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Properties of multilayer nonuniform holographic structures

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Abstract. **Experimental results and analysis of properties of multilayer nonuniform holographic structures formed in photopolymer materials are presented. The theoretical hypotheses is proved that the characteristics of angular selectivity for the considered structures have a set of local maxima, whose number and width are determined by the thicknesses of intermediate layers and deep holograms and that the envelope of the maxima coincides with the selectivity contour of a single holographic array. It is also experimentally shown that hologram nonuniformities substantially distort shapes of selectivity characteristics: they become asymmetric, the local maxima differ in size and the depths of local minima reduce. The modelling results are made similar to experimental data by appropriately choosing the nonuniformity parameters.**

Keywords: *holography, multilayer structures, nonuniformities, photopolymer material.*

1. Introduction

The properties of volume transmission and reflection holographic arrays were thoroughly studied by Kogelnik [1] and other authors who showed that such arrays possess, in addition to high diffraction efficiency, high angular and spectral selectivity. The structures comprising several volume gratings separated by optically uniform intermediate layers are characterised by specific properties determined by an interference of the waves reconstructed from each grating and provide possibility to control the shape of selective response. Similar structures may be used to design the elements for optical interfaces, multiplexers/demultiplexers in optical communication lines, spectral filters, and sensors and therefore, they are objects of particular investigations [2–6].

Nordin [2] considered the structures consisting of thin gratings separated by intermediate layers, presented an analytical model for calculating their diffraction efficiency, and discussed a possible employment of these gratings as spectral filters. In [3, 4], similar structures are called SVHOE (Stratified Volume Holographic Optical Elements). The author of [4] used the photopolymer material (PPM) produced by DuPont (USA) to obtain holograms. Experimental and simulation data are compared for multilayer structures consisting of thin holograms and it is assumed that data discrepancy is explained by hologram nonuniformities; however, no study of the nonuniformities was performed. It is shown in [5] that the characteristics of angular selectivity of the structures under study have a series of local maxima, the width of which is determined by the sum width of the whole system (including the intermediate layers), and the envelope of the maxima coincides with the selectivity contour of a single array. In [6], similar structures are called MVHG (Multilayer Volume Holographic Grating) and their diffraction properties are considered as applied to propagation of ultrashort (~50 fs) light pulses. It is shown that the spectral distribution of diffracted light substantially depends on MVHG parameters.

Unfortunately, in the mentioned works insufficient attention is paid to the influence of nonuniform parameters of used volume holograms [7, 8], which is often important in practice, for example, in the photopolymer materials where the sensitive layer width shrinks and light absorption varies along the hologram depth during exposure.

The work is aimed at studying the MVHG-type structure with nonuniform volume transmission holograms.

2. MVHG modelling

To analyse the structure properties of MVHG consisting of nonuniform volume transmission holograms we will use results [9], where nonuniform volume holograms are presented as a set of layers from uniform holograms with their particular transfer characteristics and the amplitude of the resulting output wave is determined by multiplying the transfer matrices of each layer.

An MVHG structure before and after writing is schematically shown in Fig. 1, where T_1 is the thickness of the first detection layer (hologram); t_m is the thickness of mth intermediate layer; R and S are the reference and signal beams; R' is the reconstructing beam; $\bar{n}$ is the mean refractive index of the material and intermediate layer.

It is known that the amplitudes of passed (reconstruction) and object (reconstructed) beams of a uniform volume transmission hologram are expressed in the following way [1]:

$$\mathrm{R} = \mathrm{e}^{-\mathrm{i}\xi}\left[\cos(\xi^2+\nu^2)^{1/2} + \frac{\mathrm{i}\xi}{(\xi^2+\nu^2)^{1/2}}\sin(\xi^2+\nu^2)^{1/2}\right], \quad (1)$$

$$\mathrm{S} = -\mathrm{i}\left(\frac{c_\mathrm{R}}{c_\mathrm{S}}\right)^{1/2}\nu\frac{\sin(\xi^2+\nu^2)^{1/2}}{(\xi^2+\nu^2)^{1/2}}\mathrm{e}^{-\mathrm{i}\xi}, \quad (2)$$

where

$$\nu = \frac{\pi\Delta n T}{\lambda_0(c_\mathrm{R}c_\mathrm{S})^{1/2}}; \quad \xi = \frac{T\vartheta}{2c_\mathrm{S}};$$

