CONTROL OF LASER RADIATION PARAMETERS

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Radiation phase locking in an array of globally coupled fibre lasers

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Abstract. **A model of an array of globally coupled fibre lasers, with the same fraction of the total output beam returned to each laser, is considered. The basic element of the model is a single laser controlled by an external signal. The output power of each laser in the array is found as a function of small-signal gain and frequency detuning. The maximum efficiency of phase locking and minimum fraction of output radiation that is necessary to form a feedback are calculated as functions of the number of lasers in the array. It is shown that gain saturation increases the efficiency of coherent beam summation in arrays containing up to 20 lasers.**

Keywords: *fibre laser, phase locking, global coupling.*

1. Introduction

Phase locking of arrays of coherent radiation sources makes it possible to obtain high-intensity output beams. Currently, the output power of fibre lasers with a beam divergence close to the diffraction limit can be increased to 6 kW using multistage systems of amplifiers with a single-mode fibre of large diameter in the last stage [1]. Actually the ultimate power is determined by the nonlinear processes in the active medium, which limit the power density of single-mode generation in an amplifier [2]; thus, a further increase in the power of the output beam with diffraction-limited divergence can be achieved by coherent or incoherent summation of the beams [3].

To date, a number of different types of laser arrays with coupled elements have been investigated [4]. Radiation phase locking in array elements at the same frequency makes it possible to obtain narrow-spectrum radiation; in addition, it is more appropriate (in comparison with the spectral beam summation on diffraction grating) for locking two-dimensional arrays. There are two main approaches to summation of radiation beams in an array: active control of radiation parameters for each laser in the array and passive phase locking of the total array radiation. The locking methods based on active control [5] are compatible with output-beam control systems and are convenient for users. However, the additional optical equipment and electronic control units make the system more complicated and increase its cost and failure probability [6]. Therefore, passive phase locking is a reasonable alternative for laser systems with a high-intensity output beam, because it is based on internal physical properties of the system.

In turn, phase locking methods use either distributed optical coupling in multicore fibres or coupling through some external spatial radiation filter (or a combination of filters; see review [7]). Laser arrays with an external filter are more complicated in design but can be more easily scaled, whereas the system with distributed optical coupling [8] are limited by the total power of radiation propagating through the common multimode fibre.

To make phase locking stable, it is necessary to provide global coupling [9], which implies coupling of each laser in array with all others. One of appropriate versions is coupling through fibre X-couplers (2×2 couplers [10 – 12]), in which one of the outputs is used for a feedback, while the other suppresses the out-of-phase radiation. The drawbacks of this architecture are extraction of the radiation of all array lasers into a single-mode fibre and lasing instability [13]. Locking with an external Talbot filter was successfully applied in multicore fibre lasers [14 – 18]. The difference in the field profiles of array optical modes in the far-field zone can also be used (for example, applying the transition to the Fourier plane of the external optical system). In particular, a high degree of locking was reached in a linear array of seven fibre amplifiers with an external mirror, mounted at a distance equal to its focal length [19]. Such an external filter can provide global-like coupling between active fibres, but only for a small number of elements or at a low filling factor [20].

At the same time, determination of the array architecture that would provide phase locking for a large number of lasers remains an urgent problem. The main limiting factor for passive phase locking is the spread of optical path lengths of active elements, which leads to a significant difference in the spectra of longitudinal modes for individual lasers. Nevertheless, experiments showed a high degree of locking even for fibre lengths differing by a few centimeters [11, 12, 21]. Concerning the factors determining the locking efficiency in such arrays, the radiation frequency tuning in the spectral gain band (to ensure a maximum difference between the gain and losses) has been studied most thoroughly [11, 13, 22 – 24]. Specifically, this approach was used to estimate the maximum number of locked channels in some recent studies [25 – 27].

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Another possible mechanism of phase locking is the nonlinear dependences of the gain and refractive index of the medium on the radiation intensity. The effect of refractive index nonlinearity was considered for the first time in [28] within a simplified model of a laser array with nearest coupled neighbours. It was found that the optical coupling coefficient μ of lasers with a medium having a Kerr nonlinearity coefficient n_2 must satisfy the conditions $\mathrm{Re}\mu > 0$ and $n_2\mathrm{Im}\mu < 0$ to ensure stable locking. Concerning fibre lasers, the effect of optical nonlinearity on locking in a multicore fibre was discussed in [29] in terms of the dependence of the refractive index on the fraction of excited Yb ions [30]. The effect of refractive index nonlinearity on phase locking in active elements operating in the regenerative-amplifier regime was considered in [31, 32] in the approximation of constant radiation intensity along the element length, taking into account the multiplicity of longitudinal modes in a nonlinear Fabry–Perot cavity at a specified external signal intensity [33] (which makes it possible to choose a mode minimally detuned from the master frequency).

