; 42.60.Mi
DOI: 10.1070/QE2010v040n04ABEH014236

On the mechanism of transverse-mode beatings in a Fabry – Perot laser

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***Abstract*. The mechanism of emergence of fundamental-mode and first-mode beatings in the case of a step-wise increase in the pump rate is studied under the stationary single-mode lasing conditions. Investigation is based on the numerical solution of nonstationary wave equations in a resonator in the quasi-optic approximation and on the equation for a relaxation-type medium as well as on the use of the first two Hermite – Gaussian polynomials $\psi_{0,1}(x)$ to obtain the distribution projections $I_{0,1}(t)$, $g_{0,1}(t)$ of the radiation intensity and gain, respectively. It is shown that the transverse-mode beatings emerge at early stages of two-mode lasing, the appearance of radiation intensity oscillations in the active medium preceding the development of the gain oscillations. The time of the passage of two-mode lasing to the stationary regime is determined. The phase shift $\pi/2$ between the oscillations $I_1(t)$ and $g_1(t)$ is found for the established beating regime and the modulation depth ΔI averaged over the output aperture of the radiation intensity in the established two-mode regime is shown to be proportional to the pump rate excess k over the single-mode lasing threshold. A scheme for controlling the mode composition of laser radiation is proposed, which is based on the rules for determining $I_{0,1}(t)$ by the sensor signals. The efficiency of the scheme is studied. The scheme employs two field intensity sensors mounted inside the resonator behind the output aperture.**

Keywords: *laser, Fabry – Perot resonator, lasing dynamics, numerical simulation, control of mode composition of radiation.*

1. Introduction

Complex dynamics of laser radiation determined by the interaction of transverse resonator modes was discovered soon after the advent of lasers [1]. Interest in its investigation is still high and caused by a variety of wave and oscillation regimes of laser generation [1 – 10], the importance of the transverse-mode dynamics for technological applications [11, 12], appearance of new fields of application of laser radiation and means for controlling its mode composition [13 – 15].

At present, ring generators with a stable confocal resonator partially filled with a two-level active medium have been thoroughly studied analytically [1, 6 – 8]. In particular, stability regions of single-mode and two-mode lasing of class-A and class-B ring lasers have been found, the parameters of the established mode-intensity oscillations and dynamics of the intensity distributions in three – six-mode lasing regimes have been determined [6, 8]. Analytic investigations of the dynamics of Fabry – Perot lasers are less complete. These studies have been performed for lasers with a closed Fabry – Perot resonator (FPR) (this resonator consisted of four mutually perpendicular mirrors; the mirrors mounted parallel to the optical axis are assumed to be ideally reflecting) [3, 4, 16]. In this case, the authors of [3, 4, 16] determined the conditions for exciting single-mode and two-mode generation, amplitudes of the established mode oscillations, and some characteristics of transient regimes. In particular, the authors of paper [3] showed that the time dependence of the radiation intensity when two adjacent transverse FPR modes with identical axial indices are generated is close to harmonic for pulsations with a small intensity modulation depth ΔI, and the oscillation period increases with increasing ΔI. Multimode generation in unstable-resonator lasers have been studied numerically [9, 10].

The results obtained in [3, 4, 16] allow one to understand some physical phenomena accompanying multimode generation in FPRs. However, to extend these results to lasers with an open FPR as well as to use them in practical application, for example, in designing schemes for controlling the mode composition of radiation, is rather difficult. In this case, multimode generation can be studied using numerical simulation. It allows the transition processes between different lasing regimes to be investigated, including those in the presence of nonstationary perturbations of distributions of the pump rate, losses, and optical medium inhomogeneities.

In this paper, we study numerically the beatings of two transverse mode with identical axial transitions and a passage from the established single-mode lasing to two-mode one after a sudden increase in the pump rate. We also present a scheme for controlling the mode composition of radiation, allowing one to determine the characteristics of the single-mode regime and the appearance of a passage to the two-mode regime, as well as the beating parameters. Nonstationary inhomogeneous perturbations of the pump rate distributions or the losses in the active medium layer

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Received 20 October 2009; revision received 8 February 2010
Kvantovaya Elektronika **40** (4) 363 – 367 (2010)
Translated by I.A. Ulitkin

lead to transitions from the established single-mode lasing to two-mode lasing at the pump rate k such that $k < k_t$ (k_t is the two-mode lasing threshold). Investigation of such transitions in the case of subthreshold pump rates is the subject of a separate study.