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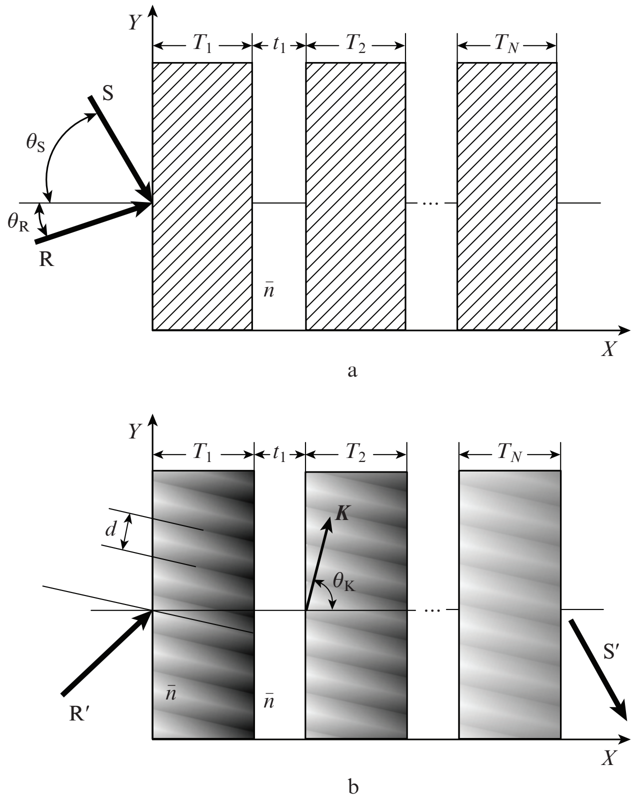

Figure 1. Schemes of MVHG recording (a) and reconstruction (b).

$$\vartheta = \Delta\theta K \sin(\theta_K - \theta_0) - \Delta\lambda \frac{K^2}{4\pi\bar{n}}; \quad \theta = \theta_0 + \Delta\theta;$$

$$\lambda = \lambda_0 + \Delta\lambda; \quad \theta_0 = \frac{|\theta_R - \theta_S|}{2}; \quad \theta_K = \frac{\pi}{2} - (\theta_0 - \theta_R);$$

$$\beta = \frac{2\pi\bar{n}}{\lambda_0}; \quad n = \bar{n} + \Delta n \cos(\boldsymbol{K}\boldsymbol{y}); \quad 2d\sin\theta_0 = \frac{\lambda_0}{\bar{n}}; \tag{3}$$

$$|\boldsymbol{K}| = \frac{2\pi}{d}; \quad c_R = \cos\theta_R; \quad \eta = \frac{|c_S|}{c_R} SS^*;$$

$$c_S = \cos\theta_R - \frac{K}{\beta}\cos\theta_K;$$

$\bar{n}$ and Δn are the mean value and amplitude of the spatial modulation of the medium refractive index; λ_0 is the wavelength in air; T is the hologram thickness; d is the grating period; $\boldsymbol{K}$ is the grating vector; θ_K is the grating vector angle (the angles are measured in medium); θ_R and θ_S are the incident angles for the reference and object waves, respectively; θ_0 is the Bragg angle; $\Delta\theta$ is the detuning from the Bragg angle; η is the diffraction efficiency.

The transfer matrix for the hologram is written in the form

$$M = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}. \tag{4}$$

In view of [8, 9] we obtain

$$M = \begin{bmatrix} \left(\cos\phi + \mathrm{i}\frac{\xi\sin\phi}{\phi}\right)\mathrm{e}^{-\mathrm{i}\xi} & -\mathrm{i}\left(\frac{c_S}{c_R}\right)^{1/2}\frac{\nu\sin\phi}{\phi}\mathrm{e}^{-\mathrm{i}\xi} \\ -\mathrm{i}\left(\frac{c_R}{c_S}\right)^{1/2}\frac{\nu\sin\phi}{\phi}\mathrm{e}^{-\mathrm{i}\xi} & \left(\cos\phi - \mathrm{i}\frac{\xi\sin\phi}{\phi}\right)\mathrm{e}^{-\mathrm{i}\xi} \end{bmatrix}. \tag{5}$$

Here, $\phi = (\xi^2 + \nu^2)^{1/2}$.

