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Acceleration of neutrons in a scheme of a tautochronous mathematical pendulum (physical principles)

L.A. Rivlin

***Abstract*.** **We consider the physical principles of neutron acceleration through a multiple synchronous interaction with a gradient rf magnetic field in a scheme of a tautochronous mathematical pendulum.**

Keywords: *quantum nucleonics, synchronous acceleration of neutral particles, low-divergent beams of fast ultracold neutrons.*

1. Introduction

Directional low-divergent and monoenergetic beams of neutrons could be of pragmatic interest, for example, in localised therapeutic intervention, remote diagnostics of different targets, etc. One of the possible ways to produce such beams is acceleration of ultracold neutrons, while maintaining their thermodynamic temperature. In fact, a beam with a directional velocity of 10^7 cm s^{-1} and chaotic velocities of ~ 100 cm s^{-1} (ultracold neutrons) would be relatively monoenergetic ($\sim 10^{-10}$) and divergent ($\sim 10^{-5}$ rad), i.e., would have the values comparable with similar parameters of optical lasers. The above-said can motivate the present research aimed at the analysis of the physical principles of neutron acceleration through a multiple synchronous and phased interaction with an alternating magnetic field in a scheme of a mathematical pendulum.

This analysis is based on the analogy of the considered phenomena with the known fact that the frequency of a nonrelativistic charged particle rotation along a circular trajectory in a magnetic field is independent of its energy. This trajectory tautochronism provides the basis for the process of particle acceleration through its multiple phased and synchronous interaction with an alternating electric field in a cyclotron, which resulted in the appearance of a ramified family of syncronous accelerators [1].

A similar tautochronism of a mathematical pendulum, i.e., independence of its oscillation frequency from the energy, can be used to accelerate synchronously the neutrons (and generally speaking – neutral particles), which cannot move along a circular trajectory in a magnetic field because of the charge absence.

At the same time, a chargeless neutron with a magnetic dipole moment $|\mu_n| \approx 0.95 \times 10^{-23}$ erg Gs^{-1} experiences the action of the force $\boldsymbol{F} = \boldsymbol{\mu}_n \times \mathrm{grad}\boldsymbol{B}$ in a spatially inhomogeneous magnetic field with the induction $\boldsymbol{B}$. Despite the smallness, this force in the case of its multiple application can prove sufficient to impart the directional velocity to neutrons, this velocity exceeding markedly the neutron chaotic thermal velocities [2].

2. A mathematical pendulum

As is known, this pendulum is a body with mass M, that performs oscillations $z = z_a \sin \Omega t$ at a frequency $\Omega = (b/M)^{1/2}$ according to the equation $\mathrm{d}^2 z/\mathrm{d}t^2 = -bz/M$ ($b = \mathrm{const}$, t is the time, z_a is the oscillation amplitude). If the oscillating body is a neutron, the vectors $\boldsymbol{B}$ and $\boldsymbol{\mu}_n$ are parallel to the z axis and the induction depends parabolically on the coordinate,

$$B = B(z = 0) + \frac{M\Omega^2 z^2}{2|\mu_n|}, \qquad (1)$$

the restoring force is $bz = |\mu_n| \mathrm{d}B/\mathrm{d}z$.

The frequency of the pendulum oscillations Ω and the neutron energy W are expressed by the highest (but nonrelativistic) neutron velocity $(\mathrm{d}z/\mathrm{d}t)_0 \ll c$ at the point $z = 0$:

$$\Omega = \frac{1}{z_a}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)_0, \qquad (2)$$

$$W = \frac{M_n}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)_0^2 = \frac{M_n z_a^2 \Omega^2}{2}, \qquad (3)$$

where c is the speed of light; $M_n \approx 1.67 \times 10^{-24}$ g is the neutron mass.

