

Measurement of angular parameters of divergent optical radiation by light diffraction on sound

V.M. Kotov, S.V. Averin, G.N. Shkerdin

Abstract. A method is proposed to measure the scattering angle of optical radiation, the method employing two Bragg diffraction processes in which divergent optical radiation propagates close to the optical axis of a uniaxial crystal, while the acoustic wave – orthogonally to this axis. The method does not require additional angular tuning of the acousto-optic cell. We suggest using a mask to measure the light divergence that is larger than the angle of Bragg scattering. The method can be used to measure the size of the polished glass plate inhomogeneities.

Keywords: acousto-optic diffraction, Bragg regime, divergent optical radiation.

Measurement of scattered radiation parameters is one of the efficient methods for studying inhomogeneous media, which can represent a rough surface [1, 2], an ensemble of scattering particles [3, 4], optically inhomogeneous composites [5], etc. In particular, angular parameters of the scattered field give information about characteristic dimensions of scatterers [6].

To evaluate the angular parameters of the scattered field, special devices – nephelometers [3, 7] – were designed to measure the scattered light intensity in the angular range from 0 to 360°; later, their different modifications were developed (see, for example, [7–9]). We will be interested in measurements in a comparatively narrow angular range (–20...+20°). This range is sufficient for measuring the scattering intensity, for example, by large particles [3, 6]. In this case, the measurements can be performed without using such a complicated device as a nephelometer.

There exist some standard methods for measuring the angular parameters of the field scattered at a comparatively small angle, in particular, the method for measuring the laser radiation divergence [10]. Among all the methods we single out two methods, which can be used here: the aperture calibration method and two cross-section method [10]. The aperture calibration method is used to assess the aperture diameter through which the specified energy

fraction propagates. The aperture together with a wide-aperture photodetector makes it possible to find the beam diameter. Clearly, this method allows one to determine not the beam divergence but its diameter in the given cross section. To measure the light divergence, it is necessary to find the beam diameter in two different cross sections (i.e., to use the two cross-section method).

We suggest using acousto-optic (AO) Bragg diffraction to measure the divergence angle. Diffraction is very attractive because it is related, by its nature, to the angular space, and the diffraction orders are the ‘natural’ angular markers for divergent radiation. In this case, it is unimportant in which cross section the measurements are taken because the proportions between the characteristics of the light field and diffraction orders are retained in any cross section.

In this paper, we analyse divergent beams using variants of Bragg diffraction where several AO interaction processes proceed. As will be clear below, such regimes can be achieved by using ordinary anisotropic AO cells whose crystals are cut in the planes traditionally used in practice. A specific feature of this approach is provisioning of synchronism not with the entire radiation beam but with some of its rays. The diffraction orders are the angular markers, which, in fact, make it possible to measure the angular divergence of radiation. The number of markers and distance between them are determined by the AO medium characteristics and the sound frequency, while the marker ‘brightness’ is determined mainly by the AO interaction efficiency. Note also that the markers can be placed where necessary, based, for example, on the relationship of the intensities in them and in the maximum of the light field (see details below).

To illustrate the above-said, Fig. 1 shows the vector diagram of AO interaction of divergent radiation in a TeO₂ single-crystal. Radiation (dark region in Fig. 1) propagates inside the crystal near its optical axis Z. The acoustic wave with the wave vector \mathbf{q} propagates orthogonally to the axis Z. Separate rays from the radiation beam diffract on the wave \mathbf{q} . In Fig. 1, these are rays with the wave vectors \mathbf{k}_1 and \mathbf{k}_2 , located symmetrically to the axis Z and incident at angle β to it. We assume that light anisotropically diffracts on sound [11–13]. Rays \mathbf{k}_1 and \mathbf{k}_2 diffract in the directions \mathbf{k}'_1 and \mathbf{k}'_2 , respectively. Thus, we have four markers determined by the propagation directions of rays \mathbf{k}_1 , \mathbf{k}_2 , \mathbf{k}'_1 , \mathbf{k}'_2 . Another variant is possible when ray \mathbf{k}_1 diffracts in the direction of ray \mathbf{k}'_2 , while \mathbf{k}_2 – in the direction \mathbf{k}'_1 . Then, we also have four markers, the angles between them being smaller because diffraction occurs at a higher frequency. Indeed, divergence of diffracted radiation is determined in this case

V.M. Kotov, S.V. Averin, G.N. Shkerdin V.A. Kotel'nikov Institute of Radio Engineering and Electronics, Fryazino Branch, Russian Academy of Sciences, pl. Akad. Vvedenskogo 1, 141190 Fryazino, Moscow region, Russia; e-mail: vmk277@ire216.msk.su