By multiplying the layer transition matrices of each volume hologram we obtain the transition matrix corresponding to a nonuniform hologram without absorption. An intermediate layer of thickness t yields a phase incursion. If we assume the refractive index of the intermediate layer is equal to that of the hologram then the transition matrix for such layer is as follows:

$$D = \begin{bmatrix} 1 & 0 \\ 0 & \exp\left(-\mathrm{i}\frac{\Delta\theta K\sin(\theta_K - \theta_0)t}{c_S}\right) \end{bmatrix}. \tag{6}$$

To find the transition matrix for the whole structure we have to multiply the transition matrices of all the layers:

$$\prod_{j=1}^{N} M_j \times D_j = M_N \times D_N \times \ldots \times D_j \times M_j \times \ldots \times D_1 \times M_1,$$

$$\begin{bmatrix} \mathrm{R_{out}} \\ \mathrm{S_{out}} \end{bmatrix} = \prod_{j=1}^{N} M_j \times D_j \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{7}$$

The types of volume hologram nonuniformities depend on the properties of recording material (in particular, on the PPM), exposure duration and conditions [8]. Variation of the refractive index modulation in the depth of hologram in writing to PPM caused by light intensity reduction in accordance with the Bouguer–Lambert–Beer law is expressed in the form

$$\Delta n(x) = \Delta n_0 \exp(-cx/T), \tag{8}$$

where c is a constant and Δn_0 is the initial modulation of the refractive index.

In addition to the allowance made for a nonuniform variation in the refractive index modulation it is often required to introduce the parameters describing the distortion of the hologram spatial structure (variations of the period and K_x-component of the grating vector $\boldsymbol{K}$) due to the longitudinal and transversal shrinkage of a sensitive layer:

$$K_x(x,p,q) = K_y\left[-\frac{K_0}{K_y} + p\left(\frac{0.5T - x}{T}\right) + q\left(\frac{0.5T - x}{T}\right)^2\right],$$

$$y(x,p,q) = \int_T^x \frac{K_x(x,p,q)}{K_y}\,\mathrm{d}x, \tag{9}$$

where K_0 is the initial value of grating vector in a material, K_y is the constant y-component of the grating vector, and $y(x,p,q)$ is the equation for interference fringes.

In modelling the selective properties of nonuniform MVHG we can use the widely known calculation method FDTD (Finite-Difference Time-Domain), which is often employed for calculating forbidden zones in photon crystals [10]. However, in our case the method requires too much time on preliminary work and on calculation of reflection coefficients or diffraction efficiency of holographic gratings. For this reason, we used the specialised Hologram Properties Modelling software (version 4.00) developed earlier, which comprises the models of multilayer nonuniform volume transmission and reflection holograms [11]. The results obtained were displayed by means of standard graphical software.

In analysing the structure comprising nonuniform holograms we consider the following typical cases:

(i) the variation in the refractive index modulation in the hologram depth is an exponent function of type (8);

(ii) the longitudinal and transversal shrinkage of a recording layer is taken into account in accordance with (9).

These cases will be considered under the varying thicknesses and number of intermediate layers. Each intermediate layer we separate to five sublayers, which are assumed uniform. Suppose, the thickness of a single hologram is 55 μm, recording wavelength is 635 nm, incident angles (in air) of the recording beams are $\theta_R = -\theta_S = 18°$, the refractive indices of the intermediate layer and external protective film are 1.48, and the refractive index of the recording layer is 1.5.

As an example of MVHG modelling, consider the structure formed by two uniform holograms with the modulation of the refractive index $\Delta n_0 = 0.0013$ in each hologram and different thicknesses of the intermediate layer. From Fig. 2, one can see that at a small thickness of the intermediate layer (t = 10 μm) the selectivity contour of the whole structure is close to the characteristic of a single hologram [curve (*1*)]; however, at the greater thickness (t = 175 μm) in agreement with the conclusions [4, 5] additional local maxima and minima of light intensity arise due to interference effects, the total diffraction efficiency remaining constant [curve (*2*)]. In this case, the local minima fall to zero.