2. Generation model

Passage from single-mode lasing to two-mode one was studied based on the numerical model [9, 15]. In the plane geometry in the small-angle approximation of the scalar diffraction theory, the electric field inside the resonator was represented in the form of counterpropagating plane waves modulated by smooth envelopes:

$$E(x,z,t) = [F(x,z,t)\exp(ik_0 z) + B(x,z,t)\exp(-ik_0 z)]\exp(-i\omega_0 t). \quad (1)$$

Here, ω_0 is the carrier frequency; $k_0 = \omega_0/c$; the z axis is directed along the resonator axis and the x axis – parallel to the mirror plane (see Fig. 1). The envelope dynamics of forward $[F(x,z,t)]$ and backward $[B(x,z,t)]$ waves was described by the equations

$$2ik_0\left(\frac{1}{c}\frac{\partial B}{\partial t} - \frac{\partial B}{\partial z}\right) + \frac{\partial^2 B}{\partial x^2} - ik_0 g B = 0, \quad (2)$$

$$2ik_0\left(\frac{1}{c}\frac{\partial F}{\partial t} + \frac{\partial F}{\partial z}\right) + \frac{\partial^2 F}{\partial x^2} - ik_0 g F = 0, \quad (3)$$

where $g(x,z,t)$ is the medium gain. We used the spectral approach to solve equations (2), (3). The number of network elements was 8192, and the number of elements on the mirror was 512. On the resonator mirrors the waves met the reflection conditions

$$F(x,0,t) = -B(x,0,t)r_1, \quad (4)$$

$$B(x,L,t) = -F(x,L,t)r_2. \quad (5)$$

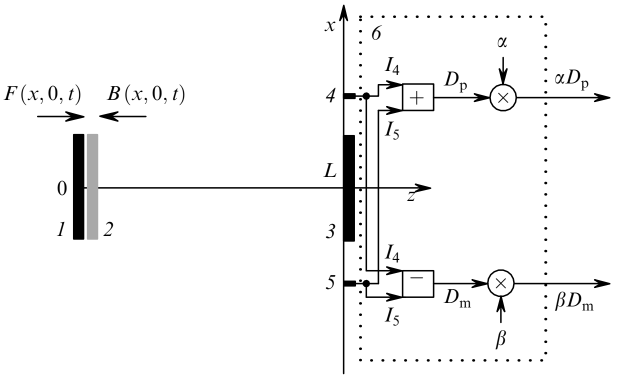

Figure 1. Fabry – Perot laser and the system controlling the mode composition of laser radiation: (*1*) highly reflecting mirror; (*2*) active medium layer; (*3*) output mirror; (*4,5*) sensors measuring the intensity amplitude (located symmetrically); (*6*) processor [includes addition (D_p) and subtraction (D_m) of sensor signals, extraction of multipliers α and β from the database, and multiplication of the sum and difference of sensor signals by α and β, respectively]; $F(x,0,t)$, $B(x,0,t)$ are the forward and backward waves in the active medium layer; L is the resonator length; I_4, I_5 are signals from sensors (*4*) and (*5*).

Here, $r_1 = 1$ and $r_2 = 0.8$ are the reflectivities of the highly reflecting and output mirrors with the radii $a = 1$ cm, spaced by $L = 150$ cm. The Fresnel number N_F of the Fabry – Perot resonator was 6.25. The threshold fundamental-mode gain g_t was equal to 1.5322×10^{-3} cm^{-1}; the excess k of the pump rate over the threshold was determined as g_0/g_t, where g_0 is the small-signal gain.

The active medium represented a thin layer located near the highly reflecting mirror (Fig. 1), which corresponded to the scheme [9] dividing the intracavity space into many thin layers by neglecting diffraction in the first layer filled with the active medium. Other layers were not filled with the active medium.