Finally, the complete theory of phase locking of laser array radiation must take into account the spike character of lasing in such systems [13]. A numerical simulation of a globally coupled 100-element laser array [34] showed that an increase in the spread of optical path lengths above some threshold leads to spike generation with an axial brightness of $\sim 30\,\%$ of the maximum value. This effect can be interpreted by introducing the concept of spectral pulse width. A sufficient condition for phase locking is that eigenfrequencies fall in a finite spectral range, determined by the radiation frequency uncertainty.

In this paper, we report the results of the theoretical study of passive phase locking of an array of globally coupled fibre lasers, with pumping exceeding the threshold level. The main attention is paid to the mechanism of tuning the generated mode frequency in the gain band, with allowance for the gain saturation.

2. Approach to the analysis of an array of globally coupled fibre lasers

The model design under consideration is shown in Fig. 1. It is a one-dimensional array of fibre lasers, each of which maintains one transverse mode (the results of this study can be directly applied to two-dimensional arrays). Each laser (an active fibre with a length randomly varied over the array) is placed in a Fabry–Perot cavity with highly reflecting and semitransparent mirrors (the role of the latter is played by the fibre end face, which provides Fresnel reflection). The output radiation of each laser is split into two parts: a beam extracted from the system and a beam supplied to the external feedback system. It is assumed that the beams are coherently added and redistributed in the external system, and the same fraction of the total split-off beam returns to each laser. If the external signal frequency is close to the laser eigenfrequency, this feedback can provide the injection-locked regime for the laser. An overlap of the locking ranges of individual lasers at some lasing frequency makes it possible to generate a collective array mode at this frequency. Thus, the key element for analysing the lasing of the entire array is the consideration of a single laser with a field controlled by the external signal.

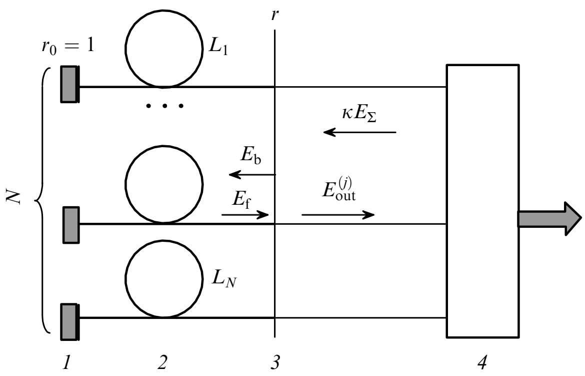

Figure 1. Schematic of an array of globally coupled fibre lasers: (*1*) highly reflecting mirrors, (*2*) active fibres, (*3*) output mirrors, and (*4*) feedback and radiation extraction system.

It was shown that, using the Rigrod dependence for the gain in a medium, $g = g_0/(1+I)$ (g_0 is the small-signal gain and I is the radiation intensity normalised to the saturation intensity), one can adequately describe the mode gain for a single-mode core by the dependence $g = g_0/(1+P)$, where $P = P_{\mathrm{f}} + P_{\mathrm{b}}$ is the total radiation field power in the fibre, normalised to the mode saturation power (which is determined separately) and $P_{\mathrm{f(b)}} = |E_{\mathrm{f(b)}}|^2$ and $E_{\mathrm{f(b)}}$ are the amplitudes of the waves propagating along the fibre in the forward and backward directions, respectively). Within this analysis we neglect the polarisation effects and the influence of the interference of counterpropagating waves on the medium.

Generally, the fibre refractive index depends on the radiation intensity. The phase shift that is caused by the Kerr effect is small, except for the cases of very long amplifiers or very high intensities ($n_2 \approx 3\times10^{-20}$ m^2 W^{-1}). However, the refractive index of doped glasses contains also a resonant term, which is related to the gain through the Kramers–Kronig relation and is proportional to the population inversion [29, 30, 35]. The corresponding phase shift can approximately be written as $\alpha\int_0^L g\mathrm{d}z$, where the factor α depends on the radiation frequency in the spectral gain band.

With the assumptions on the amplitudes of the waves propagating along the fibre, we can write the equation

$$\frac{\mathrm{d}E_{\mathrm{f(b)}}}{\mathrm{d}z} = \mathrm{i}\beta E_{\mathrm{f(b)}} \pm \frac{1}{2}g(z)(1+\mathrm{i}\alpha)E_{\mathrm{f(b)}} \pm \mathrm{i}kn_2PE_{\mathrm{f(b)}}. \qquad (1)$$

Here, β is the propagation constant and the plus and minus signs correspond to the forward and backward directions, respectively. Having separated the real and imaginary parts, we obtain the equation for the radiation power:

$$\frac{\mathrm{d}P_{\mathrm{f(b)}}}{\mathrm{d}z} = \pm g(z)P_{\mathrm{f(b)}} \qquad (2)$$

and, after integrating over the fibre length, relate the power $P_{\mathrm{b}}(L)$ of the radiation field propagating from the output face into the laser bulk and the single-pass integral gain $G = \int_0^L g(P,z)\,\mathrm{d}z$:

$$G_0 - G = P_{\mathrm{b}}(L)[\mathrm{e}^{(2G)} - 1], \qquad (3)$$

and derive the expression for the nonlinear phase difference [36]:

$$\varphi_{\rm NL} = n_2 k \int_0^L P{\rm d}z + \alpha \int_0^L g{\rm d}z = \frac{n_2 k L}{G_0} P_{\rm b}(L)$$

$$\times [{\rm e}^{2G} - 1 + P_{\rm b}(L){\rm e}^{-2G}(2G + \sinh 2G)] + \alpha G, \quad (4)$$

where $G_0 = g_0 L$ is the integral gain of weak signal.

