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Peculiarities of two-photon optical nutation of biexcitons in semiconductors taking into account elastic interparticle interactions

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***Abstract*. The peculiarities of two-photon nutation in a system of coherent biexcitons in CuCl semiconductors are considered taking into account the elastic biexciton – biexciton interaction. It is shown that depending on the system parameters, optical nutation represents a process of periodic transformation of photon pairs into biexcitons and vice versa. The possibility of exercising the phase control of the optical nutation process is predicted.**

Keywords: *two-photon optical nutation, biexcitons in semiconductors, elastic interparticle interactions.*

1. Introduction

Optical nutation is a periodic variation in the initial state of the system under the action of the external electromagnetic wave field, the variation leading to the medium radiation modulation [1, 2]. Burshtein et al. [3] presented the theory of optical nutation in a system of two-level atoms interacting with the finite number of photons in a resonator. The authors of papers [4 – 9] developed the theory of optical nutation in the exciton spectral region. They showed that at low excitation levels the nutation frequency is determined by a constant of the exciton–photon interaction, while at high excitation levels it becomes dependent on the exciton density. The authors of papers [7 – 10] also studied nutation in a system of coherent excitons, photons, and biexcitons in the region of the M-band luminescence. They showed that in the limit of the specified photon (exciton) density, the nutation frequency is proportional to the electromagnetic (material) wave amplitude.

The authors of papers [8 – 14] constructed the theory of two-photon nutation in a system of coherent biexcitons. Khadzhi et al. [13, 14] showed that the nutation frequency, even when the interparticle interaction is neglected, markedly depends on the photon and biexciton density. They predicted that nutation as a process is determined by the initial difference in photon and biexciton phases, which indicates the possibility of exercising the phase control of two-photon nutation. However, a natural question arises as to the effect of interparticle interactions on the nutation dynamics. This problem is especially urgent at high excitation levels when the biexciton density is rather high and the processes of elastic biexciton – biexciton interactions come to the fore. In this connection, we study in this paper the peculiarities of two-photon nutation in a system of coherent biexcitons, taking into account elastic interparticle interactions.

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2. Basic equations of the theory of two-photon nutation

Consider two-photon optical nutation in a system of coherent photons and biexcitons in semiconductors under the action of ultrashort resonant laser pulses. We assume that the pulse duration τ_p is much smaller that the biexciton relaxation time τ_{rel} ($\tau_p \ll \tau_{rel}$). In this case, the biexciton relaxation processes can be neglected because they have no time to come into action during the pulse. For this reason, below we take into account only the processes of stimulated emission and absorption of radiation with participation of biexcitons. Assuming that the spectral width of the pulses is much smaller than the binding energy of biexcitons (in the CuCl crystal it is 30 – 40 meV [15 – 17]), we can neglect the optical exciton – biexciton conversion and exciton – photon interaction because the above processes are characterised by a large detuning of its resonance energy with the photon energy, providing two-photon generation of biexcitons. The optical nutation phenomenon under study consists in pairwise transformation of identical photons into biexcitons and in radiative recombination of biexcitons with the formation of photon pairs (Fig. 1). The interaction Hamiltonian has the form

$$H_{\rm int} = h\mu\left(\hat{b}^+\hat{c}\hat{c} + \hat{c}^+\hat{c}^+\hat{b}\right) + \frac{1}{2}\,h v\hat{b}^+\hat{b}^+\hat{b}\hat{b}, \qquad (1)$$

where $\hat{b}$ ($\hat{b}^+$) and $\hat{c}$ ($\hat{c}^+$) are the operators of biexciton and photon annihilation (production), respectively; μ is the constant of two-photon excitation of a biexciton from the ground state of the crystal; v is the constant of the elastic biexciton – biexciton interaction.

We will use the mean field approximation where the mean value of the operators is nonzero: $\langle\hat{b}\rangle = b \neq 0$, $\langle\hat{c}\rangle = c \neq 0$. Here, b and c are the complex amplitudes of the material and electromagnetic fields. By averaging the

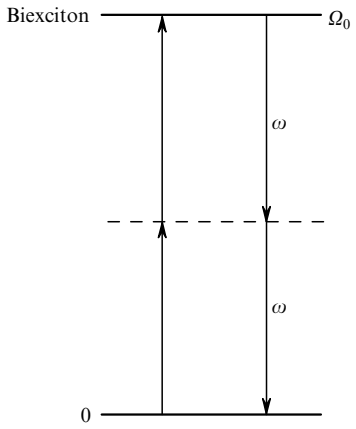

Figure 1. Scheme of quantum transitions from the ground state of the crystal into the biexciton state.

