PACS numbers: 42.55.Tv; 42.60.Da
DOI: 10.1070/QE2011v041n01ABEH014307

High-intensity super-radiance pulses generated by an active medium placed in a two-dimensional Bragg structure

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***Abstract*. The possibility is shown of using two-dimensional Bragg structures for generating short super-radiance (SR) pulses of high intensity. The structures provide spatial synchronisation of the radiation in two orthogonal directions in the active medium with large Fresnel parameters. In contrast to the SR effects without Bragg structure, an efficient generation of SR arises in an active medium with the dimensions not exceeding the inverse increment with the intensity by several orders of magnitude greater than the SR intensity of the same medium in a free space.**

Keywords: *super-radiance, distributed feedback laser, two-dimensional Bragg cavity.*

1. Introduction

In [1 – 3], the possibility is studied of generating a high-power spatially coherent radiation in lasers with the two-dimensional distributed feedback (2D DF), which is realised by using two-dimensional Bragg structures. In such structures, unlike traditional one-dimensional (single-periodic) Bragg structures, the coupling and mutual scattering of four, rather than two, wave fluxes arises, which propagate in the directions $\pm z$ and $\pm x$. Thus, it becomes possible to synchronise radiation of planar two-dimensional active media characterised by large Fresnel parameters in the orthogonal directions. In preceding works, the dynamics of 2D DF lasers was studied in the balance approximation under the assumption that the phase relaxation times of the active medium are short in the scale of the times of energy release and relaxation of inversion. In this case, at a constant pump intensity, a stationary single-frequency regime of generation is formed in a wide range of parameters.

However, presently, a number of active media are known in which a spectrally narrow optical or IR radiation is amplified many-fold per single pass. These media are used in various types of injection quantum-well heterolasers and in fibre lasers with doped active centres under the conditions of a balanced regime of stationary generation. Nevertheless, at a sufficiently weak relaxation of polarisation, such media operate in the regimes typical for Dicke super-radiance (SR), in which short (with the duration in the range from nanoseconds to sub-picoseconds) light pulses are generated [4 – 8].

It worth noting that in a series of theoretical and experimental studies the possibility of initiating SR processes by traditional single-periodic Bragg structures was investigated [4, 9]. The transversal dimensions of an active medium and, respectively, the power of radiation in the case of its quasi-single-dimension propagation are limited by the necessity of transversal synchronisation of generated radiation. This synchronisation can be provided due to the natural diffraction spreading of a wave packet at small values of the Fresnel parameter: $l_x^2/(l_z\lambda) < 1$ (l_z is the length of the active sample, l_x is its width, and λ is the wavelength).

In the present study we investigate the possibility of increasing the integral (with respect to the spatial coordinates x and z) power of SR pulses in the active media with $l_x^2/(l_z\lambda) \gg 1$. In this case, the spatial synchronisation is provided due to employment of two-dimensional Bragg structures.

2. Model and principal equations

Let us consider the simple model of an inverted active medium placed in a two-dimensional Bragg structure, in which the active medium in the y direction is assumed homogeneous and infinite, whereas in the x and z directions it occupies a rectangular area with the dimensions l_x and l_z, respectively (see Fig. 1). The Bragg structure is a dielectric with a double-periodic modulation of the real part of the dielectric constant:

$$\varepsilon(x,z) = \varepsilon_0 + \frac{\varepsilon_1}{2}\{\cos[\bar{h}(x-z)] + \cos[\bar{h}(x+z)]\}, \qquad (1)$$

where ε_0 is the average permittivity of the medium; ε_0 is the amplitude of modulation; $\bar{h} = 2\pi/D$; D is the period of modulation in the x and z directions. In the Bragg resonance conditions $h = \bar{h}$ (h is the wavenumber), two-dimensional Bragg structure (1) provides coupling and mutual scattering of the four partial wave fluxes propagating in the directions $\pm z$ and $\pm x$ of the vector-potential A of an electromagnetic field:

$$A = \mathrm{Re}[(C_z^+ e^{-ihz} + C_z^- e^{ihz} + C_x^+ e^{-ihx} + C_x^- e^{ihx})\, e^{i\omega t}]. \qquad (2)$$

