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Mirror with reflectance variable in amplitude and phase. 1. Modelling of a mirror with reflectance variable in amplitude and phase

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***Abstract*. An operator model of a mirror as a reflector with a reflectance variable over the aperture is developed on the basis of a Fabry–Perot interferometer. The main characteristics of the interferometer (mirror) are studied in the geometric, diffraction, and modal approximations. The differences in the calculation results obtained using the conventional geometric approximation and the diffraction approach proposed in this study are discussed. It is shown that, when the effective Fresnel numbers are comparable to unity, it is necessary to use the diffraction approximation. The possibility of using an interferometer-based mirror as a selector of transverse laser modes is demonstrated.**

Keywords: *mirror with reflectance variable in amplitude and phase, Fabry–Perot interferometer, laser resonator, microchip laser, transverse mode selection.*

1. Introduction

The mirrors with reflectance variable over the aperture have found wide application in laser systems [1–3]. The introduction of apodized mirrors into a laser cavity allows one to decrease the beam divergence by a factor of 1.2–2.5 and to achieve the beam quality close to the diffraction limit [2–4]. Variations in the mirror reflectance over the mirror aperture can be achieved by several methods, namely, by depositing profiled dielectric coatings [5] or by using birefringence elements [2] or Fabry–Perot interferometers with nonplanar mirrors [1]. The latter method possesses a wide scope of possibilities for forming various reflectance distributions. In addition, this method seems to be the only method that can be used in modern miniature and microchip lasers, which have a small transverse size of the output beam. The formation of a field amplitude distribution in a multibeam Fabry–Perot interferometer is described, as rule, considering the wave propagation only in the geometric approximation [1, 6]. However, an important role in the formation of fields by a mirror interferometer in, for example, microchip lasers with an aperture of ~ 100 μm can be played by diffraction effect.

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There are works devoted to the study of the role played by diffraction in laser cavities (see [7, 8] and references therein). At the same time, the role of diffraction in Fabry–Perot interferometers with nonplanar mirrors is not sufficiently studied.

In the present paper, we propose a method of diffraction calculation of a cavity mirror based on a Fabry–Perot interferometer composed of nonplanar mirrors. The differences in the calculation results obtained using the diffraction and the conventional geometric approaches are discussed. The possibility of using an interferometer-based mirror as a selector of transverse laser modes is demonstrated.

2. Mathematical model of a mirror interferometer

Let us consider an interferometer that is schematically shown in Fig. 1. The interferometer is composed of two semitransparent reflectors with given reflectances variable in amplitude and phase as $R_q \exp(\mathrm{i}\alpha_q)$ [$q = 1$, 2 for reflectors (*1*) and (*2*), respectively]. Here, R_q is the reflectance; α_q is the phase component, which depends on the mirror shape and appears in the field distribution function inside the interferometer due to the reflection of radiation from this mirror; U_0 is an arbitrary radiation field incident on the interferometer from the side of reflector (*1*); $U_{01} = U_0 T_1$; T_1 is transmittance of reflector (*1*); U_1 is the radiation field reflected from mirror (*1*) and propagating toward reflector

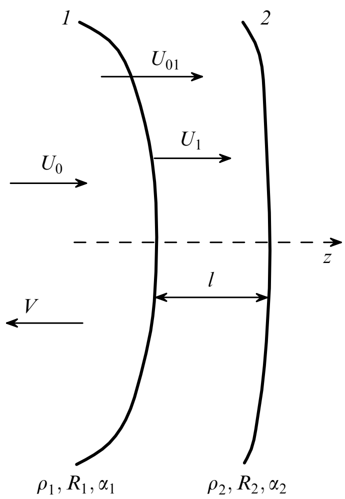

Figure 1. Interferometer scheme.

(*2*), which is determined in the aperture plane of mirror (*1*); and V is the field reflected from the interferometer. In the general case, α_q, U_0, U_{01}, U_1 and V depend on the coordinates of the point on the corresponding reflector aperture; R_1 and R_2 are taken to be real constants; and the semitransparent reflectors are assumed to be infinitely thin.

