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LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

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Laser acceleration of neutrons (physical foundations)

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***Abstract*. The concept of neutron acceleration in a gradient magnetic field of a 'drifting' standing electromagnetic wave is presented. The promising fields of application of an accelerated directional beam of ultracold neurons, in particular, remote initiation of nuclear reactions, are suggested.**

Keywords: *quantum nucleonics, synchronous acceleration of neutral particles in a 'drifting' standing wave, optical potential wells, ultracold neutrons, remote initiation of nuclear reactions.*

1. Introduction

Electrons or ions in known linear accelerators are accelerated by the electric field of the electromagnetic wave travelling synchronously with charged particles (see, for example, [1]). Can this principle of synchronous acceleration be applied to neutral particles, in particular, to neutrons when the magnetic field strength $F \sim |\mu|\mathrm{grad}B$ with the induction B acts on the dipole moments μ? If the magnetic field is represented by a wave with the amplitude B_0 and the spatial period λ, the maximum gradient is equal to $2\pi B_0/\lambda$ and the maximum effective force is

$$F_{\max} = 2\pi|\mu|B_0/\lambda. \tag{1}$$

It immediately follows that the optical range has an overwhelming advantage over the microwave range (by 12 – 15 orders of magnitude), which allows one to achieve a higher force (1) both due to its inverse proportionality to the wavelength λ and to the possibility of concentrating the electromagnetic radiation in smaller (of the order of λ^2) beam cross section for obtaining a higher B_0.

For a neutron with $\mu = \mu_{\mathrm{n}} = -1.9\mu_0 = -0.95\times 10^{-23}$ erg G^{-1} (μ_0 is a nuclear magneton) [2], we have

$$|F_{\max}| = 6\times 10^{-23}(B_0/\lambda)\ \text{dyne}, \tag{2}$$

where B_0 is measured in Gausses and λ – in centimetres. The present orders of the quantities can be inferred from the estimates of the magnetic induction modulus $B_0 \sim (4\pi S/c)^{1/2} \approx 2\times 10^{-5}S^{1/2}$ of the electromagnetic wave with the flux density S (erg cm^{-2}s^{-1}) of the Poynting vector and force (2): $B_0 \approx 300$ kG at $S \approx 20$ TW cm^{-2} (the value which can be achieved by focusing a short laser pulse with $\lambda = 5\times 10^{-5}$ cm and peak power $\sim$6 MW to a spot of diameter 5×10^{-4} cm) and $F_{\max} = 3\times 10^{-13}$ dyne. This force can endow a neutron having the mass M_{n} at the length L with the velocity

$$V = (2|F_{\max}|L/M_{\mathrm{n}})^{1/2} = 1.1\times 10^{12}(|F_{\max}|L)^{1/2}\ \text{cm s}^{-1} \tag{3}$$

i.e., for example, the velocity $V \approx 10^7$ cm s^{-1} is achieved at the length $L \approx 10$ m. In this case, the ultracold neutrons [3] acquire the directed velocity that is five orders of magnitude larger than their chaotic thermodynamic velocities. This estimate indicates that even extremely weak (of the order of the picodyne fraction) magnetic gradient forces can lead to a well-noticeable macroscopic result, and motivates the attempt to analyse the outlooks for designing a laser linear accelerator of neutrons, in particular, ultracold ones.

A neutron in a magnetic field can occupy two possible quantum state with an oppositely oriented magnetic moment, the energy difference $\varepsilon \approx |\mu|B_0$, and the ratio of the populations of the order $\exp(-\varepsilon/k_{\mathrm{B}}T)$ (T is the neutron temperature, k_{B} is the Boltzmann constant). Therefore, ultracold neutrons with $T \sim 1$ mK [2] in the field with the $\sim$ 300-kG induction are present almost in the ground state with the magnetic moment parallel to the induction vector. Because the de Broglie wavelength λ_{dB} of an ultracold neutron, equal to $\sim 10^{-6}$ cm in the order of the quantity, is inferior in the potential well dimensions in the optical standing wave, other specific quantum effects are beyond our consideration.