In the absence of scattering loss one can relate the power $P_{\rm inj}$ injected into the laser and the output beam power $P_{\rm out}$:

$$P_{\rm out} = |E_{\rm out}|^2 = P_{\rm inj} + G_0 - G. \quad (5)$$

In a laser controlled by external radiation, the external signal decreases the integral gain below the threshold, so that the condition $r{\rm e}^G < 1$ (r is the reflectance of the output face) is satisfied. In the opposite case, the laser can generate not only at the external-signal frequency but also at the frequencies of several longitudinal modes of the intrinsic cavity. The following expressions are valid in the regime of stable locking by the injected signal:

$$P_{\rm b}(L) = \frac{t^2 P_{\rm inj}}{(1 - r{\rm e}^G)^2 + 4r{\rm e}^G \sin^2\varphi} \quad (6)$$

for the radiation power returned to the fibre (after reflection from the output mirror and summation with the transmitted injection signal) and

$$E_{\rm out} = \frac{{\rm e}^G - r{\rm e}^{(-2{\rm i}\varphi)}}{{\rm e}^{(-2{\rm i}\varphi)} - r{\rm e}^G} \sqrt{P_{\rm inj}} \quad (7)$$

for the output field amplitude ($\varphi = \beta L = \varphi_0 + \varphi_{\rm NL}$ is the single-pass phase difference).

Using expressions (3)–(7), one can calculate the output field amplitude for each laser in terms of the injected signal power; radiation wavelength; and the following laser parameters: length, small-signal gain, and nonlinearity coefficients. Thus a set of output beam fields is found for a fixed realisation of laser array. After splitting off a specified fraction of radiation from each beam and summing fields in the coupling system, a set of identical beams is formed at the output of the system to be injected into separate lasers. The steady-state radiation power in the array at a given frequency can be found by numerical iterations. If the locking condition is not satisfied in a particular laser, one must take into account the generation of cavity eigenmodes to determine exactly the power. This procedure must be performed over a dense frequency spectrum in the gain band of the medium (which makes it possible to find the frequencies of power peaks).

3. Results of numerical simulation of a globally coupled array

To understand the factors determining the phase locking efficiency, we will consider in more detail an array of lasers controlled by a specified external signal (the refractive index nonlinearity is neglected). It follows from Eqns (3) and (6) that the integral gain in a laser is below the threshold $G_{\rm th} = \ln(1/r)$ if the phase difference due to the detuning of injected signal from the longitudinal mode frequency satisfies the condition [37]

$$\sin^2\varphi \leqslant \frac{t^4}{4r^2} \frac{P_{\rm inj}}{G_0 - G_{\rm th}}, \quad (8)$$

where $t^2 = 1 - r^2$. This regime is stable at any detuning if the locking ranges characteristic of the neighboring longitudinal modes merge; this situation corresponds to the critical power of the external signal:

$$P_{\rm cr} = \frac{4r^2}{t^4}(G_0 - G_{\rm th}). \quad (9)$$

For a typical Fresnel reflection coefficient of the fibre face ($r^2 = 0.04$) the factor before the parentheses in (9) is 0.39; thus, the threshold gain is ~ 1.61.

The phase difference of the radiation field passing through the cavity is proportional to the frequency and changes significantly within the spectral gain band. Let us illustrate the effect of frequency tuning in the gain band by the example of the total radiation power of a five-laser array