One of the main physical characteristics of an interferometer is reflectance. The radiation field reflected from an interferometer is

$$V = \exp(-\mathrm{i}\alpha_1)[(T_1/R_1)U_1 - R_1 U_0], \tag{1}$$

and the power reflectance of an interferometer is

$$R_{\text{int}}^2 = \int |V|^2 \mathrm{d}S \Big/ \int |U_0|^2 \mathrm{d}S; \tag{2}$$

where integration is performed over the aperture of reflector (*1*).

The field inside an interferometer can be considered as a superposition of the wave directly transmitted through the reflector and the waves formed as a result of all multiple reflections. The output field is determined as a sum of all the fields taking into account the corresponding phase incursions and amplitude variations. The effective number F of round-trips in an interferometer (the number of interfering beams) depends on the parameter $|R_1R_2|$: $F \approx \pi\sqrt{|R_1R_2|}/(1-|R_1R_2|)$. [7]. The amplitude–phase distributions of fields in different interfering beams are different due to their diffraction and geometric divergences. The diffraction divergence must be taken into account when the effective Fresnel number is $N_{\text{eff}} = N_{\text{int}}/2F \lesssim 1$. Here, $N_{\text{int}} = d^2/(4\lambda l)$ is the Fresnel number for the interferometer (d is the beam diameter, λ is the radiation wavelength, and l is the interferometer length).

In [1, 9], the characteristics of an interferometer with spherical mirrors were studied in detail in the geometric approximation. Let us derive a relationship for the field U_1 in the diffraction approximation. Taking into account the results of studies [10, 11] devoted to multipass telescopes, we will consider an interferometer as a stationary system with a self-reproducing field. The field U_1 reflected from mirror (*1*) is summed with the field U_{01}, thus forming the field $U_1 + U_{01}$ propagating to reflector (*2*). After a round trip in the interferometer, this field transforms into U_1. Let a round trip be described by the propagation operator K. Then, the operator equation of the interferometer takes the form

$$K(U_1 + U_{01}) = U_1. \tag{3}$$

In the absence of nonlinear field effects, we have

$$KU_1 + KU_{01} = U_1. \tag{4}$$

In this equation, U_1 is the sought field. Solving Eqn (4) with respect to U_1, from (1) we find the reflected field

$$V = \frac{\exp(-\mathrm{i}\alpha_1)}{R_1}\left(\frac{T_1^2 K}{E-K} - R_1^2\right)U_0, \tag{5}$$

where E is the unit operator.

In the diffraction approximation, the operator K is the field integral transform with a known kernel [8, 12]. Passing to a discrete lattice of individual sub-apertures of the reflector, expression (5) takes a matrix form. In this case, E is the $N \times N$-dimension unit matrix, N^2 is the number of nodes in the discrete lattice, K is the $N \times N$-dimension square matrix, and $\exp(-\mathrm{i}\alpha_1)$ is the diagonal matrix of the same dimension.

In the geometric quasi-plane wave approximation, the round-trip operator is given by the relationship $K = R_1R_2\exp(2\mathrm{i}kl)\exp[\mathrm{i}(\alpha_1+\alpha_2)]$, which enters (4) as a factor. Here, $k = 2\pi/\lambda$ is the wave number and $E = 1$.

Using relations (5), (1), and (2), we can calculate the main characteristics of an interferometer as a laser mirror. However, the analysis of its selecting properties is rather complicated. Let us consider the interferometer using the laser resonator theory. We will solve Eqn (4) by expanding into modes. We introduce into consideration the orthonormal eigenfunctions Ψ_n (EFs) and the eigenvalues γ_n ($n = 0, 1, 2, \ldots$) (EVs) of the propagation operator $K/(R_1R_2)$. Then,