Important parameters limited by the experimental capabilities are the difference of the magnetic inductances between the points $z = z_a$ and $z = 0$

$$|\Delta B| = \frac{M_n}{2|\mu_n|}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)_0^2 \qquad (4)$$

and the magnetic induction gradient

$$\left|\frac{\mathrm{d}B}{\mathrm{d}z}\right| = \frac{M_n}{|\mu_n| z_a}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)_0^2. \qquad (5)$$

L.A. Rivlin Applied Physics Laboratory, Moscow State Institute of Radioengineering, Electronics and Automation (Technical University), prosp. Vernadskogo 78, 119454 Moscow, Russia;
e-mail: lev_rivlin@mail.ru

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At the maximal amplitude and velocity, $z_a = z_{max}$ and $(dz/dt)_0 = (dz/dt)_{max}$, both

$$|\Delta B| = |\Delta B|_{max} = \frac{M_n}{2|\mu_n|}\left(\frac{dz}{dt}\right)^2_{max}$$

and

$$\left|\frac{dB}{dz}\right| = \left|\frac{dB}{dz}\right|_{max} = \frac{M_n}{|\mu_n| z_{max}}\left(\frac{dz}{dt}\right)^2_{max}$$

become maximal. The obtained estimates are as follows: $z_{max} = 1000$ cm, $|\Delta B|_{max} = 1$ MGs, $|dB/dz|_{max} \approx 2$ kGs cm^{-1}, $\Omega \approx 11$ kHz, $W_{max} \approx 5$ µeV, $(dz/dt)_{max} \approx 1.1 \times 10^7$ cm s^{-1} = 110 km s^{-1}.

3. Neutron acceleration

This process occurs when the neutron multiply interacts with a spatially inhomogeneous magnetic field, changing periodically in time with the frequency Ω. If, for example, the interaction region with the length that is much smaller than the maximal amplitude z_{max}, is placed in the vicinity of a zero point of the pendulum $z = 0$, the acceleration takes place twice during an oscillation period $2\pi/\Omega$ at moments when the magnetic induction of the accelerating field achieves the maximum B_A. The total duration of the multiple acceleration events should not exceed the lifetime of a free neutron $\tau_n \approx 900$ s, i.e., the number of the events is limited by the inequality $N \leqslant \Omega\tau_n/\pi$ (for the data of the previous example, $N < 1.5 \times 10^6$). At each passage through the accelerating region of width A, the neutron acquires the energy

$$\Delta W \approx |\mu_n|\left(\frac{dB_A}{dz}\right)A \approx |\mu_n| B_A,$$

while the total energy W is achieved by the neutron after

$$\frac{W}{\Delta W} = \frac{W}{|\mu_n| B_A}$$

acceleration events whose number should not exceed N, the magnetic induction of the accelerating field being expressed as

$$B_A > \frac{\pi W}{|\mu_n|\Omega\tau_n} \approx \frac{\pi M_n z_{max}^2 \Omega}{2|\mu_n|\tau_n}.$$

This indicates the necessity to have a substantial concentration of an rf accelerating field (for the previous example, $B_A > 3$ MGs).

The neutron accelerator is loaded by injecting the neutrons (in particular, ultracold neutrons) in the pendulum channel near $z = 0$, while the high-speed neutron output is performed by switching of the magnetic field in the output arm of the pendulum. Neutrons, which are in the corresponding rf half-period and constitute approximately half their total number, participate in acceleration. The second half of the out-of-phase neutrons experience deceleration and can get lost.

4. A levitating neutron in a 'zero-gravity state'

During quite a long acceleration process the neutron trajectory can experience noticeable perturbations under the action of the gravitational force. This can be prevented by placing vertically the longitudinal axis z of the pendulum and by introducing a compensating gradient of the vertical component B_z of the magnetic induction $\mathrm{grad}_z B_z = dB_z/dz = M_n g/|\mu_n| \approx 175$ Gs cm^{-1}. As a result, the neutron finds itself in a 'zero-gravity' state without any gravitational perturbation of its motion.

5. Conclusions

The performed analysis of the synchronous rf acceleration of neutrons in a scheme of a mathematical pendulum indicates its physical consistency and the possibility of generation of fast neutron beams with the velocities above hundreds of kilometres per second and small divergence (in the case of ultracold neutrons – with the divergence close to radiation divergence of optical lasers).

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