The equation for the gain in the active medium included the processes of stimulated emission and relaxation with constant time τ:

$$\tau\frac{\partial g}{\partial t} = g_0 - g(1 + I). \quad (6)$$

Here, $I(x,0,t) = |F(x,0,t)|^2 + |B(x,0,t)|^2$ is the radiation intensity in the active medium layer averaged over the interference beatings of counterpropagating waves and normalised by the saturation intensity [17]. The relaxation time τ was assumed equal to 6.0×10^{-6} s. We solved equation (6) by using the implicit scheme of the second-order approximation.

The initial conditions were the envelopes $F(x,0,t)$ and $B(x,L,t)$ and the gain distribution $g(x,0,t)$ for the established single mode lasing at the pump rate g_{0u}, which is 10 % smaller than its threshold value for two-mode lasing. The perturbation was introduced at the time instant $t^* = 5$ μs and represented a jump in the pump rate by $\Delta g =$ const so that the total value of $g_0 = g_{0u} + \Delta g$ exceeded the threshold of two-mode lasing emergence.

Two sensors measuring the radiation intensity, whose readings were used in the scheme controlling the mode composition of radiation, were placed symmetrically with respect to the resonator axis in the plane $z = L$ in the region $|x| > a$ (Fig. 1). We assumed that the radiation intensity was measured periodically, the period between them being much shorter than the period of transverse-mode beatings.

3. Investigation of the mechanism of emergence of transverse-mode beatings

Passage from the fundamental-mode generation regime to the regime of simultaneous generation of the fundamental and first modes was studied by projecting the intensity distributions and the gain on the first two Hermite – Gaussian polynomials $\psi_{0,1}(x)$ in expressions

$$I_i(t) = \int_{-a}^{a} I(x,0,t)\psi_i(x)\mathrm{d}x \Big/ \int_{-a}^{a}\psi_i(x)\psi_i(x)\mathrm{d}x,$$
$$g_i(t) = \int_{-a}^{a} g(x,0,t)\psi_i(x)\mathrm{d}x \Big/ \int_{-a}^{a}\psi_i(x)\psi_i(x)\mathrm{d}x, \quad (7)$$

where $i = 0,1$. The choice of Hermite – Gaussian polynomials is explained by the convenience of expanding the real variable functions $I(x,0,t)$ and $g(x,0,t)$ in them and by a wide use of these polynomials in analytic research. The distribution projections on $\psi_0(x)$ (intensity I and gain g amplitudes of the fundamental modes) yield information on the dynamics of symmetric parts of the distributions [Fig. 2a shows only the dependence $I_0(t)$], while the projections on $\psi_1(x)$ (intensity I and gain g amplitudes of the first modes) – on the dynamics of the asymmetric parts of the distributions, which is related to the passage to two-mode lasing [Fig. 2b shows only the dependence $I_1(t)$]. The radiation intensity $I_{av}(t)$ averaged over the output aperture coincided with the intensity $I_\psi(t)$ recovered with the help of time dependences $I_{0,1}(t)$, with an accuracy no worse than 10 % (Fig. 2c).