2. Optical potential well for neutrons in the field with a finite longitudinal magnetic component

To accelerate a neutral particle by a magnetic dipole force in the laser radiation field, the particle should be trapped in an optical potential well moving at an increasing sublight speed. The possibility of producing an optical field with such properties is inferred from the model example of the superposition of two pairs of plane waves B_i ($i = 1, 2, 3, 4$) with equal amplitudes $B_0/4$ each and frequencies $\omega_{1,2}$ and $\omega_{3,4}$, counterpropagating at angles $\vartheta_{1,2} = \pm\vartheta$ and $\vartheta_{3,4} = \mp\vartheta$ to the z axis ($0 \leqslant \vartheta \leqslant \pi/2$).

In polarising the waves with the magnetic induction vector lying in the xz plane, and with the equal frequencies $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_0$, the resultant standing wave has

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on the z axis the transverse vector components $B_x = B_y = 0$ and the finite longitudinal component

$$B_z = B_0 \sin(\omega_0 t) \cos(k_\vartheta z) \sin\vartheta, \quad (4)$$

where $k_\vartheta = 2\pi \cos\vartheta/\lambda_0$; $\lambda_0 = 2\pi c/\omega_0$; c is the speed of light; t is the time (in the theory of hollow-core metal waveguides [3] this field is classified as TE or M).

The cosine multiplier in (4) describes the spatial configuration of one of the separated segment of the standing wave with a maximum at point $z = 0$ (the phase is $\phi = 0$) and step $\Lambda = \lambda_0/2\cos\vartheta > \lambda_0/2$ between the points $z = \pm\lambda_0/4\cos\vartheta$ ($\phi = \pm\pi/2$), while the sine multiplier – the pulsation of this segment in time with the frequency w_0.

As was noted above, an ultracold neutron in a magnetic field with the induction amplitude B_0 of the order of hundreds of kilogauss is, as a rule, in the ground state and its moment in each field half-period (4) turns to be parallel to the vector B_z, changing its orientation after time π/ω_0. This reorientation occurs in the vicinity of the time instant with the phase $\omega_0 t = 0, \pi$, when the field is equal to zero and the dipole orientation becomes in fact free and is easily subjected to the action of the changing induction B_z. Therefore, the magnetic dipole in field (4) follows after the change in the induction sign, B_z, thereby, constantly retaining orientation parallel to it.

As a result, a potential well (pulsating in time at a frequency $2\omega_0$) having the length $2\Lambda = \lambda_0/\cos\vartheta$ between the points $k_\vartheta z = 0$ and $k_\vartheta z = 2\pi$, the bottom at $k_\vartheta z = \pi$ and the maximum depth

$$\Phi_{\max} = |\mu_n| B_0 \sin\vartheta \quad (5)$$

is produced to trap a neutron. The ability of the potential well to trap neutrons is determined by the inequality

$$B_0 > \frac{k_B T}{|\mu_n| \sin\vartheta}, \quad (6)$$

which, for example, yields $B_0 > 10$ kG for ultracold neutrons with $T \approx 0.001$ K.

In the potential well of the standing wave the neutron is trapped by the longitudinal force with the maximum absolute value

$$F_{z\max} = |\mu_n| k_\vartheta B_0 \sin\vartheta \quad (7)$$

at points $k_\vartheta z = \pi/2$ and $k_\vartheta z = 3\pi/2$ (at the well edges at points $k_\vartheta z = 0$ and $k_\vartheta z = 2\pi$, the force is $F_z = 0$). Therefore, in the field of a standing high-intensity wave (4) there appears a chain of potential wells capable of trapping the neutrons.