$$K\Psi_n = (R_1R_2)\gamma_n\Psi_n. \tag{6}$$

The solution of Eqn (6) with respect to Ψ_n, γ_n coincides with the solution of (6) for a laser cavity [8] with mirrors whose shape coincides with the shape of the interferometer reflectors with the reflectance equal to unity. In (4), we will use the EF expansion of the incident field U_0 and of the field U_1:

$$U_0 = \sum U_0^{(n)}\Psi_n, \quad U_1 = \sum U_1^{(n)}\Psi_n. \tag{7}$$

The expansion coefficients $U_0^{(n)}$ are calculated from the known functions U_0 and Ψ_n, while $U_1^{(n)}$ are the sought coefficients. Substituting (7) into (4), we find

$$U_1^{(n)} = \frac{R_1R_2\gamma_nT_1}{1-R_1R_2\gamma_n}U_0^{(n)}. \tag{8}$$

Thus, the field (1) reflected for the interferometer is written in the form

$$V = \exp(-\mathrm{i}\alpha_1)\sum V^{(n)}\Psi_n,$$

where $V^{(n)} = R_{\text{int}}^{(n)}U_0^{(n)}$;

$$R_{\text{int}}^{(n)} = \frac{R_2\gamma_n - R_1}{1-R_1R_2\gamma_n} \tag{9}$$

is the field reflectance of the interferometer for the nth mode, $\gamma_n = |\gamma_n|\exp(\mathrm{i}\varphi_n)$; $|\gamma_n| \leqslant 1$; $\varphi_n = 2kl - \Delta\varphi_n$; $\Delta\varphi_n$ is the correction to the geometric phase incursion per one round trip [12]. The power of the nth mode field is

$$|V^{(n)}|^2 = |R_{\text{int}}^{(n)}|^2|U_0^{(n)}|^2, \tag{10}$$

where

$$|R_{\text{int}}^{(n)}|^2 = \frac{(R_1 - R_2^{(n)})^2 + 4R_1R_2^{(n)}\sin^2(\varphi_n/2)}{(1-R_1R_2^{(n)})^2 + 4R_1R_2^{(n)}\sin^2(\varphi_n/2)}$$

is the power reflectance of the interferometer for the nth mode and $R_2^{(n)} = |\gamma_n|R_2$ is the effective reflectance of reflector (*2*) for the nth mode. In this case, the power reflectance (2) of the interferometer is

$$R_{\rm int}^2 = \frac{\sum |R_{\rm int}^{(n)}|^2 |U_0^{(n)}|^2}{\sum |U_0^{(n)}|^2}.$$

Relation (10) is similar in form to the well-known formula for the reflection coefficient of a plane interferometer. The difference is that the phase of the propagating mode takes into account not only the geometric but also diffraction incursion $\Delta\varphi_n$ [13]. In addition, in the general case, the effective reflection coefficient $R_2^{(n)}$ decreases with increasing n. The values $\Delta\varphi_n$ and $R_2^{(n)}$ are different for different modes. It is these circumstances that make it possible to use interferometers as transverse mode selectors.

Let an interferometer be a stable resonator with infinite spherical mirrors. In this case, $|\gamma_n| = 1$ and the selective properties of the interferometer are determined by the parameter $\varphi_n = 2kl - \Delta\varphi_n$. The power of a selected nth mode is maximal under the condition $kl = \pi m + \pi/2 + \Delta\varphi_n/2$, $m = 1, 2, ...$ However, if the phase distances between modes are small ($|\varphi_n - \varphi_{n+1}| \ll 1$), it is reasonable to choose the interferometer length taking into account the rate of change of function (10) with $\Delta\varphi_n$.