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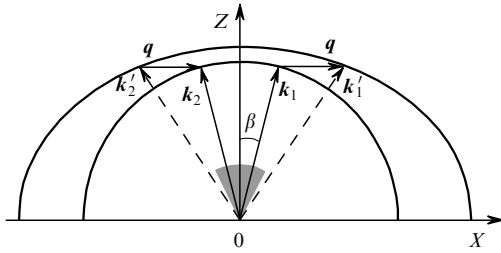


Figure 1. Vector AO diffraction diagram of divergent optical radiation in a uniaxial gyrotropic crystal.

by the sound divergence [13] $\varphi_{ac} \approx \Lambda/L = V/(fL)$, where Λ and f are the sound wavelength and frequency, V is the sound speed, L is the AO interaction length. One can see that as the frequency f increases, the sound divergence decreases (as well as the divergence of diffracted radiation).

The positions of markers k_1 and k_2 can be chosen such that the light intensity in the central part of each marker, k'_1 or k'_2 , be equal to $I_0/2$ for the 100% diffraction efficiency, where I_0 is the light intensity in the optical field maximum. Then, the distance between the markers k_1 and k_2 determines the angular width of divergent radiation at the level $I_0/2$. It is clear that in this case the polarisation of incident radiation should coincide with the polarisation of the ‘fast’ wave propagating in the crystal. If the crystal eigenwaves have the same polarisation in all the directions, this can be easily done: it is necessary to place the required phase plate or polariser in front of the crystal. However, when the TeO_2 single crystal with a strong gyrotropy near the [001] direction is used, the eigenwave polarisation changes with increasing the deflection angle of light propagation direction from the optical axis. In this case, it is impossible to establish the ‘required’ polarisation with the help of one element. Here we can act as follows: to establish polarisation of incident radiation with the help of an external polariser at an angle of 45° to the direction of sound propagation. Then, radiation splits into two elliptically polarised eigenwaves with identical intensities inside the crystal, only one of them diffracting by the acoustic wave. The angular width of optical radiation at the $I_0/2$ level is determined as angular distance between zero diffraction orders when the diffraction efficiency of the first orders is equal to $I_0/4$.

Figure 2 presents the distribution pattern of the field intensity on a screen placed behind the AO cell. The divergent radiation is shown by a dark spot at the centre. Inside the spot there are zero-order diffraction regions k_1 and k_2 shown in the form of elongated light vertical strips. The regions of the first diffraction orders k'_1 and k'_2 whose shape coincides with that of zero diffraction orders can be located both inside and outside the spot. Figure 2 also shows the scheme of diffraction $k_1 \rightarrow k'_2, k_2 \rightarrow k'_1$, which is more preferable than the scheme $k_1 \rightarrow k'_1, k_2 \rightarrow k'_2$ from the above considerations.

The diffraction orders k'_1 and k'_2 can be both outside and inside the optical field region. Consider first the case when the orders are located outside the field region. Then, measurements are simpler but the light divergence should be within some limits. Let us find these limits.

We assume for simplicity that the angular distribution of the radiation intensity obeys the Gaussian law, i.e., is described by the expression $I = I_0 \exp(-\alpha^2/\omega^2)$, where α is the angle of radiation deflection from the axis and ω is the

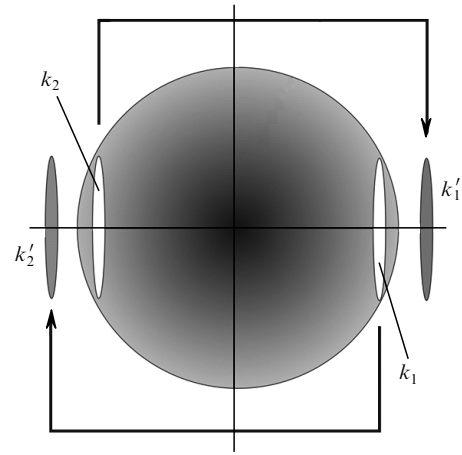


Figure 2. Optical field intensity distribution observed on the screen placed behind the AO cell.

angular half-width of radiation. Note that the half-width of this distribution at the $I_0/2$ level is $\sim 0.8\omega$. Calculations show that in the directions for which $|\alpha| \geq 2\omega$ the radiation intensity does not exceed 2%. We will use the angle 1.2ω as a criterion to estimate the minimal angle of Bragg scattering. Then, the zero order will propagate at an angle $\sim 0.8\omega$ to the axis, while the first order – at an angle 2ω . Taking into account the fact that the Bragg scattering angle is $\varphi_{sc} \approx \lambda/\Lambda$ (where λ is the light wavelength), we will write the above condition in the form $\varphi_{sc} \geq 1.2\omega$.

