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OPTICAL PARAMETRIC OSCILLATORS

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Optimal feedback in efficient single-cavity optical parametric oscillators

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Abstract. **An approach based on the description of competition of quadratic processes of merging and decomposition of quanta resulting in the formation of cnoidal waves on an effective cascade cubic Kerr-type nonlinearity is used to optimise the scheme of a single-cavity optical parametric oscillator. It is shown that the use of a feedback circuit (cavity) decreases the period of cnoidal waves produced in a nonlinear crystal, while the optimisation procedure of the transfer constant of this circuit (reflectivity of the output mirror of the cavity) is reduced to matching this period with the nonlinear crystal length.**

Keywords: *quadratic and cascade cubic nonlinearities, optical parametric oscillator, period of cnoidal waves, nonlinear crystal length, optimisation of the transfer constant of the feedback circuit.*

1. Introduction

The authors of paper [1] showed that periodic solutions of the nonlinear Schrödinger equation (NSE) – cnoidal waves – play a key role in one of the classical problems of nonlinear optics, namely, in describing the parametric frequency conversion on a quadratic nonlinearity [2]. It was established that the problem of interaction of three plane monochromatic waves, modes with the frequencies ω_1, ω_2 and $\omega_3 = \omega_1 + \omega_2$ is conveniently solved by increasing the order of the system of truncated equations, which is reduced, in this case, to three independent NSEs coupled by boundary conditions. This corresponds to the description of competition of quadratic processes of quantum merging ($\omega_1 + \omega_2 \to \omega_3$) and decomposition ($\omega_3 \to \omega_1 + \omega_2$) by the effective cascade cubic Kerr-type nonlinearity [3]. In paper [4], the same approach was used to consider cascade parametric processes. Papers [5, 6] presented the solutions describing the parametric generation (including cascade generation) in a scheme of a travelling-wave generator in which the feedback (cavity) is absent and, therefore, a weak input noise is parametrically amplified in a nonlinear crystal.

Below, we will use a similar approach to solve the problem of optimisation of a single-cavity optical parametric oscillator (OPO) scheme [2]. We will show that in this case the use of the feedback circuit (cavity) in the oscillator decreases the period of the cnoidal waves being produced and the optimisation procedure of the transfer constant of this circuit (reflectivity of the output mirror of the cavity) is reduced to matching this period with the nonlinear crystal length.

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2. Parametric conversion and NSE

As in paper [1], consider collinear interaction of three plane monochromatic waves: two waves – at the fundamental frequency (amplitudes $A_{1,2}$, frequencies $\omega_{1,2} = \omega$, wave vectors $\boldsymbol{k}_{1,2}$) and one wave – at the second harmonic frequency (amplitude A_3, frequency $\omega_3 = 2\omega$, wave vector $\boldsymbol{k}_3$), propagating from the plane $z = 0$ along the z axis in a medium with a quadratic nonlinearity, i.e., in a nonlinear crystal. Neglecting anisotropy and absorption, we assume first of all that the nonlinear crystal occupies the half-space $z \geqslant 0$ and realises nondegenerate (due to orthogonal linear polarisations of modes at frequencies $\omega_{1,2}$) parametric process of the so-called II type (oee interaction), which is described by the well-known system of equations for the amplitudes of three coupled fields – modes [2]:

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = -\mathrm{i}\beta A_2^* A_3 \exp(-\mathrm{i}\Delta z), \tag{1a}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}z} = -\mathrm{i}\beta A_1^* A_3 \exp(-\mathrm{i}\Delta z), \tag{1b}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = -\mathrm{i}2\beta A_1 A_2 \exp(+\mathrm{i}\Delta z), \tag{1c}$$

where β is the nonlinear coupling constant; $\Delta = k_1 + k_2 - k_3$ is the wave detuning. System (1) has two second-order integrals of motion:

$$\begin{gathered} I_1(z) + I_2(z) + I_3(z) = I_{10} + I_{20} + I_{30}, \\ I_1(z) - I_2(z) = I_{10} - I_{20}, \end{gathered} \tag{2}$$

describing the law of conservation of the energy flux density and Manly – Row relations [2]. Here, $I_i(z) = A_i(z)A_i^*(z)$ is a variable proportional to the intensity of the ith ($i = 1 - 3$) wave, which we will call below the intensity; $I_{i0} = I_i(z = 0)$.