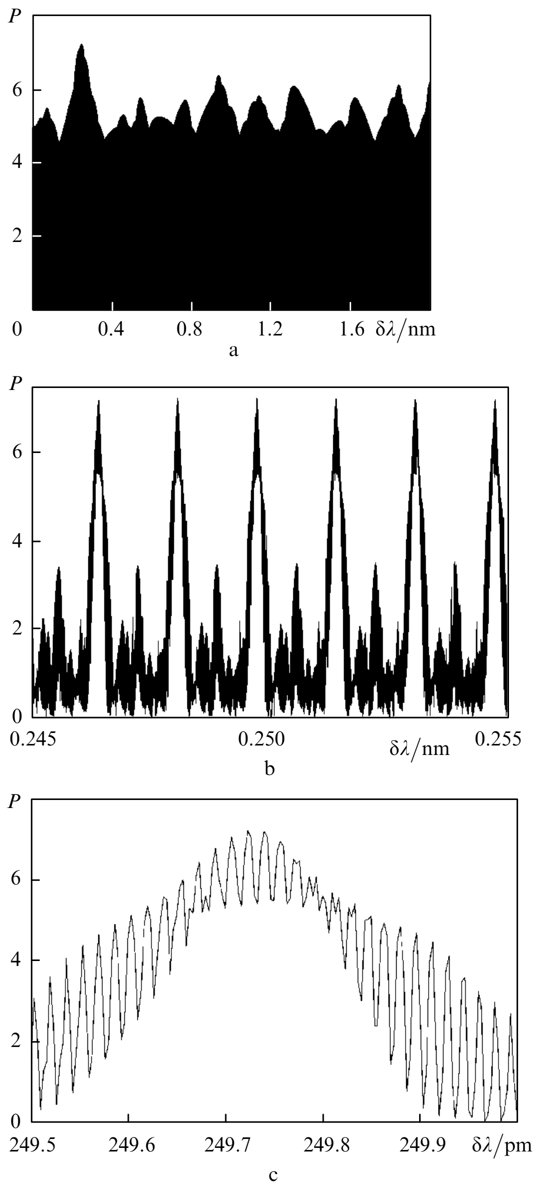

Figure 2. Total field power for an array of five fibre lasers with the fibre length variance $\sigma(\delta l_m) = 1$ mm, $P_{\rm inj} = 0.16$, and $G_0 = 2.5$.

with the same signal injected into each laser. The laser lengths (in meters) are given by the formula $L_m = (10 + 0.1m + \delta l_m)$, where m is the laser number and δl_m is a random spread. The phase difference in the mth fibre in the absence of radiation is determined as $\varphi_0 \approx 2\pi n L_m/\lambda$, where the refractive index $\varphi_0 \approx 2\pi n L_m/\lambda$, $n = 1.5$ and the radiation wavelength $\lambda = 1.05$ μm. The output radiation field is determined as $E_\Sigma = N^{-1/2}\sum_j E_{\rm out}^{(j)}$, where the fields emitted by individual lasers are calculated from formula (7).

The dependence of the total array power P on the frequency of the injected signal, whose power satisfies condition (9), is shown in Fig. 2.

The structure of the laser spectral response is complex and exhibits three different scales, which are caused by the random spread of fibre optical path lengths (on the order of few millimeters); a regular increment of the fibre length, multiple of 10 cm; and, finally, beats of modes with different longitudinal indices. The random spread of fibre lengths leads to oscillations of the envelope with a characteristic spectral width of ~ 0.1 nm (Fig. 2a). The regular change in the fibre length leads to spectral oscillations with a period of ~ 1.5 pm, which can be seen in Fig. 2b. The longitudinal-mode beats form oscillations with a period less than 0.1 pm (Fig. 2c). Under specified conditions the frequencies with a high locking efficiency fall in the spectral gain band, which is in agreement with the experimental results for few-laser arrays [12, 25, 38].

Using the integral small-signal gain $G_0 = 3.7$, we performed several series of calculations to determine the maximum efficiency of summation beams in the spectral gain band, $\eta = \max P(\delta\lambda)/P_0$, where P_0 is the total power in the absence of laser length spread. The obtained dependences for the average efficiency and its variance on the number of elements are shown in Fig. 3 for injected signal powers $P_{\rm inj}$ equal to $P_{\rm cr}$ and $0.5P_{\rm cr}$ (the halved critical power corresponds to the phase-locking band $[-\pi/4, \pi/4]$). The latter situation is of interest, because it allows for splitting off a smaller fraction of output radiation to provide phase locking. These data were obtained by averaging over 200 realisations of the laser array. It can be seen that the formally calculated locking efficiency decreases by approximately 5 % with a decrease in the injected power by half.

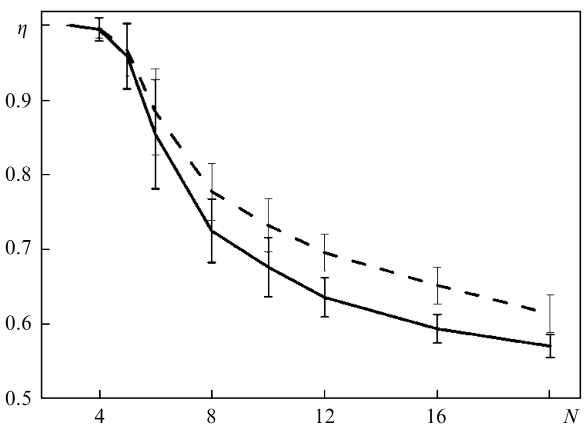

Figure 3. Average efficiency of phase locking and its variance as functions of the number of elements in the array [averaging over 200 realisations with $\sigma(\delta l_m) = 1$ mm] at $G_0 = 3.7$ nm and the spectral gain linewidth $\delta\lambda_{\max} = 4$ nm; $P_{\rm inj} = P_{\rm cr}/2$ (solid curve) and $P_{\rm inj} = P_{\rm cr}$ (dashed curve).