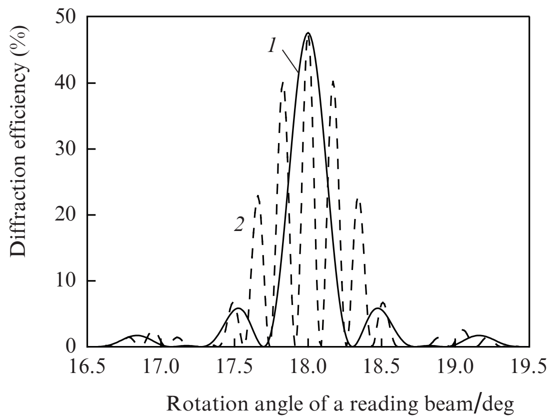

Figure 2. Angular selectivity of uniform MVHG structure at the intermediate layer thickness t = 10 μm (*1*) and 175 μm (*2*).

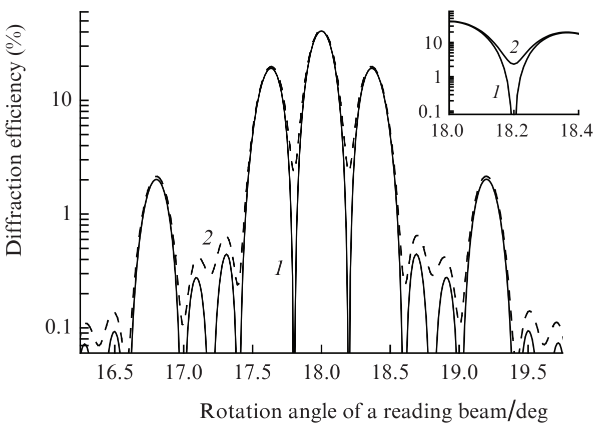

Figure 3. Angular selectivity of the nonuniform MVHG structure formed by two uniform (*1*) and two nonuniform (*2*) holograms.

Selective response of the MVHG structures formed by uniform and nonuniform holograms is presented in Fig. 3 (the diffraction efficiency is plotted in a logarithmic scale), where curve (*1*) corresponds to the structure comprising two uniform holograms with $\Delta n_0 = 0.0013$ and curve (*2*) corresponds to the structure with two nonuniform holograms having $\Delta n_0 = -0.0022$ and c = 1. One can see that if a nonuniform variation in the refractive index modulation (8) is taken into account then the depth of local minima reduces not reaching zero (see the inset in Fig. 3).

3. Experimental results

To study experimentally the selective properties of NVHG structure we used the stand [12] for recording volume transmission holograms, which is schematically shown in Fig. 4. The stand has a 635-nm diode laser and standard optical elements for forming the reference and object beams, the incident angles of which relative to the recording plane normal are symmetric and equal to 18° (in some experiments the incident angles were not symmetric). As the recording medium we used the photopolymer material developed at the Scientific Institute of Organic Chemistry, Siberian Branch of Russian Academy of Sciences and the holographic photopolymer material Bayfol HX TP presented by Bayer Material Science AG (Germany). To compare the selective characteristics we developed and studied the two-layer structures similar to those shown in Fig. 1a and usual (single-layer) volume transmission holograms. Characteristics of angular selectivity of the struc-

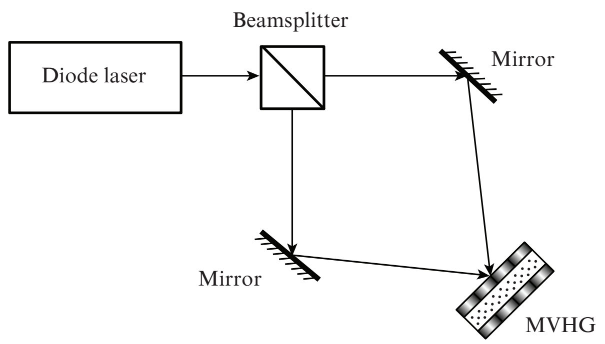

Figure 4. Stand for recording MVHG structures.