To analyse in detail the frequency-angular characteristics of AO diffraction in paratellurite, we will use the model in which the refractive indices of the anisotropic gyrotropic crystal are found from the expression [14]

$$n_{1,2}^2 = (1 + \tan^2 \varphi) \left\{ \frac{1}{n_o^2} + \frac{\tan^2 \varphi}{2} \left(\frac{1}{n_o^2} + \frac{1}{n_e^2} \right) \pm \frac{1}{2} \left[\tan^4 \varphi \left(\frac{1}{n_o^2} - \frac{1}{n_e^2} \right)^2 + 4G_{33}^2 \right]^{1/2} \right\}^{-1}, \quad (1)$$

where n_o, n_e are the main refractive indices of the crystal; φ is the angle between the optical axis of the crystal and the wave vector of the light wave; G_{33} is the component of the gyration pseudotensor. We will use in calculations the following constants corresponding to He–Ne-laser radiation propagating in the TeO_2 single-crystal: $\lambda = 0.63 \times 10^{-4}$ cm, $n_o = 2.26$, $n_e = 2.41$, $G_{33} = 2.62 \times 10^{-5}$. The moduli of the wave vectors k_1 and k_2 (because of their symmetric position with respect to the Z axis) are equal and amount to $2\pi n_1 \lambda^{-1}$, where n_1 is calculated from (1) at $\varphi = \beta$ (with the sign ‘+’ in the denominator). Then, the modulus of the sound-wave vector, q , has the form

$$q_{1,2} = \frac{2\pi}{\lambda} |n_{x1} \pm n_{x2}|, \quad (2)$$

where the sing ‘-’ corresponds to the diffraction processes $k_1 \rightarrow k'_1, k_2 \rightarrow k'_2$; the sign ‘+’ – to the processes $k_1 \rightarrow k'_2, k_2 \rightarrow k'_1$; n_{x1} ; and n_{x2} are the roots of a biquadratic equation

$$R_1 n_x^4 + P_1 n_x^2 + Q_1 = 0. \quad (3)$$

Here

$$R_1 = (n_o n_e)^{-2}; \quad P_1 = \left(\frac{1}{n_o^2} + \frac{1}{n_e^2} \right) \left(\frac{z_0^2}{n_o^2} - 1 \right);$$

$$Q_1 = z_0^4 \left(\frac{1}{n_o^4} - G_{33}^2 \right) - 2 \frac{z_0^2}{n_o^2} + 1; \quad z_0 = n_1 \cos \beta. \quad (4)$$

The angle between the vectors k_1 and k_2 outside the crystal is $\alpha_1 = 2n_o \arctan(n_{x1}/z_0)$, and the angle between k_1 and k'_1 (as well as between k_2 and k'_2) outside the crystal has the form

$$\alpha_2 = n_o \left| \arctan \frac{n_{x2}}{z_0} - 2 \arctan \frac{n_{x1}}{z_0} \right|,$$

where we assume that $n_{x1} < n_{x2}$.

Figure 3 shows the dependences of the sound frequency f on the angles α_1 , α_2 and the maximal angular half-width ω_{\max} , which can be measured in the case under study. The dependence $\omega_{\max}(f)$ is obtained from the above criterion $\varphi_{sc} \approx 1.2\omega_{\max}$. We took into account here that $\alpha_2 = \varphi_{sc}$. We assumed in calculations that diffraction results from the ‘slow’ sound wave propagating in the TeO₂ crystal at a $V = 0.617 \times 10^5 \text{ cm s}^{-1}$. One can see that α_1 monotonically increases, while α_2 decreases with increasing frequency. The light divergence measurement is, in fact, limited by the angles not exceeding 1.5° at which $\alpha_1 < \omega_{\max}$. On the other hand, the light divergence should be greater than radiation divergence in the diffraction orders $\varphi_{ac} \approx V/(fL)$. With the sound frequency $f = 35 \text{ MHz}$ and the AO interaction length $L = 0.6 \text{ cm}$ (the conditions of the experiment, see below), the divergence is $\varphi_{ac} \approx 11'$. For the light divergence angle $\omega = 1.5^\circ$, the measurement accuracy, as can be readily appreciated, is $\sim 10\%$. In the case of smaller ω , the accuracy decreases.