After substituting $A_i(z) \to \tilde{A}_i(z)\exp(-\mathrm{i}\alpha_i z)$ and selecting such constants α_i so that

$$\alpha_1 + \alpha_2 - \alpha_3 - \Delta = 0, \tag{3}$$

system (1) is reduced to three closed second-order equations having the form of a NSE:

$$\frac{d^2\tilde{A}_1}{dz^2} - \beta^2\left(4I_{10} - 2I_{20} + I_{30} - \frac{\Delta^2}{4\beta^2} - 4A_1A_1^*\right)\tilde{A}_1 = 0, \tag{4a}$$

$$\frac{d^2\tilde{A}_2}{dz^2} - \beta^2\left(-2I_{10} + 4I_{20} + I_{30} - \frac{\Delta^2}{4\beta^2} - 4A_2A_2^*\right)\tilde{A}_2 = 0, \tag{4b}$$

$$\frac{d^2\tilde{A}_3}{dz^2} + 2\beta^2\left(I_{10} + I_{20} + I_{30} + \frac{\Delta^2}{8\beta^2} - \tilde{A}_3\tilde{A}_3^*\right)\tilde{A}_3 = 0 \tag{4c}$$

with boundary conditions supplemented by the values of the first derivatives $\tilde{A}_i$ at $z = 0$

$$\tilde{A}_{10} = A_{10},\ \left.\frac{d\tilde{A}_1}{dz}\right|_{z=0} = i\frac{\Delta}{2}\tilde{A}_{10} - i\beta\tilde{A}_{20}^*\tilde{A}_{30}, \tag{5a}$$

$$\tilde{A}_{20} = A_{20},\ \left.\frac{d\tilde{A}_2}{dz}\right|_{z=0} = i\frac{\Delta}{2}\tilde{A}_{20} - i\beta\tilde{A}_{10}^*\tilde{A}_{30}, \tag{5b}$$

$$\tilde{A}_{30} = A_{30},\ \left.\frac{d\tilde{A}_3}{dz}\right|_{z=0} = -i\frac{\Delta}{2}\tilde{A}_{30} - i2\beta\tilde{A}_{10}\tilde{A}_{20}, \tag{5c}$$

which follow from system (1). The set of constants α_i in equations (4a), (4b), and (4c) is different:

$$\alpha_1 = \Delta/2,\ \ \alpha_2 - \alpha_3 = \Delta/2, \tag{6a}$$

$$\alpha_1 - \alpha_3 = \Delta/2,\ \ \alpha_2 = \Delta/2, \tag{6b}$$

$$\alpha_1 + \alpha_2 = \Delta/2,\ \ \alpha_3 = -\Delta/2, \tag{6c}$$

and can be combined only in pairs [7].

3. Nondegenerate parametric generation in the absence of feedback

System (4) can be conveniently solved by selecting preliminary the real amplitudes X_i and phases φ_i of the modes interacting in the nonlinear crystal with the help of substitution of the variables $\tilde{A}_i \to X_i \exp(i\varphi_i)$. Then, it would be logical to assume that $I_{10,20} = 0$ and $I_{30} \neq 0$. However, taking (6) into account the required solutions of (4) will be trivial in this case ($\tilde{A}_{1,2} \equiv 0$ and $\tilde{A}_3 \equiv \tilde{A}_{30}$) and unstable with respect to fluctuations $A_{10,20}$. Therefore, we will have to consider below a more complicated situation when seed radiation with the frequency ω_1 and intensity $I_{30} = X_{30}^2 \gg I_{10} = X_{10}^2 \neq 0$ is present at the nonlinear crystal input, where $X_{i0} = X_i\,(z = 0)$. Then, the waves at the frequencies ω_1 and ω_2 will be conditionally (due to the symmetry of subscript permutation $1 \leftrightarrow 2$) called idle and signal waves, respectively.