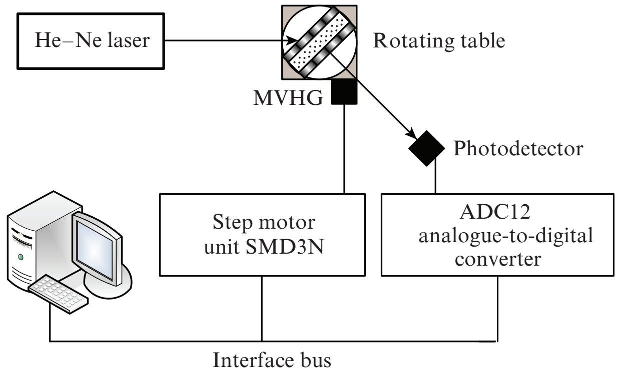

Figure 5. Stand for measuring an angular selectivity of MVHG structure.

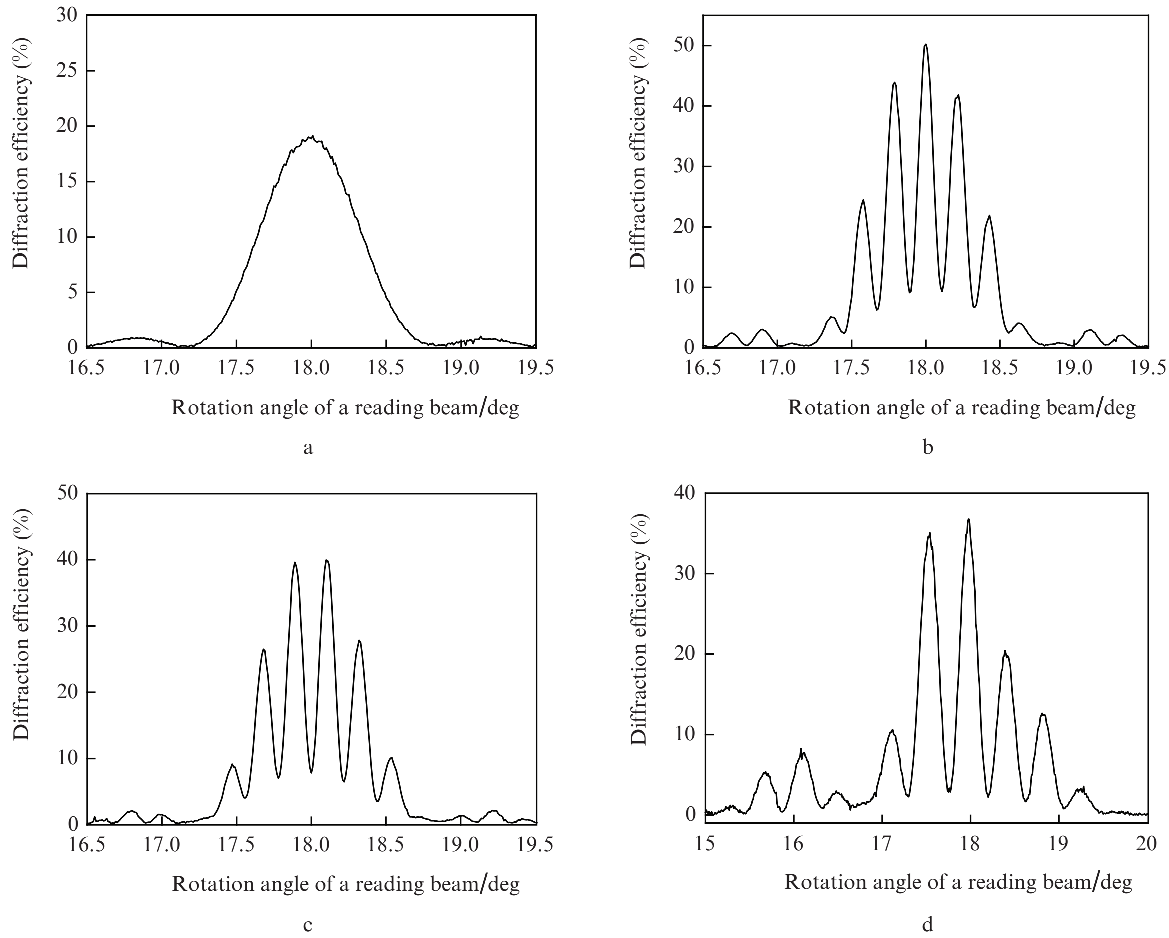

Figure 6. Experimental angular selectivity of a single hologram (a), of the nonuniform MVHG structure with symmetric beam geometry (b, c) and of the nonuniform MVHG structure with asymmetric beam geometry in recording (d).

tures were measured on the stand [12], which is schematically shown in Fig. 5.