Heisenberg equations of motion for the operators $\hat{b}$ and $\hat{c}$, we obtain in this approximation the equations of motion for the corresponding amplitudes b and c. We assume that all photons are coherent. They have the same frequency, wave vector, and polarisation, these characteristics remaining the same during the pulse action. The produced biexcitons are also coherent. Thus, when the system is completely coherent, we can factorise the mean value from the product of some operators in the form of a product of mean values of each operator. The system [derived from Hamiltonian (1)] of nonlinear differential equations, which describes the temporal evolution of the material and electromagnetic field amplitudes, has the form

$$\mathrm{i}\dot{b} = \Omega_0 b + \mu c c + v b^* b b, \tag{2}$$

$$\mathrm{i}\dot{c} = \omega c + 2\mu c^* b, \tag{3}$$

where Ω_0 is the biexciton eigenfrequency; ω is the photon frequency. Because the states of the photons and biexcitons are coherent and macrofilled, we assume the parameters b and c to be the time functions. We will supplement systems of equations (2) and (3) by initial conditions

$$b|_{t=0} = b_0 \exp(\mathrm{i}\varphi_0), \quad c|_{t=0} = c_0 \exp(\mathrm{i}\psi_0), \tag{4}$$

where each variable is characterised by the initial amplitude (b_0, c_0) and phase (φ_0, ψ_0).

Let us consider the particle densities $N = |b|^2$, $f = |c|^2$ and 'polarisation' components $Q = \mathrm{i}(b^* c c - c^* c^* b)$ and $R = b^* c c + c^* c^* b$. Using (2) and (3) we can easily obtain a system of nonlinear evolutional equations for the introduced functions:

$$\dot{N} = -\mu Q, \quad \dot{f} = 2\mu Q, \tag{5}$$

$$\dot{Q} = (\Delta - vN)R + 2\mu f(4N - f), \tag{6}$$

$$\dot{R} = -(\Delta - vN)Q, \tag{7}$$

where $\Delta = 2\omega - \Omega_0$ is the resonance detuning. Using (4) we can obtain the initial conditions for the particle and polarisation densities:

$$N|_{t=0} \equiv N_0 = |b_0|^2, \quad f|_{t=0} \equiv f_0 = |c_0|^2,$$

$$Q|_{t=0} \equiv Q_0 = 2f_0\sqrt{N_0}\sin\Theta_0, \tag{8}$$

$$R|_{t=0} \equiv R_0 = 2f_0\sqrt{N_0}\cos\Theta_0,$$

where $\Theta_0 = \varphi_0 - 2\psi_0$ is the initial phase difference.

Solving system (5)–(7) with allowance for (8), we obtain the integral of motion for the particle density

$$2N + f = 2N_0 + f_0 \tag{9}$$

and the expression for Q:

$$Q^2 = 4N(2N_0 + f_0 - 2N)^2$$

$$-\left[\frac{\Delta}{\mu}(N - N_0) - \frac{v}{2\mu}\left(N^2 - N_0^2\right) + 2f_0\sqrt{N_0}\cos\Theta_0\right]^2. \tag{10}$$

Expression (9) represents the law of conservation of the number of particles in the system. Then, using the equation $\dot{N} = -\mu Q$ from (5) and expression (10) for Q, we can obtain the solution for the function $N(t)$. We will be interested below in the temporal evolution of the biexciton density $N(t)$ at different parameters of the system. The temporal evolution of the photon density $f(t)$ can be easily found from (9).

It follows from (5) and (10) that if photons are absent in the system ($f_0 = 0$, $N_0 \neq 0$) at the initial instant of time, then $N(t) = N_0 = \mathrm{const}$ and, hence, the two-photon decay of the biexcitons is impossible. This is caused by the fact that equations (2) and (3) take into account only induced transitions. On the other hand, if biexcitons are absent in the system ($N_0 = 0$, $f_0 \neq 0$) at the initial instant of time, the system evolves in time.

For convenience of further investigations we will use the normalised parameters

$$y = \frac{2N}{2N_0 + f_0}, \quad x = \frac{f}{2N_0 + f_0}, \quad y_0 = \frac{2N_0}{2N_0 + f_0},$$

$$x_0 = \frac{f_0}{2N_0 + f_0}, \quad \tau = \frac{t}{\tau_0}, \quad \tau_0^{-1} = 2\mu[2(2N_0 + f_0)]^{1/2}, \tag{11}$$

$$\alpha = \frac{v(2N_0 + f_0)^{1/2}}{8\sqrt{2}\mu}, \quad \beta = \Delta\tau_0.$$

Then, the integral of motion (9) is reduced to the form

$$x + y = x_0 + y_0 = 1, \tag{12}$$

while the basic equation for the temporal evolution of the normalised biexciton density $y(\tau)$ can be written in the form

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\tau}\right)^2 + W(y) = 0, \tag{13}$$

where

$$W(y) = -y(1 - y)^2 + \left[\beta(y - y_0) - \alpha\left(y^2 - y_0^2\right)\right.$$