Taking into account the fact that $\tilde{A}_{20} = 0$, the phase $\tilde{A}_2$ is a constant and $\varphi_2 = \varphi_{20}$ [8]. In this case, the maximal initial (at point $z = 0$) growth rate of I_2 is realised under the condition $\varphi_{20} = \varphi_{30} - \varphi_{10} - \pi/2$, which we will consider fulfilled. Then,

$$\left.\frac{dX_2}{dz}\right|_{z=0} = \beta X_{10}X_{30}. \tag{7}$$

The phases $\varphi_{1,3}$ of the amplitudes $\tilde{A}_{1,3}$ of two other waves experience nonlinear oscillations:

$$\left.\frac{d\varphi_{1,3}}{dz}\right|_{z=0} = \pm\frac{\Delta}{2}, \tag{8}$$

therefore, we will seek the required solution of the initial problem by analysing equation (4b), which, taking into account the condition $\varphi_2 = \varphi_{20}$, has now the form

$$\frac{d^2X_2}{dz^2} - \beta^2\left(-2I_{10} + I_{30} - \frac{\Delta^2}{4\beta^2} - 4X_2^2\right)X_2 = 0, \tag{9a}$$

$$X_{20} = 0,\ \left.\frac{dX_2}{dz}\right|_{z=0} = \beta\sqrt{I_{10}I_{30}}. \tag{9b}$$

To this end, the evolution of the idle-wave and pump-wave intensities $I_{1,3}(z)$ will be determined by the second-order integrals (2) of system (1).

Because equation (9a) represents a NSE with the Kerr nonlinearity of focusing type and boundary conditions (9b) are fulfilled, the desired solution should be [9] proportional to the function $\mathrm{cn}(\gamma z, k)$ shifted by a quarter of its period, equal to $K(k)$, along the z axis, i.e. [10]

$$X_2(z) = \sqrt{I_{2\max}}\,k'\frac{\mathrm{sn}(\gamma z, k)}{\mathrm{dn}(\gamma z, k)}. \tag{10}$$

Here, $\mathrm{sn}(\gamma z, k)$, $\mathrm{cn}(\gamma z, k)$ and $\mathrm{dn}(\gamma z, k)$ are elliptic Jacobi functions with the modulus k; $K(k)$ is the complete normal elliptic Legendre integral of the first kind; $k' = \sqrt{1 - k^2}$ [10]; the maximal intensity of the signal wave, $I_{2\max}$, as well as the constants γ and k are the parameters, which should be determined. Therefore, substituting (10) into (9), we find

$$I_{2\max} = \frac{2I_{10}I_{30}}{\{8I_{10}I_{30} + [I_{30} - 2I_{10} - \Delta^2/(4\beta^2)]^2\}^{1/2} - [I_{30} - 2I_{10} - \Delta^2/(4\beta^2)]}, \tag{11a}$$

$$\gamma^2 = \beta^2\left[8I_{10}I_{30} + \left(I_{30} - 2I_{10} - \frac{\Delta^2}{4\beta^2}\right)^2\right]^{1/2}, \tag{11b}$$

$$k^2 = \frac{1}{2}\times\frac{\{8I_{10}I_{30} + (I_{30} - 2I_{10} - \Delta^2/(4\beta^2)]^2\}^{1/2} + [I_{30} - 2I_{10} - \Delta^2/(4\beta^2)]}{\{8I_{10}I_{30} + [I_{30} - 2I_{10} - \Delta^2/(4\beta^2)]^2\}^{1/2}}. \tag{11c}$$

In this case, it follows from (2) that the dependences $I_{1,3}(z)$ have the same period $2K(k)$ as the dependences $I_2(z)$ and are described by expressions

$$I_1(z) = I_{10} + I_{2\max}(1 - k^2)\frac{\mathrm{sn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)}, \tag{12a}$$

$$I_3(z) = I_{30} - 2I_{2\max}(1 - k^2)\frac{\mathrm{sn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)}. \tag{12b}$$

In the case $\Delta = 0$ (the most interesting from the point of view of practical OPO realisation), the above presented relations (11) are more simplified and have the form

$$I_{2\max} = \frac{1}{2} I_{30}, \tag{13a}$$

$$\gamma^2 = \beta^2 (I_{30} + 2I_{10}), \quad k^2 = \frac{I_{30}}{I_{30} + 2I_{10}}. \tag{13b}$$