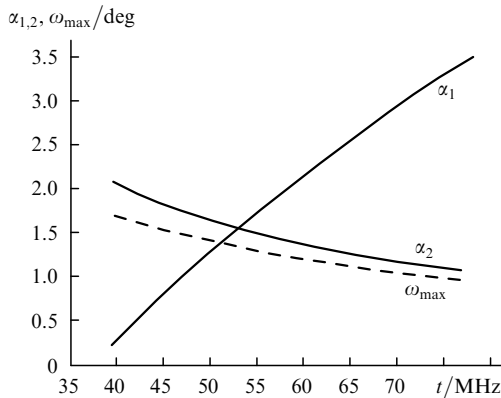


Figure 3. Dependences of the angles α_1 , α_2 , and ω_{\max} on the sound frequency f .

Consider now the case when the diffracted rays k'_1 and k'_2 lie inside the divergent radiation beam. It is clear that the accuracy of the half-width measurement increases significantly in this case because we need to take into account the intensities of both diffraction orders and the intensity of the undiffracted part of radiation. Here, for example, we can use the amplitude modulation of the acoustic wave, which makes it possible to measure the intensity of the diffraction orders with respect to the variable component, i.e., by neglecting the part of light not participating in diffraction.

In our experiments, we measured the intensity of light whose divergence is markedly greater than the Bragg scattering angle by using another method based on the employment of masks. The mask was placed in front of the AO cell so that to transmit radiation producing zero diffraction orders but to screen the regions where first diffraction orders appear. The optical scheme of the proposed method, which we tested in the experiment on measuring the divergence of laser radiation propagated through the lens is shown in Fig. 4. Linearly polarised radiation from He–Ne laser (1) propagates through polariser (2). Then, radiation is directed to lens (3), after which it becomes divergent. The beam is directed to AO cell (4) in front of which we place mask (5) in the form of a vertical wide slit. Polariser (2) is oriented at angle 45° to the direction of acoustic-wave propagation in crystal (4). Screen (6) is used to display in the horizontal plane the optical field intensity distribution. The light grey colour is the region of optical radiation propagating without the mask, while the dark grey colour is the region of radiation transmitted through the mask. The arrows originating from the AO cell show the directions of propagation of zero (dashed arrows) and first (solid arrows) Bragg orders. One can see that the first diffraction orders lie outside the screening region. They yield information only about the intensity of zero Bragg orders. The field distribution on the screen is shown schematically (the dashed curve indicates the field distribution in the absence of sound and mask, while the solid curve – in their presence). The angular distance between the zero diffraction orders is equal to the light divergence if the intensity of the first diffraction orders is 25% of its maximal intensity.

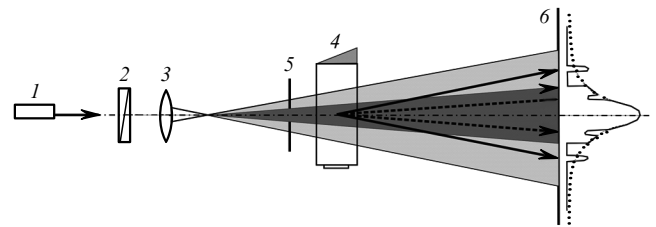


Figure 4. Optical scheme of the proposed method with the use of a mask (see details in the text).

We used in the experiment a $0.63\text{-}\mu\text{m}$ He–Ne laser. Radiation propagated through the polariser and was directed to the optical lens after which it became divergent. At a distance of $\sim 7 \text{ cm}$ from the lens we place the AO cell, which was made of the TeO₂ single crystal with dimensions $10 \times 10 \times 10 \text{ mm}$ along the directions [110], $[1\bar{1}0]$, and [001]. The piezotransducer made of LiNbO₃ was glued to the {110} face of the crystal and generated transverse oscillations propagating in the crystal in the form of a ‘slow’ acoustic wave. The AO interaction length was $L = 0.6 \text{ cm}$. The transducer frequency band lied in the range from 40 to 80 MHz at the 3-dB level. The intensity of the first diffraction orders in the case of the maximum diffraction efficiency (diffraction without overmodulation) was 25% of the radiation intensity in the central part at the sound frequency of 70 MHz. It follows from here that the angle between the zero diffraction orders is $\alpha_1 \approx 2.9^\circ$ (see Fig. 3). This angle is equal to the required radiation divergence.