3. 'Drifting' standing wave

Neutron acceleration requires potential wells to be moved synchronously at an increasing sublight velocity. This is achieved by the partial abandonment of the ideal image of the standing wave and by the transformation of field (4) into the so-called drifting standing wave in which the interacting counterpropagating pairs of travelling waves have unequal frequencies $\omega_1 = \omega_2 = \omega_0$ and $\omega_3 = \omega_4 = \omega_0(1 - 2\alpha)$, where the modulus of the frequency detuning parameter is $|\alpha| \ll 1$. The appearing resultant field, by retaining the finite longitudinal component B_z, takes the form different from (4):

$$B_z = B_0 \sin[\omega_0(1-\alpha)t - \omega_0\alpha(z/c)\cos\vartheta]$$
$$\times \cos[\alpha\omega_0 t - \omega_0(1-\alpha)(z/c)\cos\vartheta] \sin\vartheta. \quad (8)$$

At $\alpha \to 0$, this result undergoes transition to the form of the standard standing wave (4), and the smaller $|\alpha|$ the closer it to this form.

The cosine multiplier in (8) yields an instantaneous configuration of the fragment of the quasi-standing ($|\alpha| \ll 1$!) wave with the maximum when the argument is equal to zero at point

$$z^* = \frac{c\alpha t/\cos\vartheta}{(1-\alpha)}. \quad (9)$$

This maximum and the entire fragment drift on the whole along the axis with the velocity

$$V^* \equiv \frac{dz^*}{dt} = c\frac{\alpha/\cos\vartheta}{1-\alpha}, \quad (10)$$

the positive velocity V^* along the z axis appearing at $\alpha > 0$.

The argument of the cosine multiplier in (8) can be rewritten in the form $(\omega_0/c)(1-\alpha)(z - z^*)\cos\vartheta$. The maximum at point $z = z^*$ (and the entire fragment with it) pulsate at the frequency $\omega^* = \omega_0(1-2\alpha)(1-\alpha)^{-1} \approx \omega_0 \times (1-\alpha)$.

As a result, the drifting quasi-standing wave (8) can be written in the form

$$B_z = B_0 \sin[\omega_0 t(1-2\alpha)(1-\alpha)^{-1}]$$
$$\times \cos[\omega_0/c)(1-\alpha)(z - z^*)\cos\vartheta] \sin\vartheta \quad (11)$$

with a maximum of the quasi-standing fragment at point $z = z^*$ and the step between the arguments $\pm\pi/2$ of the cosine multiplier

$$\Lambda \approx \frac{\lambda_0}{2(1-\alpha)\cos\vartheta}. \quad (12)$$

Therefore, the quasi-standing wave B_z (11) drifting at a velocity V^* (10) consists of a sequence of potential wells with the length 2Λ and the finite longitudinal vector component $B_z \neq 0$, which are required for acceleration.

The numerical example ($\alpha = 5 \times 10^{-3}$, $\vartheta = 60°$, $V^*/c \approx 0.01$) indicates that even at $\alpha \ll 1$, i.e. at an insignificant deviation of the field B_z (11) from the form of the true standing wave, the relative drift velocity V^*/c (10) can tend to unity.

To increase the velocity V^* while moving the potential well along the z axis (which is necessary for synchronous acceleration) it is sufficient to produce an accelerating electromagnetic field (11) with the angle $\vartheta(z)$ increasing with increasing z. Then, the velocity V^* (10) becomes dependent on the coordinate z and there appears the 'spatial' acceleration

$$\frac{dV^*}{dz} = \frac{c\alpha}{1-\alpha}\frac{\sin\vartheta}{\cos^2\vartheta}\frac{d\vartheta}{dz} = V^* \tan\vartheta \frac{d\vartheta}{dz}, \quad (13)$$

which, when multiplied by V^*, yields the true 'temporal' acceleration

$$\frac{dV^*}{dt} = V^{*2}\tan\vartheta\frac{d\vartheta}{dz} = \frac{c^2\alpha^2}{(1-\alpha)^2}\frac{\sin\vartheta}{\cos^3\vartheta}\frac{d\vartheta}{dz}. \quad (14)$$