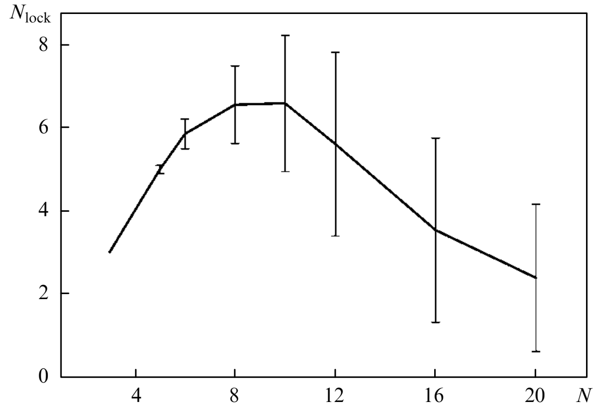

Figure 4. Dependence of the number of elements locked by the external signal at the operating frequency on the total number of elements at $G_0 = 3.7$, $P_{\rm inj} = 0.5P_{\rm cr}$ and $\delta\lambda_{\max} = 4$ nm.

If the steady-state gain G of some laser exceeds the threshold value $G_{\rm th}$ at the found lasing frequency, along with the generation at the external signal frequency, the laser can also generate at the eigenmode frequency. In practice this may lead to spike generation, which can be considered as simultaneous generation of array supermodes at several frequencies. The analysis of multimode generation, which was experimentally observed in [27], is beyond the scope of this study. For each array realisation we found the number of lasers generating under locking by the

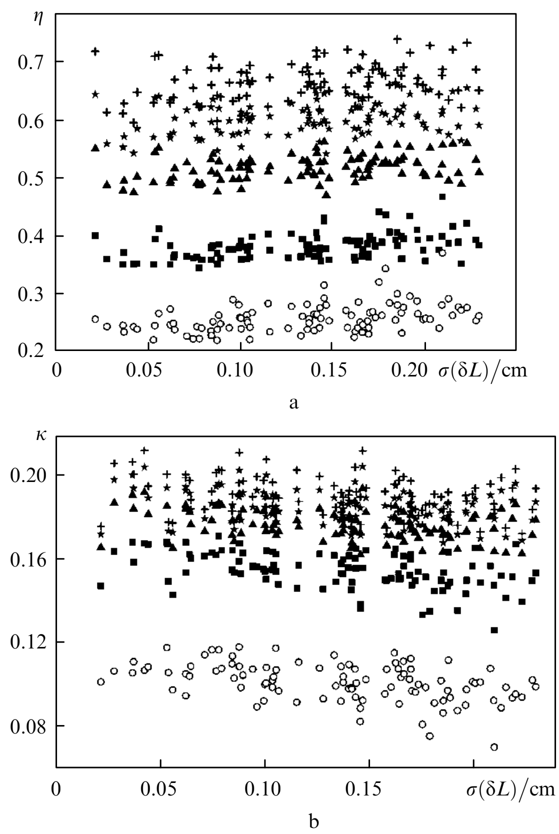

Figure 5. (a) Phase locking efficiency η and (b) the part of the output power κ that must be split off to implement feedback for a 20-laser array as functions of the variance of optical path lengths at $P_{\rm inj} = P_{\rm cr}$ and $G_0 = 1.7$ (○), 2.0 (■), 2.7 (▲), 3.7 (⋆) and 5.7 (+).

external signal. This number can be an estimate of the effective number of lasers locked in one supermode; its dependence on the total number of lasers in the array is shown in Fig. 4. Note that locking of more than 7–8 lasers in one supermode is highly unlikely without involving additional mechanisms, such as the refractive index nonlinearity.

Let us consider a 20-laser array, with a power $P_{\rm cr}$ injected into each laser. Figure 5 shows the locking efficiency and the fraction κ of the output power that is necessary for providing a feedback as functions of the variance of the random spread of fibre lengths for different G_0 values. The necessary power fraction is calculated from the formula $\kappa = P_{\rm inj}N/(\eta P_0)$. It can be seen that with an increase in the random spread variance the locking efficiency somewhat increases, which can be explained by the more accurate choice of the frequency in the gain band. To provide efficient array locking, one must use no less than 20 % output power for the feedback.

4. Results and discussion

The main factor that limits the phase locking efficiency in an array is the detuning of the collective mode frequency from the cavity eigenfrequencies. The dependence of the output power of the controlled laser on the detuning of the external field phase from resonance is shown in Fig. 6 for different small-signal gains. In each case the injected signal power is critical. It can be seen that the controlled-laser power depends strongly on the phase detuning when the small-signal gain slightly exceeds the threshold.

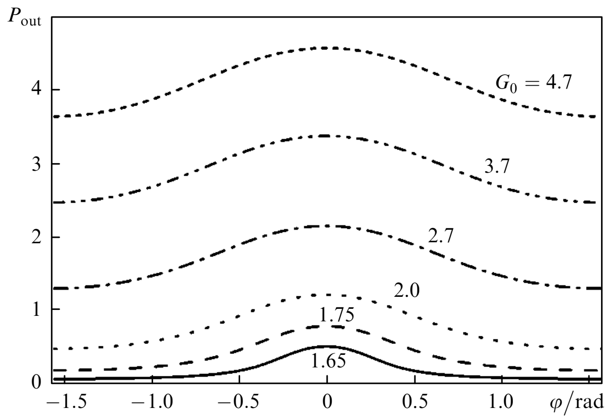

Figure 6. Dependences of the output power on the phase φ at $P_{\rm inj} = P_{\rm cr}$ and different small-signal gains.