Interaction of the field with an active two-level medium will be describe in the framework of semi-classical approx-

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Received 11 March 2010; revision received 10 November 2010
Kvantovaya Elektronika **41** (1) 34 – 37 (2011)
Translated by N.A. Raspopov

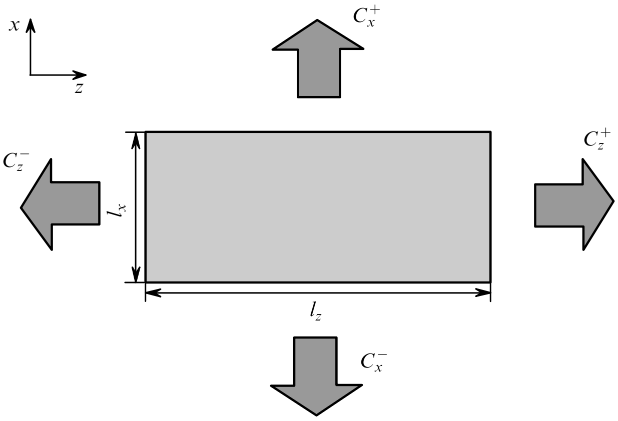

Figure 1. Model of two-dimensional Bragg structure filled by inverted active medium.

imation [10], in which electromagnetic fields are described by the Maxwell equations, and the Bloch equations are used for representing the active medium:

$$\left(\Delta - \frac{\varepsilon(x,z)}{c^2}\frac{\partial^2}{\partial t^2}\right)A = -\frac{4\pi}{c}P,$$

$$\frac{\partial^2 P}{\partial t^2} + \frac{1}{T_2}\frac{\partial P}{\partial t} + \left(\frac{1}{4T_2^2} + \omega_0^2\right)P = -\frac{2\omega_0|\mu|^2}{\hbar c}A\rho, \qquad (3)$$

$$\frac{\partial \rho}{\partial t} + \frac{\rho}{T_1} = \frac{2}{h\omega_0 c}A\frac{\partial P}{\partial t}.$$

Here, P is the polarisation of the medium; ρ is the population difference; μ is the dipole moment; T_1 and T_2 are the inversion and polarisation relaxation times of the active medium, respectively. We assume that the transition frequency ω_0 coincides with the Bragg resonance frequency: $\omega_0 = \bar{h}c$.

In view of the representation of the radiation field as four partial wave fluxes (2) we will represent the polarisation and inversion of the medium in the following way:

$$P = \mathrm{Re}[\mathrm{i}(P_z^+ \mathrm{e}^{-\mathrm{i}\bar{h}z} + P_z^- \mathrm{e}^{\mathrm{i}\bar{h}z} + P_x^+ \mathrm{e}^{-\mathrm{i}\bar{h}x} + P_x^- \mathrm{e}^{\mathrm{i}\bar{h}x})\mathrm{e}^{i\omega_0 t}],$$

$$\rho = \rho_0 + \mathrm{Re}[\rho_{2z}\mathrm{e}^{2\mathrm{i}\bar{h}z} + \rho_{2x}\mathrm{e}^{2\mathrm{i}\bar{h}x} \qquad (4)$$

$$+ \rho_{z-x}\mathrm{e}^{2\mathrm{i}\bar{h}(z-x)} + \rho_{z+x}\mathrm{e}^{2\mathrm{i}\bar{h}(z+x)}],$$

where $P_{x,z}^{\pm}$, ρ_0, $\rho_{2x,2z}$, and $\rho_{z\pm x}$ are the amplitudes slowly varying in time and space.