$$\left. + x_0\sqrt{y_0}\cos\Theta_0\right]^2. \tag{14}$$

Expression (13) can be treated as the equation of oscillations of a nonlinear oscillator, where $(dy/d\tau)^2$ and $W(y)$ are the kinetic and potential energies of the oscillator, respectively. The behaviour of the function $y(\tau)$ can be qualitatively established by studying the dependence of the potential energy W of the nonlinear oscillator on y at different values of the parameters. The function $y(\tau)$ can vary in the region of y values, where $W(y) \leqslant 0$, and we can obtain from (5) and (8) the initial condition for the rate of change of this function. The sign of the derivative $(dy/d\tau)|_{\tau=0}$ is determined only by the initial phase difference Θ_0. At $\pi(2k+1) \leqslant \Theta_0 \leqslant 2\pi(k+1)$, we have $(dy/d\tau)|_{\tau=0} > 0$, while at $2\pi k \leqslant \Theta_0 \leqslant \pi(2k+1)$, $k = 0, 1, 2, \ldots$, we have $(dy/d\tau)|_{\tau=0} < 0$. One can see that the evolution of the system is determined by the initial particle densities x_0 and y_0, the initial phase difference Θ_0, normalised resonance detuning β, and nonlinearity parameter α. The case $\alpha = 0$ was studied in [13]. Consider the peculiarities of the temporal evolution of the system at $\alpha \neq 0$ under conditions of the exact resonance $\beta = 0$. The form of the solution of $y(t)$ depends on the parameters α and y_0. Unlike the case $\alpha = 0$ [13], the potential energy $W(y)$ of the nonlinear oscillator contains, apart from the term $-y(1-y)^2$, a positively defined term $[\alpha(y^2 - y_0^2) - x_0\sqrt{y_0}\cos\Theta_0]^2$ whose presence significantly changes the behaviour of the potential energy. This term has a minimum at point $y = (y_0^2 + x_0\sqrt{y_0}\alpha^{-1}\cos\Theta_0)^{1/2}$, where it vanishes and leads to the fact that at $\alpha = 0$ the twofold square $y = 1$ of the equation $W(y) = 0$, responsible for the motion of the phase point along the separatrix, splits at $\alpha \neq 0$ to two different squares, thereby ensuring the oscillatory evolution regime. Thus, we can assert that allowance for elastic interparticle interactions in the system leads to replacement of aperiodic evolution regimes by periodic ones.

3. Basic results

We will consider first the case when the initial phase difference $\Theta_0 = \pi/2$. At small α and fixed y_0, the equation $W(y) = 0$ has four positive roots $y_{\min} \leqslant y_{\max} < y_3 < y_4$, where $y_{\min} < y_0$, $y_0 < y_{\max} < 1$, $y_4 > y_3 \gg 1$. At $\alpha \to 0$ we obtain $y_{\min} = \alpha^2 y_0^4$, $y_{\max} = 1 - \alpha(1 - y_0^2)$, $y_3 = 1 + \alpha(1 - y_0^2)$, $y_4 = \alpha^{-2}$. As α increases, the root y_3 increases, while y_4 decreases, and it turns out that

$$y_3 = y_4 \equiv y_c = 1 + \sqrt[3]{1 - y_0^2}\left(\sqrt[3]{1 + y_0} + \sqrt[3]{1 - y_0}\right)$$

at some critical value of the nonlinearity parameter $\alpha = \alpha_c$, which is determined by the parameter y_0:

$$\alpha_c^2 = 4\left[(1 + y_0)^{4/3} + (1 - y_0)^{4/3} + (1 - y_0^2)^{2/3}\right]^{-3}.$$

As α further increases, there remain only two real roots, $y_{\min}$ and $y_{\max}$, while the roots y_3 and y_4 prove complex-conjugated: $y_{3,4} = u \pm iz$. When $\alpha^2 < \alpha_c^2$, i.e., when all the four roots of the equation $W(y) = 0$ are real, the solution of equation (13) has the form:

$$y = \frac{y_{\min}(y_4 - y_{\max}) + y_4(y_{\max} - y_{\min})\mathrm{sn}^2\varphi}{y_4 - y_{\max} + (y_{\max} - y_{\min})\mathrm{sn}^2\varphi}, \qquad (15)$$

where

$$\varphi = \frac{1}{2}\alpha[(y_4 - y_{\max})(y_3 - y_{\min})]^{1/2}\tau \pm F(\varphi_0, k);$$

$$\varphi_0 = \arcsin\left[\frac{(y_4 - y_{\max})(y_0 - y_{\min})}{(y_{\max} - y_{\min})(y_4 - y_0)}\right]^{1/2};$$

$$k^2 = \frac{(y_{\max} - y_{\min})(y_4 - y_3)}{(y_4 - y_{\max})(y_3 - y_{\min})};$$

$\mathrm{sn}\varphi$ is the elliptic Jacobi function [18, 19]; $F(\varphi_0, k)$ is the inexact elliptic integral of the first kind with the modulus k [18, 19]. The oscillation amplitude A and period T of the function $y(\tau)$ are determined by the expressions

$$A = y_{\max} - y_{\min}, \quad \frac{T}{\tau_0} = \frac{4K(k)}{\alpha[(y_4 - y_{\max})(y_3 - y_{\min})]^{1/2}}, \qquad (16)$$

where $K(k)$ is the total elliptic integral of the first kind with the modulus k [18, 19]. In the limit $\alpha \to 0$, we obtain from (15) $y = \tanh^2(\tau/2 \pm \mathrm{arctanh}\sqrt{y_0})$. At some critical values of the nonlinearity parameter $\alpha = \alpha_c$, the solution of (15) is reduced to the form

$$y = \frac{y_{\min}(y_c - y_{\max}) + y_c(y_{\max} - y_{\min})\sin^2\varphi}{(y_c - y_{\max}) + (y_{\max} - y_{\min})\sin^2\varphi}, \qquad (17)$$

where

$$\varphi = \frac{1}{2}\alpha[(y_c - y_{\min})(y_c - y_{\max})]^{1/2}\tau \pm \varphi_0;$$

$$\varphi_0 = \arcsin\left[\frac{(y_c - y_{\max})(y_0 - y_{\min})}{(y_{\max} - y_{\min})(y_c - y_0)}\right]^{1/2},$$

and the oscillation period T of the function $y(\tau)$ is

$$\frac{T}{\tau_0} = \frac{2\pi}{\alpha_c[(y_c - y_{\min})(y_c - y_{\max})]^{1/2}}. \qquad (18)$$