Figure 5 presents photographs of the pattern observed on the screen. One can clearly see from Figs 5b, c the diffraction spots corresponding to the zero (dark vertical stripes) and first (light stripes) diffraction orders. They are rather narrow and their angular width at $f = 70$ MHz is equal to $\sim 6'$, i.e., amounts to $\sim 3.5\%$ of the optical beam width. One can see from Fig. 5c that the noise is absent in the region of the first diffraction orders. This makes it possible to measure with maximum accuracy the intensity of radiation 'retreating' from the zero orders.

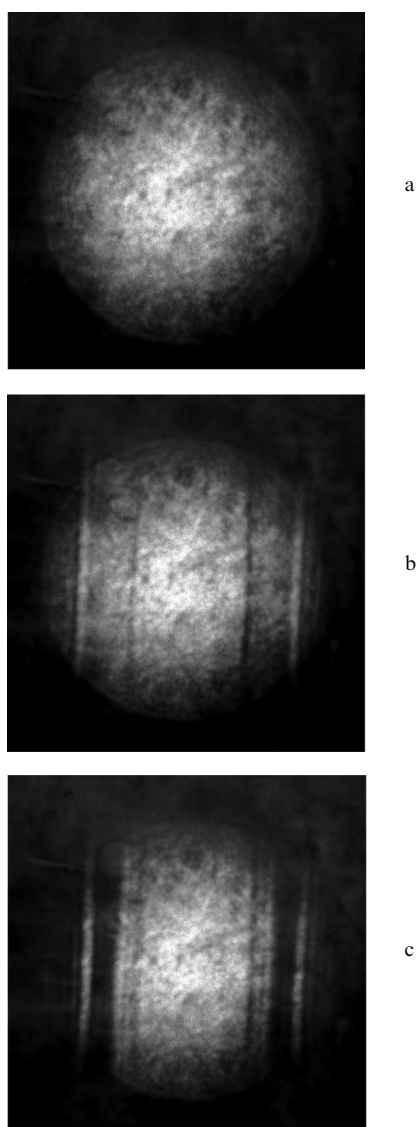


Figure 5. Photographs of the optical field, obtained on the screen: spot of divergent radiation in the absence of sound (a), in the presence of the acoustic wave (b), and in the presence of sound and the mask in the form of a wide vertical slit (c).

The method was used to measure the angular divergence of radiation scattered from the polished plate inhomogeneities. Radiation from the He–Ne laser was directed to the plate with the AO cell on the other side. The mask was placed between the plate and the AO cell. We observed on the screen the optical field intensity distribution with a small-grain structure (speckle pattern). We assumed that the envelope of the speckle pattern is described by a Gaussian

function, the maximal angular beam divergence being ensured by scattering from the smallest inhomogeneities. The divergence of scattered radiation proved rather great so that the frequency band of the AO cell did not provide the 'required' position of the diffraction orders. To decrease the divergence in half, we used a system of lenses mounted in front of the AO cell. The averaged intensity of the first diffraction orders near their maxima was $\sim 25\%$ of the intensity in the central part at $f = 78$ MHz. This frequency corresponds to the radiation divergence angle $\alpha_1 \approx 3.5^\circ$ (see Fig. 3). The light divergence without the lenses was $\omega = 7^\circ$. If the obtained angle corresponds to the angle of radiation scattering from inhomogeneities with the characteristic size D [6], it is easy to find D , by assuming that $D \approx \lambda/\omega$. In our case, $D \approx 5 \mu\text{m}$. Note that the scattering surface represented the glass plate polished with the M5 powder whose characteristic 'grains' are equal to $5 \mu\text{m}$, i.e., the size of the powder 'grain' virtually coincides in this case with that of the minimal inhomogeneity D .

Therefore, in this paper we did the following:

(i) We have proposed a method for measuring the radiation scattering angle, which is based on several Bragg diffraction processes proceeding in the crystal at different angles of light incidence on the sound wave.

(ii) We have analysed the variant when divergent optical radiation propagates near the optical axis of a uniaxial gyrotropic crystal, while the acoustic wave propagates orthogonally to this axis. The appearing diffraction processes occur symmetrically with respect to the optical axis of the crystal.

(iii) For radiation with the divergence greater than the Bragg scattering angle, we have suggested using a mask transmitting zero diffraction orders and screening those regions where the first diffraction orders are produced.

(iv) We have tested the method when measuring the divergent radiation produced by transmitting a Gaussian beam (emitted from the He–Ne laser) through a lens.

(v) We have used the method to estimate the size of the surface inhomogeneities by the angle of scattered radiation. Experiments with AO diffraction in paratellurite have made it possible to assess the characteristic minimal size of the polished glass plate inhomogeneities, which was close to the size of the polishing powder particles.

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