3. Results of numerical experiments

The main properties of the mirror interferometer were numerically studied based on relations (2) and (5) in the diffraction and geometric approximations using the matrix description of fields and propagation operators. In the numerical experiment, we considered an interferometer (see Fig. 1) designed to be used as an output mirror of a microchip laser [14]. We studied the interferometer selectivity with respect to reflectance (2) and the coincidence of calculation results obtained using the geometric and diffraction approaches. The interferometer consisted of two spherical reflectors. The phase components introduced into the field inside the interferometer due to reflection from mirrors (*1*) and (*2*) were given by the relations

$$\alpha_1(\boldsymbol{r}) = \frac{kr^2}{\rho_1}, \quad \alpha_2(\boldsymbol{r}) = \frac{kr^2}{\rho_2}, \tag{11}$$

where ρ_1, ρ_2 are the radii of curvature of reflectors (*1*) and (*2*), whose signs were chosen according to the resonator theory [8]; $\boldsymbol{r}$ is the radius vector of the point on the reflector aperture in the cylindrical coordinate system with the z axis coinciding with the optical axis of the interferometer; $\boldsymbol{r} = (x, y)$; and x, y are the Cartesian coordinates of the point.

In calculations, we assumed that the interferometer, when considered as a resonator, has a stable configuration. The incident field U_0 was given by the functions describing the TEM$_{\mu,\nu}$ modes (μ, ν are the transverse indexes) of a cavity external with respect to the interferometer. The field U_0 and the filed corresponding to Ψ_n had identical radii of curvature but different amplitude distributions. The amplitude–phase distribution of U_0 was given by the Hermite polynomial [8].

Figure 2 shows the results of calculation of the amplitude–phase distribution of a laser beam field reflected from an interferometer with the length l ($l = l_0 + \Delta l$, where l_0 is the base length of the interferometer multiple of an integer number of half-wavelengths and Δl is the variable displacement of one of the interferometer mirrors along the optical axis). We considered a one-dimensional incident beam with the TEM$_{00}$ mode field distribution. The effective Fresnel number for the interferometer is $N_{\rm eff} \approx 0.35$. In this case, as should be expected, the fields calculated in the diffraction and geometric approximations were considerably different.

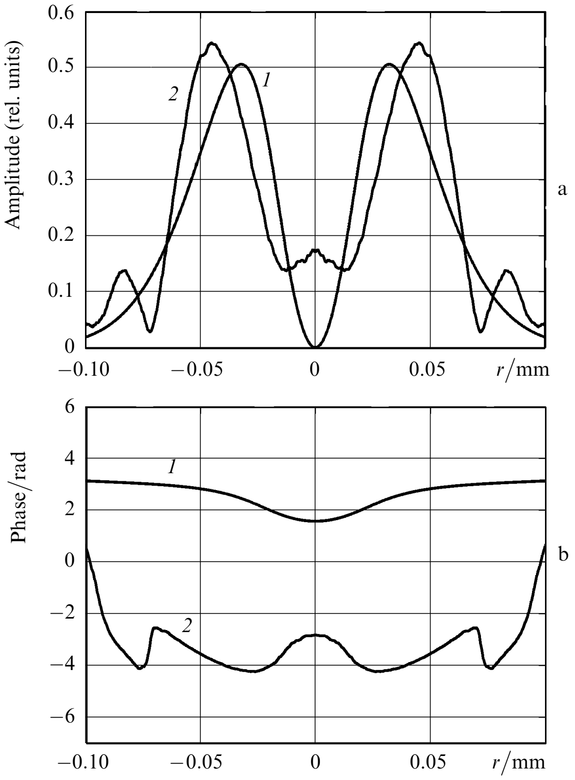

Figure 2. Amplitude (a) and phase (b) distributions of a field reflected from an interferometer calculated in the geometric (*1*) and diffraction (*2*) approximations at $\rho_1 = \infty$, $\rho_2 = 0.02$ m, $l_0 = 500\lambda$, $\lambda = 1.064$ μm; $\Delta l = 0$, $d = 140$ μm, and $R_1 = R_2 = 0.88$.

The interferometer reflectances (2) calculated for several lowest transverse modes of the stable external resonator are shown in Figs 3 and 4. The dependences in Fig. 3 are calculated for the interferometer configuration with $\rho_2 = -\rho_1$, $\rho_1 > 0$. In this case, the reflectance in the geometric approximation depends only on the interferometer length l and reflectances R_1 and R_2 of its mirrors. The interferometer affects the transmitted and reflected fields similar to a semitransparent mirror with a radius of curvature equal to the radius of curvature of reflector (*1*). However, in the diffraction approximation, the reflectances for different modes are noticeably different.