Another way to produce true 'temporal' acceleration consists in temporal variation of the frequencies $\omega_{3,4}$, i.e. the frequency detuning parameter $\alpha = \alpha(t)$, when

$$\frac{dV^*}{dt} = c\cos\vartheta\frac{d}{dt}\left(\frac{\alpha}{1-\alpha}\right) \approx c\cos\vartheta\frac{d\alpha}{dt}. \quad (15)$$

While accelerating, the neutron is subjected to force

$$F_\alpha = M_n\frac{dV^*}{dt} \approx cM_n\cos\vartheta\frac{d\alpha}{dt}, \quad (16)$$

which should not exceed the force $F_{z\,\max}$ (7) in order to trap it in the potential well. This implies limitation of the acceleration rate

$$\frac{dV^*}{dt} < M_n^{-1}|\mu_n|k_\vartheta B_0\sin\vartheta. \quad (17)$$

The numerical example: $dV^*/dt < 10^{10}$ cm s^{-2} at $B_0 \approx 10^5$ G.

An important circumstance, which reproduces the Veksler principle of synchronous acceleration of charged particles in a linear accelerator (see, for example, [1]), is the existence of an equilibrium neutron phase lying in the range $\pi/2 < (\omega_0 z^*/c)\cos\vartheta < \pi$ in the drifting potential well.

4. Sketch configuration of a laser linear neutron accelerator

Despite the internal consistency of the considered physical foundations of the laser neutron acceleration concept, its realisation is a challenging problem. The sketch scheme of a possible laser neutron accelerator is as follows.

A beam of ultracold neutrons with the temperature $T \approx 0.001$ K from a source produced by the methods of neutron optics [3] is coupled into an electromagnetic waveguide structure. The neutron optical elements of the neutron source should take into account the gravitational force $F_g = M_n g \approx 10^{-21}$ dyne (g is the gravitational acceleration) [3], which, however, can hardly affect the further acceleration process, being inferior to the magnetic dipole accelerating force $F_{z\,\max}$ (equal to 10^{-13} dyne according to the estimate) by many orders of magnitude.

The electromagnetic field (required for neutron acceleration) having the properties of the model of a drifting standing wave with the positive 'spatial' acceleration can be produced without the temporal variation of the parameter $\alpha(t)$ in hollow electromagnetic waveguides and is described as a superposition of two counterpropagating waves of the TE mode with the finite longitudinal magnetic component B_z. The drifting standing wave, as a superposition of two counterpropagating waves in the waveguide, is described by the approximate expression valid for $\alpha \ll 1$ and equivalent to (8):

$$B_z \sim \sin\left\{\omega_0 t - \frac{\omega_0 z}{2c}\sqrt{1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2}\left[1 - \frac{2\alpha}{1-(\omega_{nm}/\omega_0)^2}\right]\right\}$$

$$\times\cos\left\{\omega_0\alpha t - \frac{\omega_0 z}{2c}\sqrt{1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2}\left[1 + \frac{2\alpha}{1-(\omega_{nm}/\omega_0)^2}\right]\right\} \quad (18)$$

(ω_{nm} is the critical frequency of the TE_{nm} mode).