Finally, at $\alpha > \alpha_c$ we obtain the solution

$$y = \frac{y_{\max}n''(1 - \mathrm{cn}\varphi) + y_{\min}n'(1 + \mathrm{cn}\varphi)}{n' + n'' + (n' - n'')\mathrm{cn}\varphi}, \qquad (19)$$

where

$$\varphi = \alpha\left\{\left[(u - y_{\min})(u - y_{\max}) + z^2\right]^2 + z^2(y_{\max} - y_{\min})^2\right\}^{1/4}\tau + F(\varphi_0, k);$$

$$k^2 = \frac{1}{2}\times$$

$$\left\{1 - \frac{(u - y_{\min})(u - y_{\max})}{\left\{\left[(u - y_{\min})(u - y_{\max}) + z^2\right]^2 + z^2(y_{\max} - y_{\min})^2\right\}^{1/2}}\right\};$$

$$\varphi_0 = \arccos\frac{n''(y_{\max} - y_0) - n'(y_0 - y_{\min})}{n''(y_{\max} - y_0) + n'(y_0 - y_{\min})};$$

$$n' = \left[(u - y_{\max})^2 + z^2\right]^{1/2}; \quad n'' = \left[(u - y_{\min})^2 + z^2\right]^{1/2};$$

u and z are the real and imaginary parts of two complex-conjugated roots. The oscillation period T is determined by the expression

$$T = \frac{4K(k)}{\alpha\left\{\left[(u - y_{\min})(u - y_{\max}) + z^2\right]^2 + z^2(y_{\max} - y_{\min})^2\right\}^{1/4}}. \tag{20}$$

At some critical value of the parameter $\alpha = \alpha_c$, when $k = 0$, $z = 0$, and $u = y_c$, we again arrive to solution (17).

Figure 2 presents the dependences of the temporal evolution of the normalised biexciton density $y(\tau)$ on the nonlinearity parameter α at some values of the initial biexciton density y_0 for solutions with the sign '$-$' in the argument of the function because the solutions with the sign '$+$' differ only in the phase shift. The plots of the temporal evolution at $\beta = 0$ as a function of the parameter α are symmetric, i.e., $y(-\alpha) = y(\alpha)$. One can see from Fig. 2 that at $\alpha = 0$ the evolution of the system is aperiodic. The biexciton density $y(\tau)$ rapidly decreases with time. At $\tau = 2\text{arctanh}\sqrt{y_0}$, the density vanishes and then increases, and at $\tau \to \infty$ it asymptotically tends to unity. (Note that the solution with sign '$+$' increases with time and at $\tau \to \infty$ it also tends to unity.) Thus, at large times the solution $y(\tau)$ at $\alpha = 0$ tends to unity, which indicates that all the photons are transformed into biexcitons, thereby completing the evolutional process. At $\alpha \neq 0$ the evolution of the system is periodic with a period dependent on the parameters α and y_0. Therefore, at $\alpha \neq 0$ the optical nutation is a periodic process of pairwise transformation of photons into biexcitons and vice versa. Figure 3 shows the periods and amplitudes of the biexciton density oscillation as functions of the parameter α at a different initial biexciton density. As $|\alpha|$ increases, the amplitude and period of the biexciton density oscillations decreases at a fixed y_0, these changes being slow at small y_0. It follows from Fig. 2 that at large y_0 ($y_0 \leqslant 1$) the function $y(\tau)$ changes most drastically in the region of small values of the nonlinearity parameters $|\alpha|$. At large $|\alpha|$ and $y_0 \leqslant 1$ the oscillation amplitude A of the function $y(\tau)$ is approximately expressed by the formula $A = (1 - y_0)/|\alpha|$, i.e., it decreases with increasing α and y_0. At $y_0 = 1$ the oscillation amplitude vanishes, which means that oscillations are absent. The oscillation period T, infinite at $\alpha = 0$, slowly decreases with increasing $|\alpha|$ (Figs 2, 3). Thus, we can conclude that at $\Theta_0 = (2k + 1)\pi/2$ the account for biexciton – biexciton interactions leads to disappearance of the aperiodic evolution of the system and to a decrease in the amplitude of the biexciton density oscillations with increasing nonlinearity parameter α.