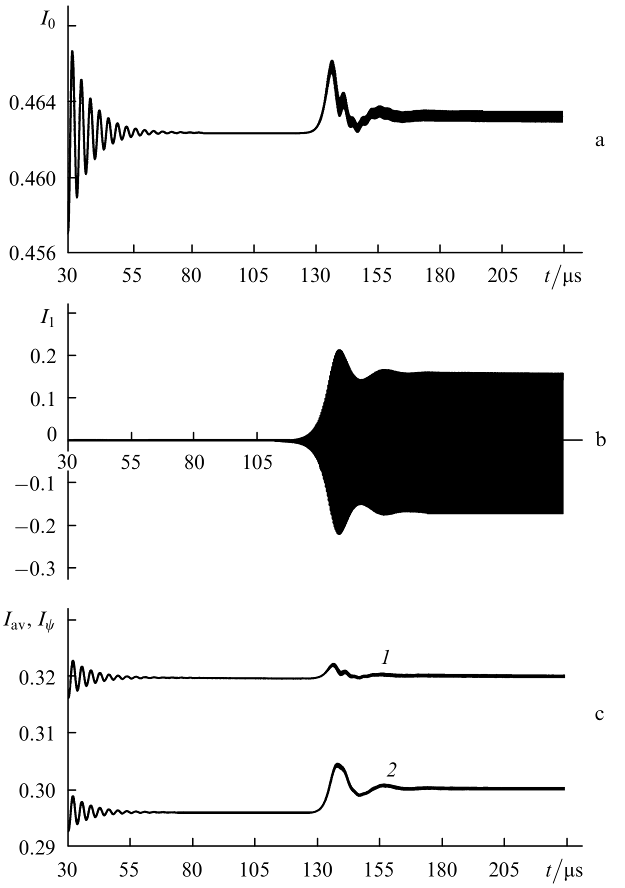

Figure 2. Dependences of the projections of intensity distributions on $\psi_0(x)$ (a) and $\psi_1(x)$ (b), as well as the dependences $I_\psi(t)$ (*1*) and $I_{av}(t)$ (*2*) (c).

The mechanism of pumping energy into the light field in the beating regime began forming immediately after a jump in the pump rate. Optical field oscillations played a leading role in this process. One can see from Figs 3a, b that after a jump in the pump rate, the amplitude of small-scale intensities $I_1(t)$ caused by beatings started increasing. In this case, the projection of the gain on ψ_1 first had a noise character (Fig. 3b). Extraction of increasing oscillations $g_1(t)$ from noise continues also at the initial stage of the exponential growth of the amplitudes of $I_1(t)$ and $g_1(t)$ oscillations. As a result, $g_1(t)$ oscillations become regular and the phase shift $\pi/2$ is established between the $I_1(t)$ and $g_1(t)$ oscillations. In this case, the local maxima $I_1(t)$ fall on zero values of $g_1(t)$, i.e. maximally asymmetric distributions of the intensity $I(x,0,t)$ correspond to the symmetric distributions of the gain $g(x,0,t)$. This means that within a quarter of a period $g(x,0,t)$ increases due to pumping that part of the active medium where $g(x,0,t)$ has smaller values. Simultaneously, $g(x,0,t)$ decreases due to stimulated emission in that part of the active medium where $g(x,0,t)$ has larger values. The regular character of the $I_1(t)$ and $g_1(t)$ oscillations and the phase shift $\pi/2$ between them is retained at the part of the exponential growth of the oscillation amplitudes and then in the region of stationary values of these amplitudes (Fig. 3c). One can see from Fig. 2c [curve (*2*)] that in the case of two-mode lasing the average radiation intensity in the layer is higher than in the case of generation of one fundamental mode.