In the phase-locking mode the field frequency is determined by the difference between the gain and losses for the entire array. It follows from Fig. 2 that variations in the output power as a function of frequency are very large and have a characteristic period, caused by the similarity of the beat frequencies of the longitudinal modes of array lasers (the variations in beat frequencies can be as high as few percent of the mean). The number M of the longitudinal modes whose frequencies fall in the gain band can be considered in terms of the probability theory as the number of independent attempts of lasing evolution for the entire array. This number is generally large: $\sim 6\times10^4$ for the chosen parameters. Obviously, the larger the number of attempts, the higher the probability of implementing a favorable situation with a small spread of phase differences.

An array with many lasers ($N \gg 1$) can be considered within the asymptotic theory. In particular, on the assumption that the phase detunings φ_j are distributed with uniform probability density in the interval $[-\pi/2, \pi/2]$, the total field is the mean of the output field:

$$E_{\rm av}(G_0) = \frac{1}{\sqrt{N}}\sum_j E_{\rm out}(G_0, \varphi_j)$$

$$= \pi^{-1}\int_{-\pi/2}^{\pi/2} E_{\rm out}(G_0, \varphi){\rm d}\varphi\,. \qquad (10)$$

Here, the transition from summation to integration implies a large number of lasers in the array, which leads to self-averaging of the sum at a random spread of phases. In the limiting case, with randomly spread phases and zero gain, the mean field of the oscillator set is zero [39]. The presence of gain leads, according to formula (7), to a phase dependence of the radiation field. Figure 7 shows the dependence of the average output power $P_{\rm av} = |E_{\rm av}|^2$ on the small-signal gain in comparison with the dependence for the power P_0 in exact resonance. It can be seen that the power of mean output field increases with an increase in gain, although with a low efficiency. In the zero-gain limit it is equal to the part of injected power that was reflected from the output mirror, $r^2P_{\rm inj}$. In the presence of gain the terms in formula (7) that correspond to the interference of external signal with that returned after passing through the cavity do not turn to zero upon averaging, even in the case of a uniform phase distribution over the entire interval. In other words, gain is favorable for locking.

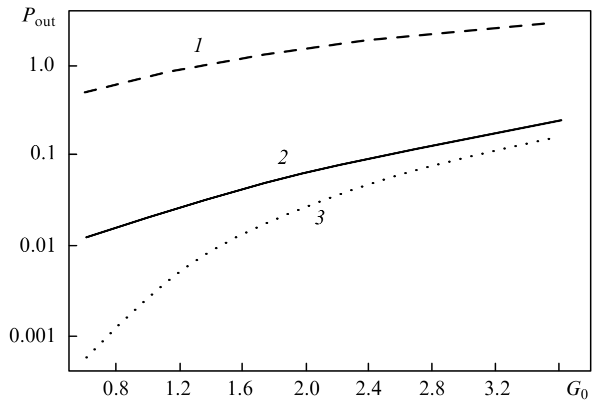

Figure 7. Dependences of (*1*) the output field power in exact resonance P_0, (*2*) the phase-averaged field power $P_{\rm av}$, and (*3*) $P_{\rm av}$ with subtracted power of the injected signal reflected from the output mirror on the small-signal gain at $P_{\rm inj} = 0.2$.

Generally, at least in one of the gaps between the longitudinal-mode frequencies within the gain band the phase differences of all lasers may fall in a narrower interval $[\varphi_0 - \delta\varphi, \varphi_0 + \delta\varphi]$, where φ_0 is the midpoint of this interval and $\delta\varphi$ is its half-width. The purpose of our study was to determine the probability of this event. The case with a specified midpoint of the interval ($\varphi_0 = 0$) and varied interval width $2\delta\varphi$ was considered in [36]. The results of the numerical calculations showed that consideration of asymmetric realisations allows one to more realistically describe phase locking.

In the limit of a large number of lasers the locking efficiency can be determined as $\eta = |E_{\rm av}(\varphi_0, \delta\varphi)|^2/P_0$, where

$$E_{\rm av}(\varphi_0, \delta\varphi) = \frac{1}{2\delta\varphi}\int_{\varphi_0-\delta\varphi}^{\varphi_0+\delta\varphi} E_{\rm out}(G_0, \varphi){\rm d}\varphi.$$

Figure 8 shows the dependence of the efficiency η (in the form of level lines) on the half-width of the averaging interval $\delta\varphi$ and its midpoint φ_0 at $G_0 = 3.7$ and critical injection power. A high locking efficiency (90 % or more) can be reached only at small magnitudes of the phase difference φ, because phase detuning reduces the output power (Fig. 6). However, for an efficiency of ~ 60 % the allowable spread of phases increases with increasing detuning of the interval midpoint from resonance. The reason is the rapid change in the output field phase [which is determined by formula (7)] as a function of φ near the resonance with the eigenmode, due to which the contribution of the range of small φ diminishes upon averaging. As a result, in the case of relatively low efficiencies, grouping of laser field phases near large detunings φ becomes more likely, which explains the numerically found decrease in the number of locked lasers at injected powers below critical (see Fig. 4).