These expressions show that in the absence of the wave detuning the change in the seed intensity I_{10}, without changing the limiting conversion efficiency of pump radiation into radiation at the signal frequency [see (13a)] allows one in essence to control the periodicity of the energy exchange processes in the nonlinear crystal [change in $K(k)$ due to the change in the parameters γ and k, see (13b)]. Because at $I_{10} \to 0$, the parameters $k^2 \to 1$ and $\gamma^2 \to \beta^2 I_{30}$ (i.e. γ remains finite), the desired solution of the problem in this limit becomes solitary (soliton-like) and its maximum is localised on infinity. Therefore, the limiting conversion efficiency can be realised only asymptotically (at $z \to \infty$).

4. Nondegenerate parametric generation and optimal feedback

Let us introduce now the feedback only in the idle wave (so-called single-cavity OPO scheme). We will assume that the nonlinear crystal has the finite length L and some part $|R|^2$ of the output (the plane $z = L$) idle-wave intensity $I_1(L)$ again returns to the converter input (in the plane $z = 0$). Here, R is the complex transfer constant of the field A_1 from the output to the input. When the frequencies are equal ($\omega_1 = \omega_2$), this means that the feedback circuit is polarisation selective. Assuming that due to the optimal choice of the phase incursion in the feedback circuit (phases of the transfer constant R), the condition $\varphi_{20} = \varphi_{30} - \varphi_{10} - \pi/2$ is still fulfilled, and taking (12a) into account, we derive the relation

$$I_{10} = \frac{|R|^2}{1 - |R|^2} I_{2\max} (1 - k^2) \frac{\mathrm{sn}^2(\gamma L, k)}{\mathrm{dn}^2(\gamma L, k)}. \tag{14}$$

Let the parameter $|R|$ have the optimal value $|R|_{\mathrm{opt}}$ at which

$$\mathrm{sn}^2[\gamma(|R|_{\mathrm{opt}})L, k(|R|_{\mathrm{opt}})] = 1, \tag{15}$$

i.e., so that

$$I_{10} = \frac{|R|^2_{\mathrm{opt}}}{1 - |R|^2_{\mathrm{opt}}} I_{2\max}, \quad I_1(L) = \frac{I_{2\max}}{1 - |R|^2_{\mathrm{opt}}}, \tag{16a}$$

$$I_{1\mathrm{out}} = (1 - |R|^2_{\mathrm{opt}}) I_1(L) = I_{2\max},$$

$$I_2(L) = I_{2\max}, \tag{16b}$$

$$I_3(L) = I_{30} - 2I_{2\max}. \tag{16c}$$

Here, I_{out} is the radiation intensity at the frequency ω_1 at the cavity output. In this case, it follows from (11), taking into account (16), that in the general case

$$I_{2\max} = \frac{1}{2}\left[I_{30} - (1 - |R|^2_{\mathrm{opt}}) \frac{\Delta^2}{4\beta^2}\right], \tag{17a}$$

$$\gamma^2 = \frac{\beta^2}{1 - |R|^2_{\mathrm{opt}}}\left[I_{30} - (1 - |R|^2_{\mathrm{opt}})^2 \frac{\Delta^2}{4\beta^2}\right], \tag{17b}$$

$$k^2 = (1 - |R|^2_{\mathrm{opt}}) \frac{I_{30} - (1 - |R|^2_{\mathrm{opt}})\Delta^2/(4\beta^2)}{I_{30} - (1 - |R|^2_{\mathrm{opt}})^2 \Delta^2/(4\beta^2)}, \tag{17c}$$

where R_{opt} is determined by the transcendental equation

$$\gamma(|R|_{\mathrm{opt}})L = K[k(|R|_{\mathrm{opt}})]. \tag{18}$$

Note that in the stationary regime, the idle-wave intensities both at the nonlinear crystal input (I_{10}) and at its output [$I_1(L)$] can significantly exceed $I_{2\max}$ due to accumulation of the energy in the cavity, which is taken away from the pump wave during the transient process.