In the case of large Fresnel parameters ($l_{x,z}^2/(l_{z,x}\lambda) \gg 1$), we will describe propagation of partial wave fluxes (2) in the geometrical optics approximation neglecting their diffraction spreading. We also omit the summands describing relaxation of medium inversion because in the SR regime this process should be substantially slower in the time scale of electromagnetic filed evolution. Under these assumptions, the amplification of wave fluxes (2) in an active medium and their mutual scattering on the Bragg lattice (1) and on the nonlinear lattice induced due to the modulation of inversion (see [11]) may be described by the combined averaged equations comprising the coupled wave equations

$$\left(\pm\frac{\partial}{\partial Z} + \frac{\partial}{\partial \tau}\right)\hat{C}_z^{\pm} + \mathrm{i}\alpha(\hat{C}_x^+ + \hat{C}_x^-) = \hat{P}_z^{\pm},$$

$$\left(\pm\frac{\partial}{\partial X} + \frac{\partial}{\partial \tau}\right)\hat{C}_x^{\pm} + \mathrm{i}\alpha(\hat{C}_z^+ + \hat{C}_z^-) = \hat{P}_x^{\pm}, \qquad (5)$$

and by the combined equations for the amplitudes of spatial harmonics of polarisation and inversion of the active medium:

$$\frac{\partial \hat{P}_z^+}{\partial \tau} + \frac{\hat{P}_z^+}{\hat{T}_2} = 2\hat{C}_z^+\hat{\rho}_0 + \hat{C}_z^-\hat{\rho}_{2z}^* + \hat{C}_x^+\hat{\rho}_{z-x}^* + \hat{C}_x^-\hat{\rho}_{z+x}^*,$$

$$\frac{\partial \hat{P}_z^-}{\partial \tau} + \frac{\hat{P}_z^-}{\hat{T}_2} = 2\hat{C}_z^-\hat{\rho}_0 + \hat{C}_z^+\hat{\rho}_{2z} + \hat{C}_x^+\hat{\rho}_{z+x} + \hat{C}_x^-\hat{\rho}_{z-x},$$

$$\frac{\partial \hat{P}_x^+}{\partial \tau} + \frac{\hat{P}_x^+}{\hat{T}_2} = 2\hat{C}_x^+\hat{\rho}_0 + \hat{C}_x^-\hat{\rho}_{2x}^* + \hat{C}_z^+\hat{\rho}_{z-x} + \hat{C}_z^-\hat{\rho}_{z+x}^*,$$

$$\frac{\partial \hat{P}_x^-}{\partial \tau} + \frac{\hat{P}_x^-}{\hat{T}_2} = 2\hat{C}_x^-\hat{\rho}_0 + \hat{C}_x^+\hat{\rho}_{2x} + \hat{C}_x^+\hat{\rho}_{z+x} + \hat{C}_x^-\hat{\rho}_{z-x}^*, \qquad (6)$$

$$\frac{\partial \hat{\rho}_0}{\partial \tau} = -\mathrm{Re}(\hat{C}_z^+\hat{P}_z^{+*} + \hat{C}_z^-\hat{P}_z^{-*} + \hat{C}_x^+\hat{P}_x^{+*} + \hat{C}_x^-\hat{P}_x^{-*}),$$

$$\frac{\partial \hat{\rho}_{2z}}{\partial \tau} = -(C_z^{+*}P_z^- + C_z^- P_z^{+*}), \quad \frac{\partial \hat{\rho}_{2x}}{\partial \tau} = -(C_x^{+*}P_x^- + C_x^- P_x^{+*}),$$

$$\frac{\partial \hat{\rho}_{z+x}}{\partial \tau} = -(\hat{C}_z^{+*}\hat{P}_x^- + \hat{C}_z^-\hat{P}_x^{+*} + \hat{C}_x^{+*}\hat{P}_z^- + \hat{C}_x^-\hat{P}_z^{+*}),$$

$$\frac{\partial \hat{\rho}_{z-x}}{\partial \tau} = -(\hat{C}_z^{+*}\hat{P}_x^+ + \hat{C}_z^-\hat{P}_x^{-*} + \hat{C}_x^+\hat{P}_z^{+*} + \hat{C}_x^{-*}\hat{P}_z^-).$$