Away from the critical regime, the maximum of the standing wave fragment is achieved, as in (9), at the zero argument of the cosine at point

$$z^* \approx 2c\alpha t\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-1/2}\left[1+4\alpha(1-\alpha)\right.$$

$$\left.\times\left(1-\frac{\omega_{nm}^2}{\omega_0^2}\right)^{-1}\right]^{-1/2} \approx 2c\alpha t\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-1/2}. \quad (19)$$

The fragment pulsates at a frequency $\omega^* \approx \omega_0(1-\alpha)$ and, as in (10), moves with a velocity

$$V^* \approx 2c\alpha\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-1/2} \quad (20)$$

and the 'spatial' acceleration [equivalent of (13)]

$$\frac{dV^*}{dz} \approx 2c\alpha\left(\frac{\omega_{nm}}{\omega_0}\right)\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-3/2}\frac{d\omega_{nm}}{dz}. \quad (21)$$

Because for a fixed waveguide mode the critical frequency ω_{nm} is inversely proportional to the transverse waveguide cross section A, the derivative in (21) is equal to $d\omega_{nm}/dz = -\mathrm{const}\times A^{-2}(dA/dz) = -(\omega_{nm}/A)(dA/dz)$ and positive at $dA/dz < 0$, i.e. by decreasing the cross section A with increasing z. The 'spatial' acceleration dV^*/dz is transformed into the temporal one by taking the product of the velocity V^*,

$$\frac{dV^*}{dt} = V^*\frac{dV^*}{dz} \approx (2c\alpha)^2$$

$$\times\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-2}\frac{\omega_{nm}}{\omega_0}\frac{d\omega_{nm}}{dz}, \quad (22)$$

which, as shown above, allows one to realise the true acceleration without changing the parameter α in time. This, however, does not eliminate the possibility, as in (15), of producing the true temporal acceleration by varying this parameter:

$$\frac{dV^*}{dt} \approx 2c\left[1-\left(\frac{\omega_{nm}}{\omega_0}\right)^2\right]^{-1/2}\frac{d\alpha}{dt}. \quad (23)$$

Obviously, with regard to experiments the first method is more preferable because it does not require the frequency modulation of laser radiation ($\alpha = \mathrm{const}$). However, if a very low thermodynamic velocity of ultracold neutrons $V_0^* \approx 44$ cm s^{-1} [2] is used, the achievement of the large values of the total acceleration coefficient requires a very small frequency detuning parameter ($\alpha \sim 3\times10^{-8}$) at the initial segment.

To eliminate the expected electric breakdown when an intense electromagnetic wave propagates in a small-cross-section waveguide, the waveguide structure can be replaced by an analogue of the lens waveguide for Gaussian beams having a consistent variable cross section with a narrow common neutron channel on the axis of the lens sequence.

5. Conclusions

Summing up, we can make a conclusion about the potential possibility of laser neutron acceleration up to energies of the order of kiloelectronvolt. However, to give a final estimate of the efficiency of the scheme under study, it is necessary to further analyse the number of neutrons involved in acceleration, taking into account both the low intensity of the existing sources of ultracold neutrons ($\sim 10^5$ cm^{-2} s^{-1} [3]) and the requirements to maintain the transverse stability of the accelerated beam.

The central element of the acceleration scheme is the so-called drifting standing wave with the potential wells moving with an increasing velocity together with the neutrons trapped in them. The produced beam of accelerated ultracold neutrons is highly mono-kinethic (the ratio of the thermal energy of accelerated neutrons to the energy of their directed motion is $\sim 10^{-11}$) and characterised by the low divergence (for example, at $\sim 10^7$ cm s^{-1}, the deviation from the beam axis at a 1-km-long trajectory is about of 1 cm).

The directed beam of ultracold neutrons arriving to a target at a relatively short time (of the order of milliseconds) is scattered and heated in the target. If the target contains isotopes capable of interacting with neutrons, this creates prerequisite for remote initiation of nuclear reactions in the target, including (taking into account the required isotope composition of the target) chain reactions as well as for remote examination of the target composition. It is necessary to take into account the fact that while moving along the external extended trajectory, the gravitational force $F_g = M_{ng} = 10^{-21}$ dyne proves sufficient for introducing a noticeable additional beam displacement. Other fields of possible applications of accelerated beams of ultracold neutrons include neutron microscopy and other optical problems as well as localised therapeutic treatment.

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