In the same most interesting case $\Delta = 0$, expressions (17) and (18) are simplified:

$$I_{2\max} = \frac{1}{2} I_{30}, \; \gamma^2 = \frac{\beta^2 I_{30}}{1 - |R|^2_{\mathrm{opt}}}, \; k^2 = 1 - |R|^2_{\mathrm{opt}}, \tag{19}$$

where the optimal reflectivity of the coupling mirror is determined by the solution of the expression

$$(1 - |R|^2_{\mathrm{opt}})^{1/2} K[(1 - |R|^2_{\mathrm{opt}})^{1/2}] = \beta \sqrt{I_{30}} L. \tag{20}$$

Because $(1 - |R|^2_{\mathrm{opt}})^{1/2} = k$ and the dependence $K(k)$ is tabulated, the solution of (20) could be sought for graphically, which is illustrated in Fig. 1. However, at $k \ll 1$ ($|R|^2_{\mathrm{opt}} \sim 1$) and $k \sim 1$ ($|R|^2_{\mathrm{opt}} \ll 1$), we can make use of the expansions of $K(k)$ into series [10].

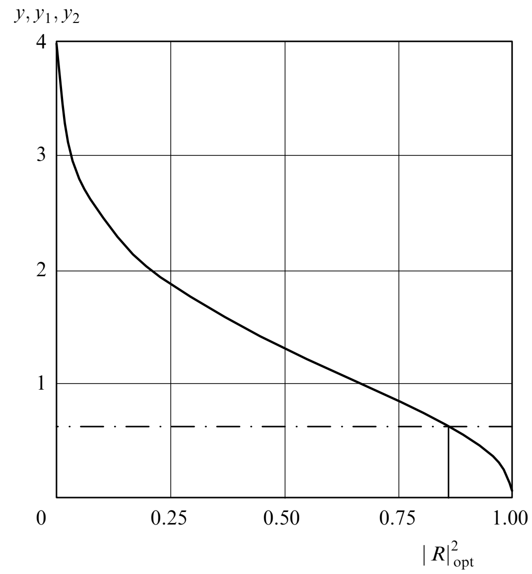

Figure 1. Graphical solution of equation (20). Dependence $y(|R|^2_{\mathrm{opt}}) = (1 - |R|^2_{\mathrm{opt}})^{1/2} K[(1 - |R|^2_{\mathrm{opt}})^{1/2}]$ is shown by a solid curve, dependence $y_1(|R|^2_{\mathrm{opt}}) = \beta\sqrt{I_{30}}\,L = \mathrm{const}$ (or $y_2(|R|^2_{\mathrm{opt}}) = [2(|\gamma_1|^2 - 2|\gamma_2|^2)I_{30}]^{1/2} \times L = \mathrm{const}$) is shown by the dash-and-dot line. The optimal reflectivity of the feedback mirror $|R|^2_{\mathrm{opt}}$ is determined by the abscissa of the point of intersection of solid and dash-and-dot lines.

5. Cascade parametric generation and optimal feedback

In analysing the cascade parametric generation, we will restrict our consideration to only one of two possible situations, which were previously discussed [6]. Consider the collinear interaction of four ($i = 1-4$) modes with multiple frequencies ω_1, $\omega_2 = 2\omega_1$, $\omega_3 = \omega_1 + \omega_2 = 3\omega_1$ and $\omega_4 = \omega_1 + \omega_3 = 4\omega_1$, the wave vectors $\boldsymbol{k}_i$ and amplitudes $A_i = X_i \exp(\mathrm{i}\varphi_i)$. Let the nonlinear processes $\omega_3 \leftrightarrow \omega_1 + \omega_2$ and $\omega_1 + \omega_3 \leftrightarrow \omega_4$ with wave detunings $\Delta_{1,2} = k_1 + k_{2,3} - k_{3,4}$ and nonlinear coupling constants $\beta_{1,2}$, respectively, proceed in a medium. We will also assume that a structure is created in the nonlinear crystal in which the signs $\beta_{1,2}$ alternate along the z axis with a period multiple of the coherent lengths $2\pi/\Delta_{1,2}$ (the quasi-phase-matching condition is fulfilled). We will also introduce the constants $\gamma_{1,2} = \langle \beta_{1,2}(z)\exp(-\mathrm{i}\Delta_{1,2}z)\rangle_z$ describing the averaged nonlinear coupling. It was shown in [6] that in this case, the passage from the initial truncated first-order equations to second-order equations yields a closed system of two NSEs for the mode amplitudes $A_{1,3}$ and that when the travelling-wave OPO is pumped by the mode at the frequency ω_3 and the seed at the frequency ω_1 is present, in the case of the optimal relation of the initial phases φ_{i0} specified by the conditions