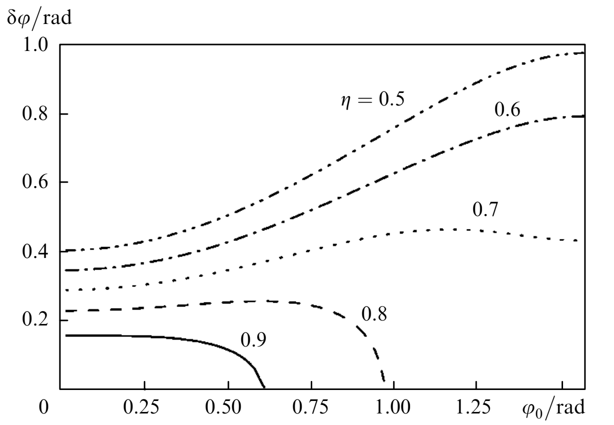

Figure 8. Laser locking efficiency η at $G_0 = 3.7$ and $P_{\rm inj} = P_{\rm cr}$ for uniform field phase distribution in lasers as a function of the midpoint of the averaging interval (φ_0) and its half-width $\delta\varphi$.

The ratio of the area under the curve in Fig. 8 to the area of the entire square $\pi^2/4$ on the assumption of uniform distribution of φ can be related to the probability p of the combination of the laser field and external field phases that is necessary for a specified efficiency. This probability depends on the small-signal gain and injected power. Assuming that the distributions of field phase differences in different lasers are independent, we find that the probability for N lasers to satisfy the locking conditions is p^{N-1}. The number of longitudinal modes of fibre lasers in the spectral gain band (M) is very high. The probability for necessary phase distribution to be implemented at least in one frequency range of longitudinal mode beats can be approximately estimated as $1-(1-p^{N-1})^M \simeq 1-\exp(-M \times p^{N-1})$. To implement phase locking for the array, this probability must be close to unity. Thus, the number of lasers that can be locked with a specified efficiency can be determined from the relation $Mp^{N-1} \approx 2$. For example, under the conditions of Fig. 2 ($\delta\lambda_{\rm max} = 2$ nm and $M \simeq 6\times 10^4$), 5 to 6 lasers can be locked with an efficiency above 90 %, which is in agreement with the calculation results in Section 3.

Note that the approach used in [25] is also based on the probability theory. The probability measure was considered to be the ratio of the effective width of the longitudinal-mode spectral line to the intermode beat frequency. The effective width was estimated as $\sqrt{2(1-R)}$, with the R value not determined explicitly but interpreted as some locking efficiency. Our approach makes it possible to estimate the phase locking efficiency as a function of the pump and design parameters.

5. Conclusions

A method was developed to determine the phase locking efficiency for an array of fibre lasers with an external global-coupling system. The theoretical model takes into account the presence of many longitudinal modes in the individual laser cavity and the nonuniform field distribution over the cavity length. The analysis performed makes it possible to determine the dependence of the efficiency on the laser parameters and predict its asymptotic behaviour for arrays with a large number of lasers. Realistic array parameters were found, at which up to 20 fibre lasers can be locked. The approach developed, which is based on the analysis of basic array element (laser controlled by external injection signal) can be applied to laser arrays with 2×2 couplers [11, 12] and Fourier coupling [19, 20].

Relations (3)–(7) were derived, which make it possible to perform extended analysis with allowance for the refractive index nonlinearity. The mode of partial phase locking of array lasers can also be of practical interest.

References

1. Gapontsev D. *Proc. Solid State Diode Laser Technol. Rev.* (SSDLTR) (Albuquerque, NM, USA, 2008).
2. Dawson J., Messerly M., Beach R., et al. *Opt. Express*, **16** (17), 13240 (2008).
3. Fan T.Y. *IEEE J. Sel. Topics Quantum Electron.*, **11**, 567 (2005).
4. Leger J.R. *Proc. SPIE Int. Soc. Opt. Eng.*, **6102**, XVII (2006).
5. Goodno G.D., McNaught S.J., Rothenberg J.E., et al. *Opt. Lett.*, **35** (10), 1542 (2010).
6. Shay T.M., Baker J.T., Sanchez A.D., et al. *Proc. SPIE Int. Soc. Opt. Eng.*, **7195**, 71951M (2009).
7. Likhanskii V.V., Napartovich A.P. *Usp. Fiz. Nauk*, **160** (3), 101 (1990).
8. Cheo P.K., Liu A., King G.G. *IEEE Photonics Technolol. Lett.*, **13**, 439 (2001).
9. Fader W.J., Palma G.E. *Opt. Lett.*, **10**, 381 (1985).
10. Lyndin N.M., Sychugov V.A., Tikhomirov A.E., Abramov A.A. *Kvantovaya Elektron.*, **21**, 1441 (1994) [*Quantum Electron.*, **24**, 1421 (1994)].
11. Shirakawa A., Saitou T., Sekiguchi T., Ueda K. *Opt. Express*, **10**, 1167 (2002).
12. Sabourdy D., Kermene V., Desfarges-Berthelemot A., et al. *Opt. Express*, **11** (2), 87 (2003).
13. Shirakawa A., Matsuo K., Ueda K. *Techn. Dig. CLEO 2004* (San Francisco, USA, 16–21 May 2004) CThGG2.
14. Wrage M., Glas P., Fischer D., Leitner M., Napartovich A.P., Vysotsky D.V. *Opt. Lett.*, **25** (19), 1436 (2000).