Consider now the case $\Theta_0 = 0$. At fixed y_0 and $\alpha < \alpha_c = (-1 + 3y_0 + 2\beta\sqrt{y_0})/(4y_0\sqrt{y_0})$, the equation $W(y) = 0$ has four real positive roots: $y_{\min} < y_0 < y_3 < y_4$. At $\alpha_c < \alpha < \alpha_d$, this equation has four real roots: $y_0 < y_{\max} < y_3 < y_4$. Note that the parameter α_c is determined

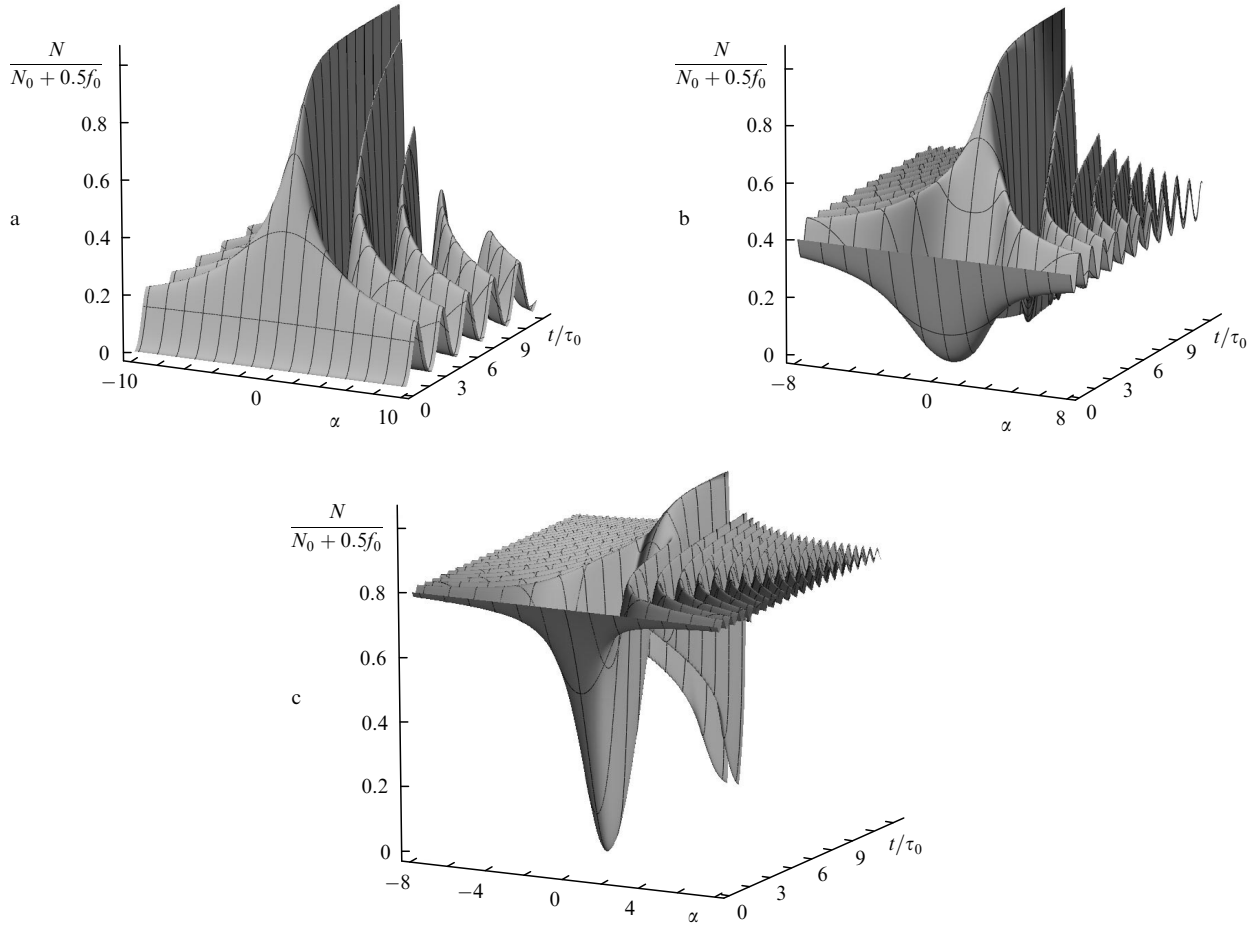

Figure 2. Temporal evolution of the biexciton density $N(t)$ as a function of the parameter α at $\Theta_0 = (\pi/2)(2k + 1)$ ($k = \pm 0, 1, 2, \ldots$) and $y_0 = 0$ (a), 0.4 (b), and 0.8 (c).