$$\begin{aligned} &\varphi_{10} + \varphi_{20} - \varphi_{30} - \varphi_{\gamma_1} + \frac{\pi}{2} = 0, \\ &\varphi_{10} + \varphi_{30} - \varphi_{40} - \varphi_{\gamma_2} - \frac{\pi}{2} = 0, \end{aligned} \tag{21}$$

the maximal initial (at point $z = 0$) growth rate of $X_{2,4}$ is provided. Here, $\varphi_{\gamma_{1,2}}$ are determined by the relations $\gamma_{1,2} = |\gamma_{1,2}|\exp(\mathrm{i}\varphi_{\gamma_{1,2}})$. At $|\gamma_1|^2 > 2|\gamma_2|^2$ and $I_{20} = I_{40} = 0$, the cascade parametric generation with up conversion corresponds to the solution

$$X_1 = \sqrt{I_{10}}\,\frac{1}{\mathrm{dn}(\gamma z, k)},\ X_3 = \sqrt{I_{30}}\,\frac{\mathrm{cn}(\gamma z, k)}{\mathrm{dn}(\gamma z, k)}, \tag{22a}$$

$$\begin{aligned} I_2 &= \frac{1 - \mathrm{dn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)} I_{10} + \frac{1}{3}\frac{\mathrm{dn}^2(\gamma z, k) - \mathrm{cn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)} I_{30}, \\ I_4 &= -2\frac{1 - \mathrm{dn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)} I_{10} + \frac{2}{3}\frac{\mathrm{dn}^2(\gamma z, k) - \mathrm{cn}^2(\gamma z, k)}{\mathrm{dn}^2(\gamma z, k)} I_{30}, \end{aligned} \tag{22b}$$

in which the dependences $I_{2,4}(z)$ are found from the law of conservation of the energy flux density and Manly–Row relations, and

$$\begin{aligned} k^2 &= \frac{(|\gamma_1|^2 - 2|\gamma_2|^2)I_{30}}{3(|\gamma_1|^2 + 2|\gamma_2|^2)I_{10} + (|\gamma_1|^2 - 2|\gamma_2|^2)I_{30}}, \\ \gamma^2 &= 2[3(|\gamma_1|^2 + 2|\gamma_2|^2)I_{10} + (|\gamma_1|^2 - 2|\gamma_2|^2)I_{30}]. \end{aligned} \tag{23}$$

It was found in [6] that localisation of the maxima of the dependence $I_4(z)$ at $z = z_{\max}$ corresponds to the condition $|\mathrm{sn}(\gamma z_{\max}, k)|^2 = 1$ and that

$$I_{4\max} = I_4(z_{\max}) = \frac{8}{3}\frac{|\gamma_2|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30} \tag{24}$$

does not change with I_{10}. At points $z = z_{\max}$, the pump wave is completely depleted and the remaining part of its energy flux is redistributed between two other modes:

$$\begin{aligned} I_1(z_{\max}) &= I_{10} + \frac{1}{3}\frac{|\gamma_1|^2 - 2|\gamma_2|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30}, \\ I_2(z_{\max}) &= \frac{2}{3}\frac{|\gamma_1|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30},\quad I_3(z_{\max}) = 0. \end{aligned} \tag{25}$$

It means that as in the previously considered case, the change in I_{10}, without changing the limiting conversion efficiency of pump radiation into radiation at the signal wave [see (24)], allows one to control the periodicity of the energy exchange processes in the nonlinear crystal [the change in $K(k)$ due to a change in the parameters γ and k)]. Because at $I_{10} \to 0$, the parameters $k^2 \to 1$ and $\gamma^2 \to 2(|\gamma_1|^2 - 2|\gamma_2|^2)I_{30}$ (i.e., γ remains finite), the solution of the problem in this limit also becomes solitary, its maximum is localised on infinity and the limiting efficiency can be realised asymptotically (at $z \to \infty$).