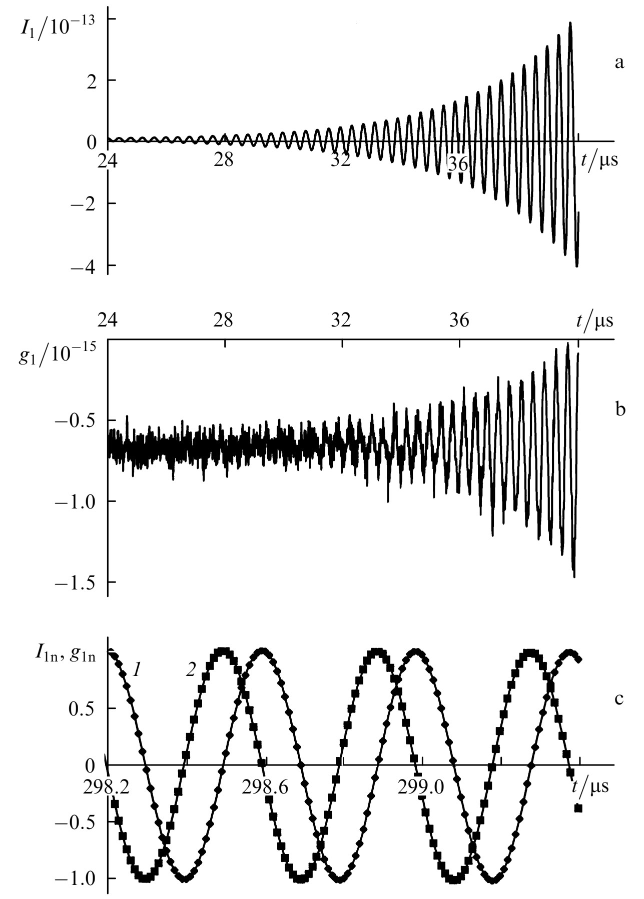

Figure 3. Dependences $I_1(t)$ (a) and $g_1(t)$ (b) at the beginning of two-mode lasing development and established dependences $I_{1n}(t) = I_1(t)/I_{1\max}$ (*1*) and $g_{1n}(t) = g_1(t)/g_{1\max}$ (*2*) in the beating regime (c) ($I_{1\max} = 0.163884$ and $g_{1\max} = 0.000279908$ are the maximum values of the corresponding quantities).

The zero lines of the $I_1(t)$ and $g_1(t)$ dependences at the beginning of the process (Figs 3a, b) are displaced from zero approximately by 5×10^{-15} and -5×10^{-16}, respectively, which is caused by a slight distortion in the symmetry of the fundamental mode distribution from which the growth starts. These displacements are a systematic error in calculations and noticeable only at the beginning of the two-mode lasing development.

Characteristics of transition from single-mode lasing to two-mode lasing depend on the excess k of the pump rate over the threshold. They can be studied within model (1) – (6). Determining the transition time T as the time of the linear growth of logarithm I_1 (i.e., the interval at which $V = \tau \mathrm{d} \lg I_1/\mathrm{d}t = \mathrm{const}$), we can find the dependence of T on

k (Fig. 4a). The transition time rapidly increases when k from the side of larger values approaches the two-mode lasing threshold (Fig. 4a). In this case, the growth rate V proves proportional to k (Fig. 4b). In the established beating regime (after the segment $V = \text{const}$ is through and relaxation processes are over), the modulation depth ΔI of the radiation intensity averaged over the output aperture is proportional to k (Fig. 4c).

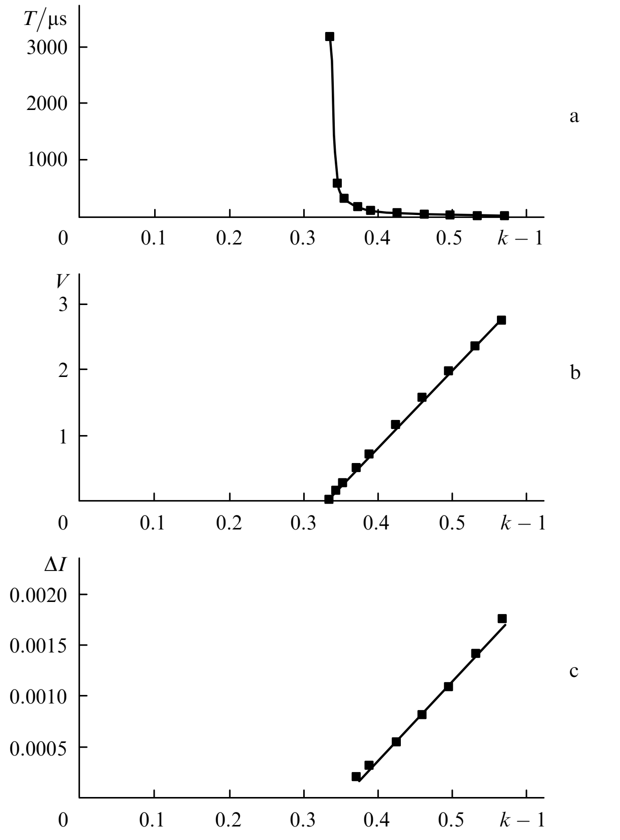

Figure 4. Dependences $T(k-1)$ of the transition time from single-mode lasing to two-mode lasing (a), of the growth rate $V(k-1)$ of the logarithm $I_1(t)$ ($V = \tau \mathrm{d}\lg I_1/\mathrm{d}t$) (b) and of the modulation depth $\Delta I(k-1)$ of the radiation intensity averaged over the output aperture in the established beating regime (c).