Figure 6 presents the experimental data obtained for the multilayer hologram (Fig. 6a) and MVHG structures recorded with various exposures (Figs 6b, c) and configurations of reference and object beams (Fig. 6d). One can see that in accordance with the theoretical predictions the angular selectivity of the considered structures has a series of local maxima, the number and width of which are determined by the thicknesses of intermediate layers and volume holograms and the envelope coincides with the selectivity contour of a single holographic grating. In addition, the experimental selectivity substantially differs from theoretical one: the local maxima may be asymmetric and have different values, the amplitude of local minima may not reach zero and so on.

4. Analysis of experimental and model data

To identify the types of nonuniformities and the degree of their influence on the selectivity characteristics of the structures under study we compared the experimental characteristics with model predictions obtained by parameter fitting.

In Fig. 7a, the corresponding experimental and calculated selectivities are shown for the single-layer volume transmission hologram with a thickness of 55 μm assuming it has a uniform structure. The best coincidence of these characteristics is obtained at the refractive index modulation $\Delta n_0 = 0.00156$; insignificant distortions of the selectivity contour are related with arising nonuniformities.

In Fig. 7b, the experimental and calculated angular selectivity is shown of the MVHG structure consisting of two nonuniform holographic gratings with a thickness of 55 μm (separated by the intermediate layer 175-μm thick). In the result of modelling it was established that in these gratings a modulation of the refractive index is given by formula (8) and a diffraction grating bend due to a longitudinal shrinkage is described by formula (9). The following layer parameters were used in the calculations: for the first layer $\Delta n_0 = 0.0035$, for the second layer $\Delta n_0 = 0.0014$, for both layers $c = 2.5$ and $p = -0.001$.

One can see from Fig. 7b that in the case of nonuniform holograms the intensities of local minima do not reach zero and a change of the grating vector results in certain asymmetry of sidelobes. The envelope of the experimental curve coincides with the selectivity contour of a single hologram.

At longer exposure time, the effect of shrinkage increases and, correspondingly, the maxima and minima move (see Fig. 8a) which results in that a local minimum arises at the point where the Bragg conditions hold (at an angle of 18°). The calculated layer parameters were as follows: for the first layer $\Delta n_0 = 0.0031$, for the second layer $\Delta n_0 = 0.0012$, for both layers $c = 2.5$ and $p = 0.005$.

Figure 8b presents the characteristics of angular selectivity for the multilayer structure comprising inclined holograms