15. Wrage M., Glas P., Fischer D., Leitner M., Elkin N.N., Vysotsky D.V., Napartovich A.P., Troshchieva V.N. *Opt. Commun.*, **205**, 367 (2002).
16. Li L., Schülzgen A., Chen S., Temyanko V.L., Moloney J.V., Peyghambarian N. *Opt. Lett.*, **31** (17), 2577 (2006).
17. Winthrop J.T., Worthington C.R. *J. Opt. Soc. Am.*, **55**, 373 (1965).
18. Antyukhov V.V., Glova A.F., Kachurin O.R., Lebedev F.V., Likhanskii V.V., Napartovich A.P., Pis'mennyi V.D. *Pis'ma Zh. Eksp. Tekh. Fiz.*, **44**, 63 (1986).
19. Corcoran C.J., Durville F. *Appl. Phys. Lett.*, **86** (20), 201118 (2005).
20. Vysotsky D.V., Napartovich A.P., Troshchieva V.N. *Kvantovaya Elektron.*, **37**, 345 (2007) [*Quantum Electron.*, **37**, 345 (2007)].
21. Bruesselbach H., Jones D.C., Mangir M.S., Minden M., Rogers J.L. *Opt. Lett.*, **30**, 1339 (2005).
22. Gerasimov V.B., Zakharov M.V., Lyubimov V.V., Makarov N.A., Orlov V.K. *Kvantovaya Elektron.*, **13**, 1278 (1986) [*Sov. J. Quantum Electron.*, **16**, 839 (1986)].
23. Vysotsky D.V., Elkin N.N., Napartovich A.P., Troshchieva V.N. *Kvantovaya Elektron.*, **32**, 271 (2002) [*Quantum Electron.*, **32**, 271 (2002)].
24. Napartovich A.P., Vysotsky D.V. *J. Modern Opt.*, **50** (18), 2715 (2003).
25. Kouznetsov D., Bisson J., Shirakawa A., Ueda K. *Opt. Rev.*, **12**, 445 (2005).
26. Chang W.Z., Wu T.W., Winful H., Galvanauskas A. *Techn. Dig. CLEO 2009* (San Francisco, USA, 2009) JTuD63.
27. Wu T.W., Chang W.Z., Galvanauskas A., Winful H.G. *Opt. Express*, **17**, 19509 (2009).
28. Golubentsev A.A., Likhanskii V.V., Napartovich A.P. *Kvantovaya Elektron.*, **16**, 730 (1989) [*Sov. J. Quantum Electron.*, **19**, 477 (1989)].
29. Bochove E.J., Cheo P.K., King G.G. *Opt. Lett.*, **28**, 1200 (2003).
30. Digonnet M.J.F., Sadowski R.W., Shaw H.J., Pantell R.H. *J. Lightwave Technol.*, **15**, 299 (1997).
31. Corcoran C.J., Pasch K.A. *IEEE J. Quantum Electron.*, **43**, 437 (2006).
32. Corcoran C.J., Durville F., Pasch K.A. *IEEE J. Quantum Electron.*, **44**, 275 (2007).
33. Felber F.S., Marburger J.H. *Appl. Phys. Lett.*, **26**, 731 (1976).
34. Kourchatov S.Yu., Likhanskii V.V., Napartovich A.P., Arecchi F.T., Lapucci A. *Phys. Rev. A*, **52**, 4089 (1995).
35. Fotiadi A.A., Antipov O.L., Megret P. *Opt. Express*, **16**, 12658 (2008).
36. Napartovich A.P., Elkin N.N., Vysotsky D.V. *Proc. SPIE Int. Soc. Opt. Eng.,* **7580**, 758021 (2010).
37. Gordon R. *IEEE J. Quantum Electron.*, **42**, 353 (2006).
38. Wang B., Mies E., Minden M., Sanchez A. *Opt. Lett.*, **34**, 863 (2009).
39. Mandel L., Wolf E. *Optical Coherence and Quantum Optics* (Cambridge: Cambridge Univ. Press, 1995; Moscow: Nauka, 2000).