4. Scheme for controlling the mode characteristics of generation

In the previous section we showed that a jump in the pump rate leads to the development of the two-mode lasing. The control scheme should indicate at a certain moment the appearance of the first mode and estimate the quantities of the projections $I_{0,1}(t)$ by the sensor signals. Let us find the rules according to which this can be done by processor (*6*) (see Fig. 1).

While generating one fundamental mode, variable signals $I_4(t)$ and $I_5(t)$ read from sensors (*4*) and (*5*), respectively, will be approximately equal, i.e. $I_4(t) - I_5(t) \approx 0$ because the distribution $I(x, z, t)$ is symmetric in this case. In the case of two-mode lasing, variable signals $I_4(t)$ and $I_5(t)$, generally speaking, will differ from each other, reflecting the change in the shape of the radiation intensity distribution during beatings, i.e. $D_{\mathrm{m}}(t) = |I_4(t) - I_5(t)| \geqslant 0$. One can see from Figs 5a, b that the dependences $\lg I_1(t)$ and $\lg D_{\mathrm{m}}(t)$ are close in shape and the latter dependence indicates the emergence of two-mode lasing. By fixing some level $L_{D_{\mathrm{m}}}$ (for example, $L_{D_{\mathrm{m}}} = 10^{-8}$), we can assume that two-mode lasing takes place at $D_{\mathrm{m}}(t) \geqslant L_{D_{\mathrm{m}}}$. In order to formulate the rules according to which the processor determines $I_{0,1}(t)$, let us find rearrangements allowing one to make the sensor signals $I_{4,5}(t)$ maximally similar to the dependences $I_{0,1}(t)$.

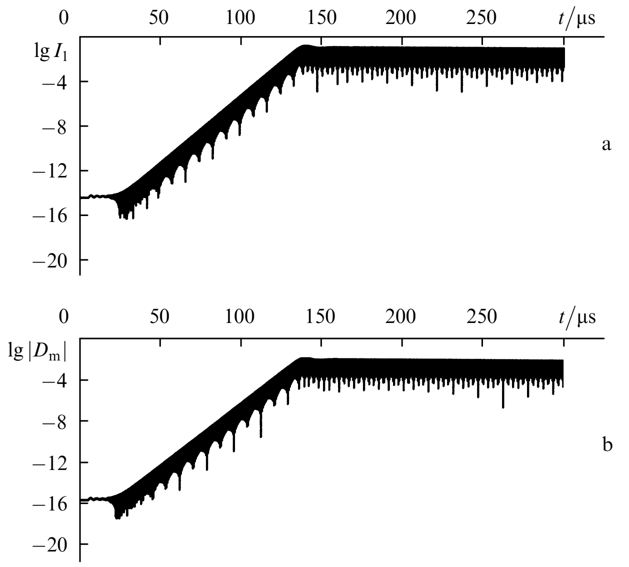

Figure 5. Dependences $\lg I_1(t)$ (*a*) and $\lg D_{\mathrm{m}}(t)$ (b) upon passage to two-mode lasing.

The total signal $D_{\mathrm{p}}(t) = I_4(t) + I_5(t)$ is close to $I_0(t)$ in the case of single-mode lasing, and by multiplying it by $\alpha = I_{\mathrm{av,st}}/D_{\mathrm{p,st}}$ – the ratio of the values $I_{\mathrm{av,st}} = I_{\mathrm{av}}(t)$ and $D_{\mathrm{p,st}} = D_{\mathrm{p}}(t)$, established at $t - t^* \gg \tau$, the dependences $I_0(t)$ and $\alpha D_{\mathrm{p}}(t)$ virtually coincide (Figs 6a, b). In the case of two-mode lasing, the signal $D_{\mathrm{p}}(t)$ has a complicated shape because relaxation oscillations are imposed on the emerging beatings. Therefore, the multiplier α can be selected in such a way that the dependence $\alpha D_{\mathrm{p}}(t)$ will trace the relaxation oscillations in the case of two-mode lasing and will differ from $I_0(t)$ after their decay (Figs 6c, d) or so that to bring $I_0(t)$ and $\alpha D_{\mathrm{p}}(t)$ together at $t - t^* \gg \tau$, by sacrificing the accuracy at large amplitudes of relaxation oscillations. The third variant is also possible: the processor uses different values of α depending on the amplitude of relaxation oscillations. The dependences $I_1(t)$ and $\beta D_{\mathrm{m}}(t)$ virtually coincide at $\beta = I_{1,\mathrm{st}}/D_{\mathrm{m,st}}$, where $I_{1,\mathrm{st}} = I_1(t)$, $D_{\mathrm{m,st}} = D_{\mathrm{m}}(t)$ at $t - t^* \gg \tau$ (Fig. 7). The multipliers α and β depend on the known excess k of the pump rate over the threshold.