Let us assume now that unlike [6] and in accordance with the above-considered procedure, the seed I_{10} is produced by a part $|R|^2$ of the output (plane $z = z_{\max} = L$) intensity $I_1(L)$, which returns to the converter input ($z = 0$). Here, R is the complex transfer constant of the field A_1 from the output to the input. Assuming that conditions (21) are still fulfilled due to the optimal choice of the phase of the transfer constant R, when selecting the optimal value $|R| = |R|_{\mathrm{opt}}$, satisfying the same condition (18), we, taking into account (24) and (25), obtain the relations

$$\begin{aligned} I_1(L) &= \frac{1}{3}\frac{1}{1 - |R|^2}\frac{|\gamma_1|^2 - 2|\gamma_2|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30}, \\ I_{1\mathrm{out}}(L) &= (1 - |R|^2)I_1(L) = \frac{1}{3}\frac{|\gamma_1|^2 - 2|\gamma_2|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30}, \end{aligned} \tag{26a}$$

$$\begin{aligned} I_2(L) &= \frac{2}{3}\frac{|\gamma_1|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30},\ I_3(L) = 0, \\ I_4(L) &= \frac{8}{3}\frac{|\gamma_2|^2}{|\gamma_1|^2 + 2|\gamma_2|^2} I_{30}, \end{aligned} \tag{26b}$$

$$k^2 = 1 - |R|^2,\ \gamma^2 = 2\frac{|\gamma_1|^2 - 2|\gamma_2|^2}{1 - |R|^2} I_{30}. \tag{26c}$$

Here, $I_{1\mathrm{out}}$ is the radiation intensity at the frequency ω_1 at the cavity output.

It is easy to see that optimisation of the feedback-circuit transfer constant allowed us again to realise the limiting conversion efficiency of the pump energy flux I_{30} into radiation at the frequency ω_4 at the finite length of the nonlinear crystal, this limiting efficiency also depending only on the nonlinear coupling constants $|\gamma_{1,2}|^2$ and Manly–Row relations. The optimal $|R|^2$ can be still determined from Fig. 1; however, the value of γ should be now found from relation (26c).

6. Conclusions

We have shown that in the case of nondegenerate parametric generation neglecting the losses, the use of a positive feedback circuit (cavity) for one of the generated modes allows one to make the period of cnoidal waves produced in the nonlinear crystal finite (to change the periodicity of the energy exchange processes). In this case, the optimisation procedure of the transfer constant of this circuit (in fact, the reflectivity of the output mirror of the cavity) is reduced to matching this period with the nonlinear crystal length and to localisation of the plane, in which the conversion efficiency is maximal, at the output face of the nonlinear crystal. The limiting conversion efficiency does not change and remains the same as in the case of a travelling-wave OPO [5, 6] (i.e., determined either by the Manly – Row relations only [5], or in addition by two constants describing the cascade nonlinear interaction [6]).

Note that the both requirements should be valid in the case of two-cavity OPOs (including degenerate type-I generation [2] in a single-cavity scheme). However, the optimisation procedure in this case can prove to be more complicated. First, the position of the plane, where the wave intensities through which the feedback is closed are minimal, is shifted with respect to the input face of the nonlinear crystal (the first derivatives of the wave intensities at $z = 0$ are not zeroed) [5]. Therefore, the optimal period of cnoidal waves should be matched with the nonlinear crystal length but will not be equal to it as was the case in two situations considered above. In principle, even an exotic situation is possible when the both the period and the afore-mentioned shift are infinite but the maximum of the output-wave intensity is localised at the back face of the nonlinear crystal (at $z = L$). Second, in two-cavity OPO schemes, the transfer constants of both feedback circuits (reflectivities of the output mirrors of two cavities) should be matched with each other.

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