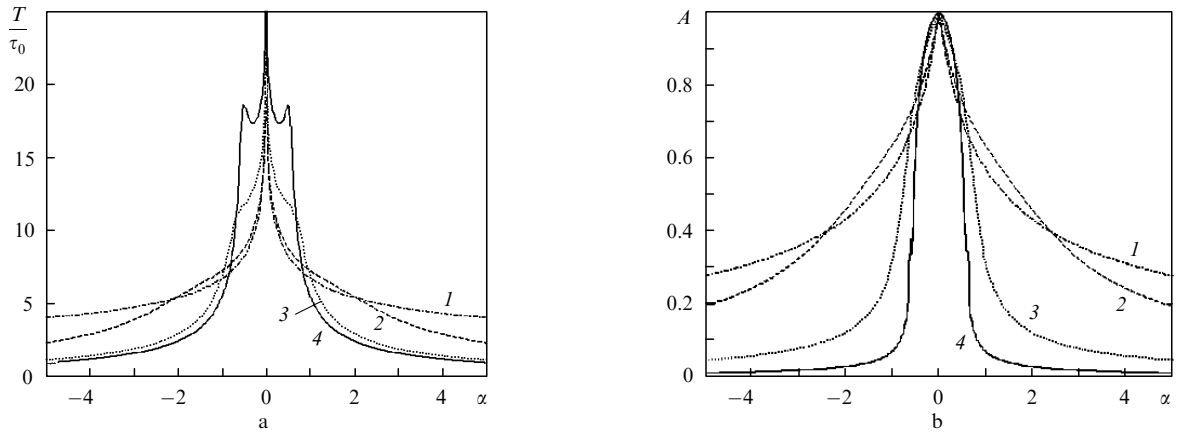

Figure 3. Dependences of the period (a) and amplitude (b) of the biexciton density oscillations at $\Theta_0 = (\pi/2)(2k+1)$ ($k = 1, 3, 5, \ldots$) on the parameter α at $y_0 = 0$ (*1*), 0.4 (*2*), 0.8 (*3*), and 0.9 (*4*).

from the condition of equality of the first two roots, $y_{\min} = y_0$, and the parameter α_d – from the condition of equality of the last two roots, i.e., $y_3 = y_4$. As the parameter α ($\alpha > \alpha_d$) increases, the equation has two real positive roots ($y_0 < y_{\max}$) and two complex-conjugated roots ($y_{3,4} = u \pm iz$). We will numerically calculate them below.

If $\alpha < \alpha_c$, the solution of equation (13) has the form:

$$y = \frac{y_{\min}(y_4 - y_0) + y_4(y_0 - y_{\min})\mathrm{sn}^2\varphi}{y_4 - y_0 - (y_0 - y_{\min})\mathrm{sn}^2\varphi}, \tag{21}$$

where

$$\varphi = \frac{1}{2}\,\alpha[(y_4 - y_0)(y_3 - y_{\min})]^{1/2}\tau;$$

$$k^2 = \frac{(y_0 - y_{\min})(y_4 - y_3)}{(y_4 - y_0)(y_3 - y_{\min})}.$$

The amplitude A and period T of the function $y(\tau)$ oscillations are found from the expressions

$$A = y_0 - y_{\min}, \quad \frac{T}{\tau_0} = \frac{4K(k)}{\alpha[(y_4 - y_0)(y_3 - y_{\min})]^{1/2}}. \tag{22}$$

When the nonlinearity parameter lies in the interval $\alpha_c < a < a_d$, the solution of equation (13) takes the form:

$$y = \frac{y_0(y_4 - y_{\max}) + y_4(y_{\max} - y_0)\mathrm{sn}^2\varphi}{y_4 - y_{\max} + (y_{\max} - y_0)\mathrm{sn}^2\varphi}, \tag{23}$$

where

$$\varphi = \frac{1}{2}\,\alpha[(y_4 - y_{\max})(y_3 - y_0)]^{1/2}\tau;$$

$$k^2 = \frac{(y_{\max} - y_0)(y_4 - y_3)}{(y_3 - y_0)(y_4 - y_{\max})};$$

$$A = y_{\max} - y_0; \quad \frac{T}{\tau_0} = \frac{4K(k)}{\alpha[(y_4 - y_{\max})(y_3 - y_{\max})]^{1/2}}. \tag{24}$$

At $\alpha = \alpha_c$ the solution of the basic equation, as follows from (21) and (23), is given by the equality $y = y_0 = \mathrm{const}$. This means that when the initial densities of biexcitons and photons are nonzero, the particle density stops oscillating at $\alpha = \alpha_c$ and $y = y_0$. In this case, upon passing through some critical point, the oscillation period changes continuously. The phase centre corresponds to this regime on the phase trajectory. As α increases, the phase trajectory (which has the shape of an ellipse-like closed curve) slowly shrinks and at $\alpha = \alpha_c$ collapses to a point. Then, at $\alpha > \alpha_c$ a new ellipse-like trajectory develops from this point, which increases with increasing α. Thus, at $\alpha = \alpha_c$ the system is at rest although the photon and biexciton densities are nonzero at the initial instant of time.

At a critical value of the nonlinearity parameter $\alpha = \alpha_d$, when $y_3 = y_4$, the solution of the basic equation is reduced to the form

$$y = \frac{y_0(y_3 - y_{\max}) + y_3(y_{\max} - y_0)\sin^2\varphi}{y_3 - y_{\max} + (y_{\max} - y_0)\sin^2\varphi},$$

where

$$\varphi = \frac{1}{2}\,\alpha[(y_3 - y_0)(y_3 - y_{\max})]^{1/2}.$$

In this case, the oscillation period is expressed as

$$\frac{T}{\tau_0} = \frac{2\pi}{\alpha[(y_3 - y_{\max})(y_3 - y_0)]^{1/2}}.$$

Finally, at $\alpha > \alpha_d$ we arrive to the solution

$$y = \frac{y_{\max}n''(1 - \mathrm{cn}\varphi) + y_0n'(1 + \mathrm{cn}\varphi)}{n' + n'' + (n' - n'')\mathrm{cn}\varphi}, \tag{25}$$

where

$$\varphi = \alpha\left\{[(u - y_0)(u - y_{\max}) + z^2]^2 + z^2(y_{\max} - y_0)\right\}^{1/4}\tau;$$

$$k^2 = \frac{1}{2}$$

$$\times\left\{1 - \frac{(u - y_0)(u - y_{\max}) + z^2}{\left\{[(u - y_0)(u - y_{\max}) + z^2]^2 + z^2(y_{\max} - y_0)^2\right\}^{1/2}}\right\};$$

$$n' = \left[(u - y_{\max})^2 + z^2\right]^{1/2}; \quad n'' = \left[(u - y_0)^2 + z^2\right]^{1/2};$$