Figure 4 presents the results of calculations of $R_{\rm int}^2$ for a mirror interferometer with $\rho_2 = -\rho_1 + 1100\lambda$. Its selectivity was found to be considerably higher than the selectivity of an interferometer with $\rho_2 = -\rho_1$. This is explained by the fact that the intermode phase differences strongly depend on ρ_1, ρ_2 (just as in stable resonators) and are larger at $\rho_2 = -\rho_1 + 1100\lambda$.

With increasing Fresnel number $N_{\rm int}$, which was varied in our calculations by changing the interferometer length, all the reflectances become close to geometric ones. In these calculations, the reflectance R_1, R_2 of the interferometer mirrors were chosen so that the effective Fresnel numbers $N_{\rm eff}$ varied from 0.2 to 1. Note that, according to the calculation results, the interferometer reflectances (2) for the TEM$_{\mu\nu}$ modes with identical sums of indices $\mu + \nu$ are almost the same. This occurs because the phase incursion

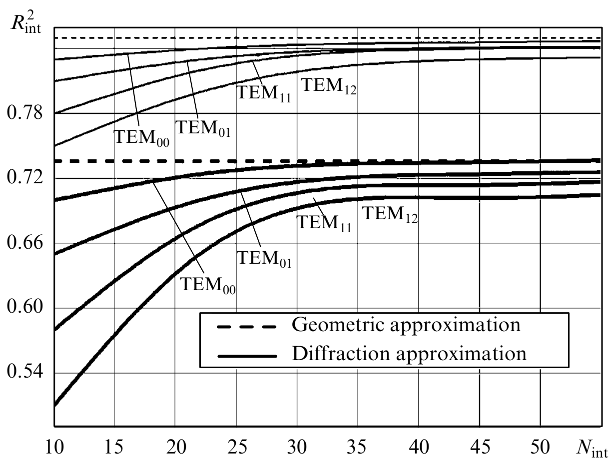

Figure 3. Dependences of the interferometer reflectances R^2_{int} on the Fresnel number N_{int} for $\Delta l = 0.05\lambda$ (thin lines) and $\Delta l = 0.03\lambda$ (thick lines) at $\lambda = 1.064$ μm, $\rho_2 = -\rho_1$, $\rho_1 = 0.07$ m, $d = 100$ μm, and $R_1 = R_2 = 0.88$.

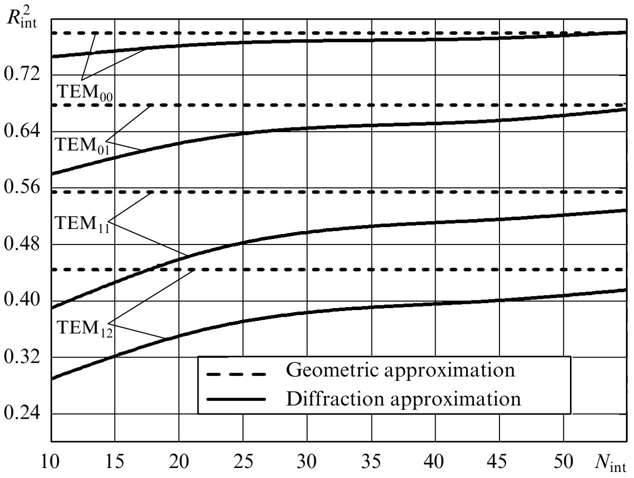

Figure 4. Dependences of the interferometer reflectances R^2_{int} on the Fresnel number N_{int} for $\Delta l = \lambda/40$, $\rho_2 = -\rho_1 + 1100\lambda$, $\rho_1 = 0.07$ m, $d = 100$ μm, $R_1 = R_2 = 0.88$, and $\lambda = 1.064$ μm.