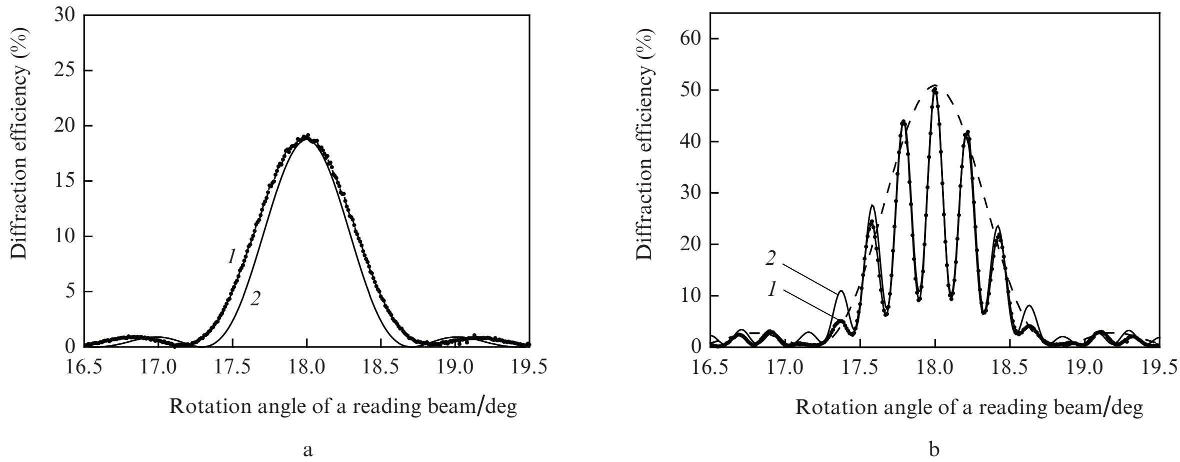

Figure 7. Comparison of the experimental (*1*) and calculated (*2*) data for a single hologram (a) and for an MVHG structure (b) at symmetric beam geometry in recording.

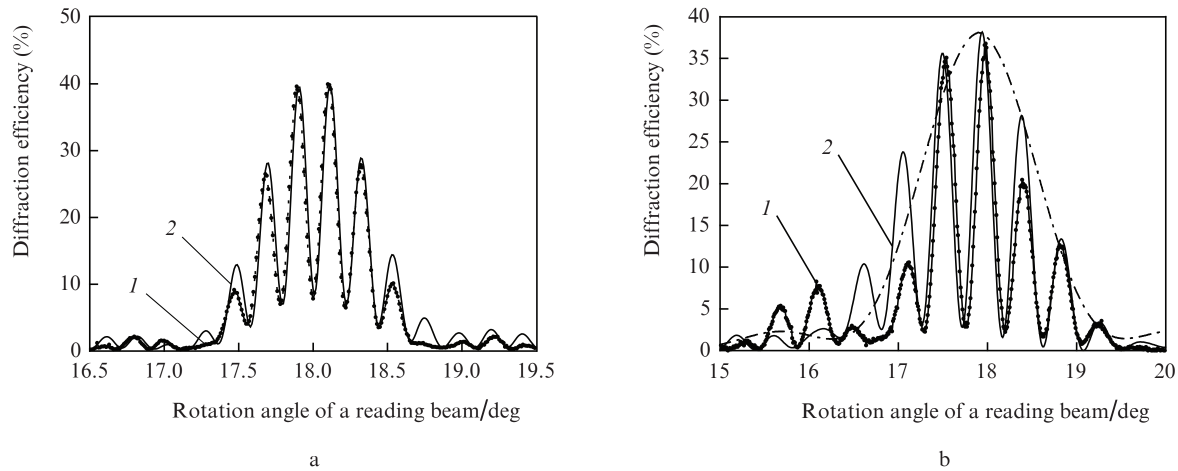

Figure 8. Comparison of the experimental (*1*) and calculated (*2*) angular selectivity of the MVHG structure with symmetric (a) and asymmetric (b) beam geometry in recording. The contour of single hologram selectivity is plotted by the dash-and-dot curve.

(the recording angles are $\theta_R = 18°$ and $\theta_S = 0$) with shrinkage. The layer parameters are: for the first layer $\Delta n_0 = 0.0025$, $c = 1.5$, $p = -0.015$ and for the second layer $\Delta n_0 = 0.0013$, $c = 2.5$. Despite the noticeable shifts of maximum and minimum positions they still fit inside the selectivity contour of a single hologram.

5. Conclusions

Selective properties of the structures consisting from two volume nonuniform transmission holograms separated by intermediate layers are investigated. The recording media were thick photopolymer materials whose properties are responsible for various types of nonuniformities of hologram structure: an exponential attenuation of the refractive index modulation in the hologram depth and a change of the vector direction and period of holographic gratings due to a longitudinal and transversal shrinkage of the recording medium.

The theoretical assumptions are confirmed that the angular selectivity of the considered structures has a series of the local maxima, the number and width of which are determined by the thicknesses of intermediate layers and volume holograms; the envelope of the maxima coincides with the selectivity contour of a single holographic grating. It was also experimentally shown that hologram nonuniformities substantially distort the shape of selectivity characteristic: it becomes asymmetric, the local maxima differ in magnitude; the depth of local minima reduces. By choosing the parameters of nonuniformities the modelling results were made similar to the experimental data.

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