Thus, to have this control scheme in operation, we should know $L_{D_{\mathrm{m}}}$ and the multipliers α and β for each k or at least for some of its rather close values. The level $L_{D_{\mathrm{m}}}$ can be associated, for example, with the sensitivity to the beating amplitude of those technological operations where the laser is employed. The multipliers α, β can be determined in numerical simulations [9] or in experiments, thereby forming a database for the scheme controlling the lasing regimes. The rules, the processor should follow during the control are rather simple: the inequality $|D_{\mathrm{m}}(t)| \geqslant L_{D_{\mathrm{m}}}$ is checked, then the value of α corresponding to the type of lasing is

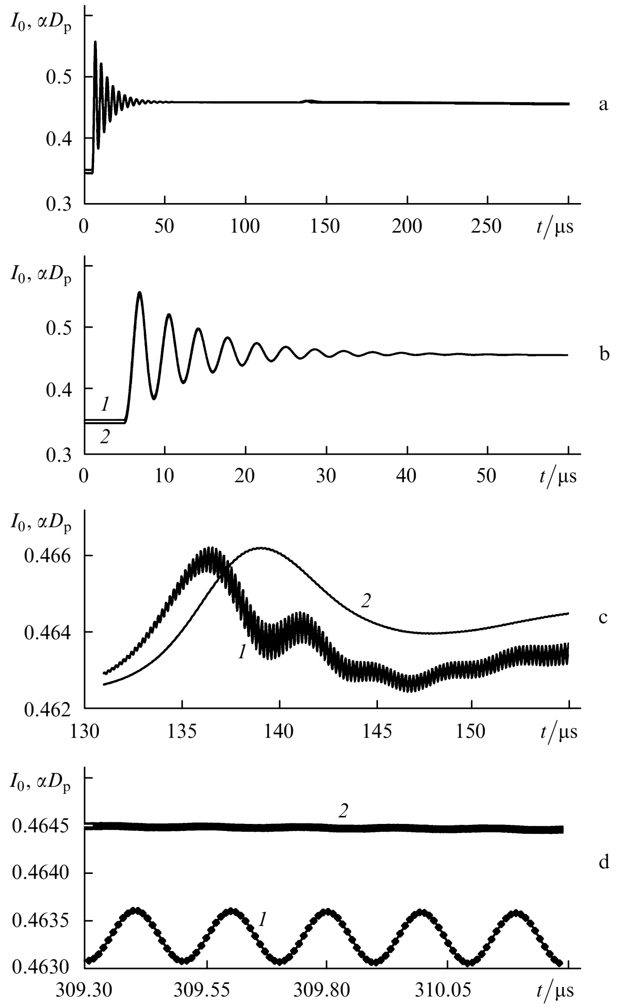

Figure 6. Dependences $I_0(t)$ and $\alpha D_p(t)$ in the case of single-mode lasing (a) and when tracing relaxation oscillations by the processor in the case of two-mode lasing (c–d); (*1*) $I_0(t)$, (*2*) $\alpha D_p(t)$.

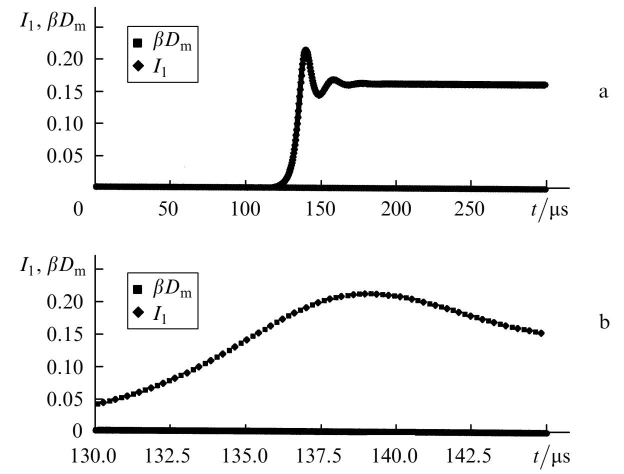

Figure 7. Dependences $I_1(t)$ (◆) и $\beta D_m(t)$ (■) in the case of two-mode lasing at different time scales.