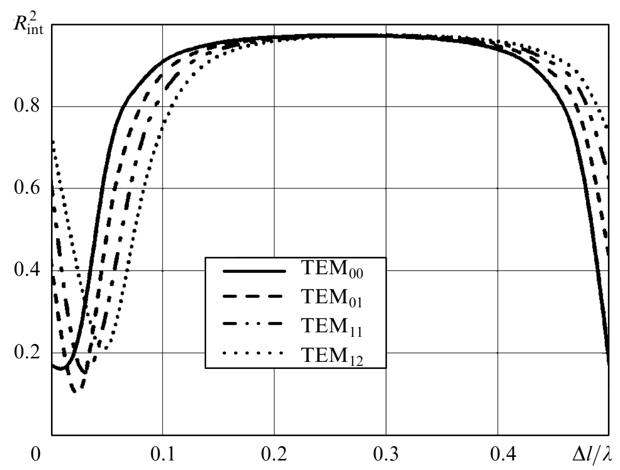

Figure 5. Dependences of the interferometer reflectances R^2_{int} on the misalignment Δl at $\rho_1 = \infty$, $\rho_2 = -0.06$ m, $l_0 = 100\lambda$, $\lambda = 1.064$ μm, $d = 200$ μm, and $R_1 = R_2 = 0.88$.

$\Delta\varphi_{\mu+\nu}$ appearing additionally to the geometric one in the case of expansion of the field $\mathrm{TEM}_{\mu\nu}$ mode in terms of $\Psi_{\mu+\nu}$ functions is identical for these modes. Therefore, Figs 3 and 4 present the curves not for all lowest transverse modes but only for the modes with different sums of indices $\mu + \nu$.

It is necessary to note that the length $l = l_0 + \Delta l$ ($l_0 = m\lambda/2$, where m is an integer number) in our calculations was discretely changed with changing m (the misalignment Δl was fixed), and, hence, the number $N_{int} = N_{int}(m)$ also varied discretely. For convenience, the dependences $R^2_{int}(N_{int})$ are shown in Figs 3 and 4 by continuous lines obtained by extrapolation of the calculated data.

The interferometer selectivity also strongly depends on the misalignment Δl (Figs 3 and 5). The data presented in Fig. 5 allow one to optimise Δl with respect to the mode selection and the parameter R^2_{int}. For example, as one can see from Fig. 5, to select the TEM_{00} mode, it is preferable to use the misalignment region $\Delta l \approx 0.05\lambda - 0.12\lambda$. In this region, the reflectances for neighbouring modes differ by a factor of 1.5–2. At the same time, in the misalignment regions $\Delta l \approx 0 - 0.01\lambda$ and $\Delta l \approx 0.43\lambda - 0.5\lambda$, the TEM_{00} mode is suppressed and the TEM_{01} becomes dominant. In the displacement region where the higher modes dominate, ΔN_{eff} decreases from the point Δl close to 0.5λ.

The presented results confirm the possibility of using an interferometer as a selector of transverse cavity modes. When optimising the interferometer parameters, it is necessary to take into account the amplitude–phase distributions of the resonance fields that form inside the external laser cavity and are incident with respect to the interferometer.

4. Conclusions

Based on the developed operator model of a mirror interferometer as a reflector with a reflectance variable over the aperture, the basic characteristics of the interferometer are studied in the geometric, diffraction, and modal approximations. It is shown that the spherical interferometer can be used as a selector of transverse laser modes. At the optimal choice of the mirror interferometer parameters (base, misalignment, curvature radius of reflectors), the relative difference in the reflectance of neighbouring modes can reach a factor of 1.5–2. In the cases when the effective Fresnel number is $N_{eff} \lesssim 1$, the mirror interferometer parameters must be optimised using the diffraction approach.

The mirror interferometer has been considered as a passive unit, which transforms the laser radiation but does not affect the laser operation. The second part of the work is to be devoted to the investigation of the properties of an interferometer and a laser cavity as a unified system.

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