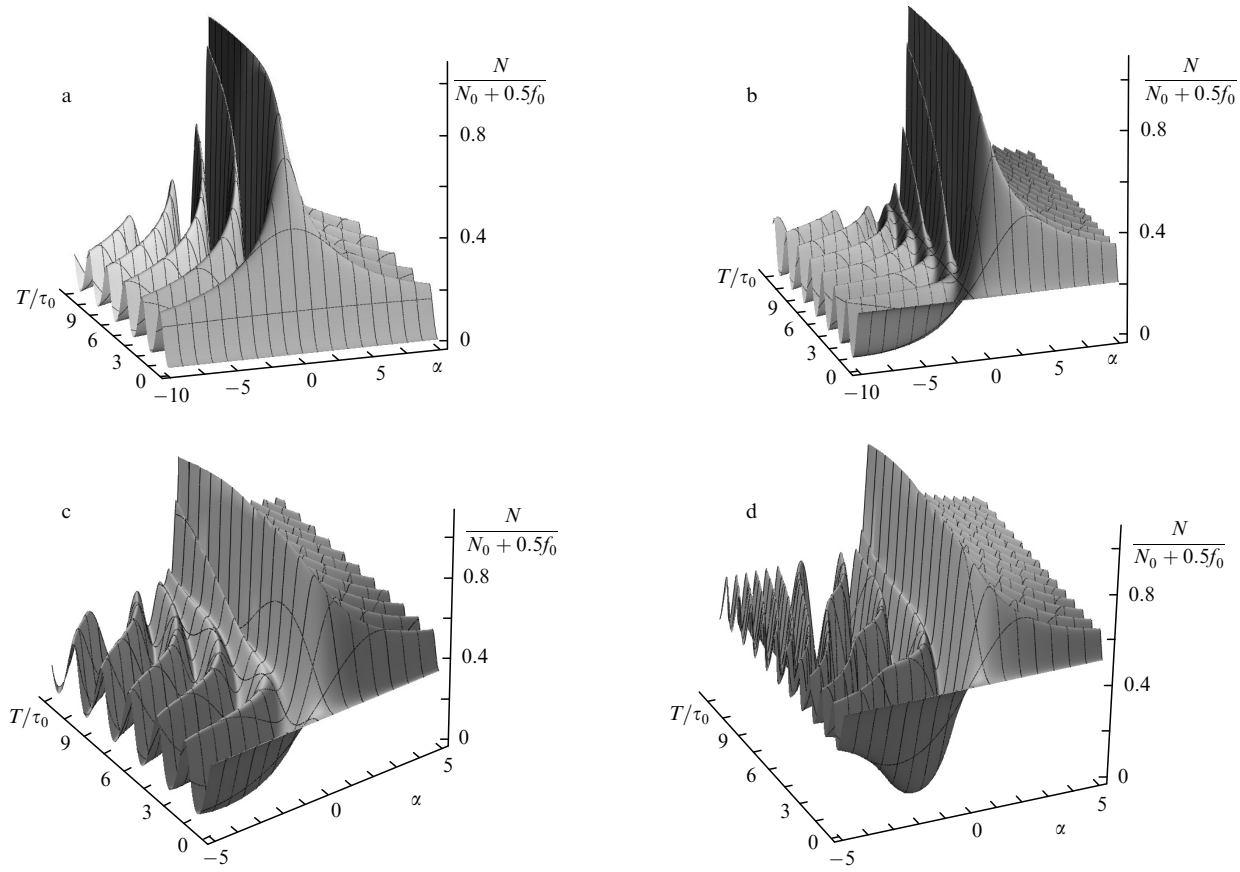

Figure 4. Temporal evolution of the biexciton density $N(t)$ as a function of the parameter α at $\Theta_0 = \pi k$ ($k = \pm 0, 1, 2, \ldots$) and $y_0 = 0$ (a), 0.2 (b), 1/3 (c), and 0.5 (d).

u and z are the real and imaginary parts of two complex-conjugated roots. The oscillation amplitude A and period T are expresses as

$$A = y_{\max} - y_0,$$
$$T = \frac{4K(k)}{\alpha\left\{\left[(u-y_0)(u-y_{\max})+z^2\right]^2 + z^2(y_{\max}-y_0)\right\}^{1/4}}. \tag{26}$$

Figure 4 shows the normalised biexciton density versus time and nonlinearity parameter α at the initial phase difference $\Theta_0 = 0$ and a different initial biexciton density. Figure 5 presents the period and amplitude of the biexciton density oscillations as a function of α at different fixed y_0. One can see that at $y_0 = 0$ and in the absence of interparticle interaction ($\alpha = 0$) the system evolves aperiodically: all photons transform into biexcitons, thereby terminating the evolution (Fig. 4a). When $y_0 \neq 0$ the aperiodic oscillation regime is displaced with respect to the zero with increasing the parameter α (Figs 4b–d and Fig. 5a).