extracted and $\alpha D_p(t)$ is calculated, after it the value of β is extracted and $\beta D_m(t)$ is calculated. If for some reasons the multipliers α and β are not obtained, the control scheme can yield important qualitative information on the presence of relaxation oscillations in the case of single-mode lasing, on the passage to two-mode lasing and the emergence of relaxation oscillations in this case, as well as on the presence of transverse-mode beatings.

5. Conclusions

Simulation of a laser transition from generation of one transverse mode to generation of two transverse modes and emergence of beatings in the case of a jump in the pump rate has allowed us to reveal some peculiar features of this process. The main peculiarities include the determining role of the optical field in forming the beating regime and the phase shift $\pi/2$ between the oscillations of projections of the gain and intensity distributions. A somewhat unexpected result is the high degree of proportionality between the difference signal of the sensors and projection of intensity distributions on the Hermite–Gaussian polynomial $\psi_1(x)$. Studying the proposed scheme for controlling the mode composition of radiation has shown that its potentials are comparable with those of the schemes from papers [15] when the lasing parameters are accurately determined.

References

1. Belousova I.M., Vinokurov G.N., Danilov O.B., Rozanov N.N. *Zh. Eksp. Teor. Fiz.*, **52**, 1146 (1967).
2. Khanin Ya. I. *Principles of Laser Dynamics* (Amsterdam, North-Holland: Elsevier, 1995; Moscow: Nauka, 1999).
3. Belenov E.M., Morozov V.N., Oraevsky A.N. *Trudy FIAN*, **52**, 238 (1970).
4. Lugiato L.A., Oldano C., Narducci L.M. *J. Opt. Soc. Am.*, **5**, 879 (1988).
5. Brambilla M., Battipede F., Lugiato L.A., et al. *Phys. Rev. A*, **43**, 5090 (1991).
6. Brambilla M., Cattaneo M., Lugiato L.A., et al. *Phys. Rev. A*, **49**, 1427 (1994).
7. Prati F., Zucchetti L., Molteni G. *Phys. Rev. A*, **51**, 4093 (1995).
8. Vladimirov A.G., Skryabin D.V. *Kvantovaya Elektron.*, **24**, 913 (1997) [*Quantum Electron.*, **27**, 887 (1997)].
9. Elkin N.N. *Mat. Model.*, **10** (4), 91 (1998).
10. Elkin N.N., Napartovich A.P. *Kvantovaya Elektron.*, **30**, 1065 (2000) [*Quantum Electron.* **30**, 1065 (2000)].
11. Arutyunov R.V., Baranov V.Yu., Bol'shov L.A., et al. V*ozdeistvie lazernogo izlucheniya na materialy* (Action of Laser Radiation on Materials) (Moscow: Nauka, 1989).
12. Nesterov A.V., Niziev V.G. *Izv. Ross. Akad. Nauk, Ser. Fiz.*, **63**, 2039 (1999).
13. Lugiato L.A., Narducci L.M., Lefever R. *Lasers and Synergetics – a Volume in Honor of the 60th Birthday of Hermann Haken* (Berlin: Springer-Verlag, 1987) pp 53 – 71.
14. Brambilla M., Broggi G., Prati F. *Phys. D*, **58**, 339 (1992).
15. Ledenev V.I. *Kvantovaya Elektron.*, **36**, 933 (2006); 38, 1033 (2008) [*Quantum Electron.*, **36**, 933 (2006); 38, 1033 (2008)].
16. Lin H., Abraham N.B. *Opt. Commun.*, **79**, 476 (1990).
17. Zvetlo O. *Principles of Lasers* (New York: Plenum Press, 1998; Moscow: Mir, 1990).