When the parameter of the interparticle interaction is nonzero, the system behaviour is mainly determined both

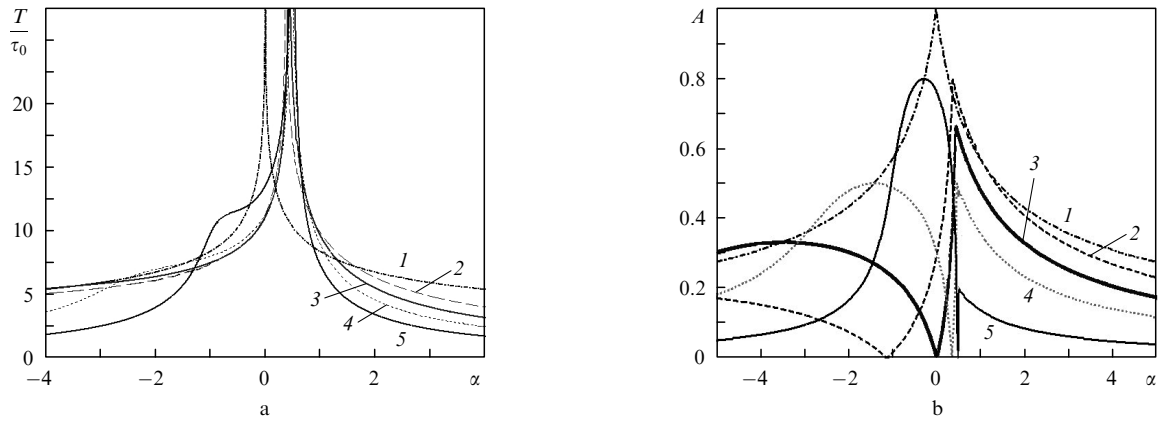

Figure 5. Dependences of the period (a) and amplitude (b) of the biexciton density oscillations at $\Theta_0 = \pi k$ ($k = \pm 0, 1, 2, \ldots$) on the parameter α at $y_0 = 0$ (*1*), 0.2 (*2*), 1/3 (*3*), 0.4 (*4*), and 0.8 (*5*).

by the initial photon density and α. One can see from Figs 4b–d that at $\alpha < \alpha_c$ the oscillations take place below the plane $y = y_0$. In this interval the oscillation amplitude first increases with increasing α and then decreases becoming zero at $\alpha = \alpha_c$ (Fig. 5b). The oscillation period at $\alpha = \alpha_c$ monotonically increases with increasing the parameter α (Fig. 5a). At $\alpha = \alpha_c$ the system is at rest, i.e., the biexciton density does not change with time. At $\alpha > \alpha_c$ the oscillations take place above the plane $y = y_0$.

At $y_0 \sim 1$ the equation $W(y) = 0$ has a twofold root $y \simeq 1$. In this case, the divergence of the oscillation period emerges at $y \sim \sqrt{y_0}/(1 + y_0)$, i.e., shifts to the positive α (Fig. 4a). At $y_0 = 1$ we obtain $\alpha = 0.5$. The parts of the curves with $T/\tau_0 \to \infty$ approach asymptotically this point. Therefore, at $y_0 \neq 0$ the aperiodic regime shifts to positive nonzero α. Thus, at $y_0 \to 1$ the system evolves in a complicated manner in the vicinity of $\alpha = 0$, while at $|\alpha| \gg 1$ the evolution is reduced to small-amplitude oscillations of the biexciton density above or below the initial density f_0. In addition, one can see from Fig. 5 that the oscillation amplitudes at different y_0 have zero values both at $\alpha < 0$ and $\alpha > 0$. Indeed, the position of the amplitude zero on the α axis is given by the expression $\alpha = \alpha_c = (1 - 3y_0)/(4y_0\sqrt{y_0})$. Of interest is also the fact that at $y_0 = 1/3$ the system is at rest when $\alpha = 0$, i.e., in the absence of interparticle interactions.

4. Conclusions

Account for interparticle interactions in two-photon nutation of biexcitons leads to the fact that the temporal evolution of the system changes qualitatively: it transforms from aperiodic at $\alpha = 0$ into periodic at $\alpha \neq 0$, the nutation amplitude and period decreasing monotonically with increasing $|\alpha|$ (see Fig. 2). Note also that as $|\alpha|$ increases, the biexciton self-trapping is absent in the temporal evolution of the system. Indeed, the self-trapping typical of the evolution of Bose-condensed atom density in a two-well potential is absent here. But nevertheless we can find some similarities. The atoms are self-trapped in one of the wells when the values of the interatom interaction parameter are larger than the critical ones. In this case, the amplitude of the atom oscillations decreases stepwise and continues decreasing with increasing this parameter [20, 21]. In our case we deal only with a monotonic change in the amplitude oscillation with increasing $|\alpha|$, which, in our opinion, is related but not identical to the self-trapping phenomenon. In this case, the larger the y_0 the faster (depending on α) the regime of small-amplitude oscillations of the system density.

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