Here, $\hat{\rho} = \rho/\rho_{\mathrm{a}}$; $\hat{P}_{x,z}^{\pm} = P_{x,z}^{\pm}(\rho_{\mathrm{a}}\hbar\omega_0\omega_{\mathrm{c}}^2\sqrt{\varepsilon_0}/2\pi)^{-1/2}$; $\hat{C}_{x,z}^{\pm} = C_{x,z}^{\pm}(2\pi c^2\rho_{\mathrm{a}}\hbar/2\pi\omega_0\sqrt{\varepsilon_0})^{-1/2}$; $\hat{T}_2 = \omega_{\mathrm{c}}T_2$; $X = \omega_{\mathrm{c}}\sqrt{\varepsilon_0}x/c$; $Z = \omega_{\mathrm{c}}\sqrt{\varepsilon_0}z/c$; $\tau = \omega_{\mathrm{c}}t$; $\alpha = \varepsilon_1\omega_0/8\omega_{\mathrm{c}}$ is the coupling factor for the partial waves on the two-dimensional Bragg structure; $\omega_{\mathrm{c}} = [\pi\rho_{\mathrm{a}}|\mu|^2/(h\omega_0\sqrt{\varepsilon_0})]^{1/2}$ is the cooperative frequency; ρ_{a} is the initial density of inverted atoms.

Assuming that there are no fluxes of electromagnetic energy passing to the system outside and no considerable partial wave reflections from the boundaries of the Bragg structure, the boundary conditions to Eqns (5) can be presented in the form

$$\hat{C}_x^{\pm}\big|_{X=\mp L_x/2} = 0, \quad \hat{C}_z^{\pm}\big|_{Z=mL_z/2} = 0, \qquad (7)$$

where $L_{x,z} = \omega_{\mathrm{c}}\sqrt{\varepsilon_0}l_{x,z}/c$. In order to describe a super-radiant regime we assume that the initial homogeneous inversion of the medium is prescribed (an external pumping is absent)

$$\hat{\rho}_0|_{\tau=0} = 1, \quad \hat{\rho}_{2x,2z,z\pm x}|_{\tau=0} = 0, \qquad (8)$$

and there are also the fluctuations of polarisation in the medium

$$\hat{P}_{x,z}^{\pm}\big|_{\tau=0} = p_0\exp[\mathrm{i}\varphi_0(X,Z)], \qquad (9)$$

which have a small initial amplitude $p_0 \ll 1$ and random phase distribution $\varphi_0(X,Z)$. Initial electromagnetic fields are absent: $\hat{C}^{\pm}_{x,z}\big|_{\tau=0} = 0$. The linear (specific) radiation power (the power per unit thickness of the system in the coordinate y) in the directions $\pm x$ and $\pm z$ can be presented in the form

$$S_z^+ = \frac{\rho_a h\omega_0 c^2}{4\omega_c\sqrt{\varepsilon_0}}\hat{S}_z^+, \quad (10)$$

where

$$\hat{S}_x^{\pm} = \int_{-L_z/2}^{L_z/2} |\hat{C}_x^+(\pm L_x/2, Z, \tau)|^2 \mathrm{d}Z,$$
$$\hat{S}_z^{\pm} = \int_{-L_x/2}^{L_x/2} |\hat{C}_x^+(X, \pm L_z/2, \tau)|^2 \mathrm{d}X \quad (11)$$

are normalised integral radiation powers with respect to the corresponding coordinates.

3. Results of modelling

In Fig. 2, the results of modelling super-radiant operation regime are shown based on combined equations (5) and (6) at various values of the normalised time of phase relaxation $\hat{T}_2$. One can see that at greater relaxation constant, the SR pulse formation time and duration reduce, and the peak power increases. In these conditions, after emitting a pulse the inversion averaged over the sample takes a negative value, which is typical for the SR regime. From Fig. 3 it follows that there is the optimal value of the coupling coefficient α, at which for the given geometrical dimensions of the active sample $L_x = 0.25$, $L_z = 0.5$ the pulse peak power reaches a maximum. It is important that at a sufficiently small, in the scale of the inverse increment, length of the sample, introduction of the Bragg structure increases the pulse intensity by several orders of magnitude. At the same parameters of the active medium and initial noises, the maximum SR pulse amplitude $\hat{C}_z^+$ in the regime of single-pass amplification without two-dimensional Bragg structure is 2×10^{-6} and the delay τ increases to 50. Correspondingly, introduction of the Bragg lattice lowers requirements to the rates of phase relaxation in an active medium, at which the SR pulse is formed.

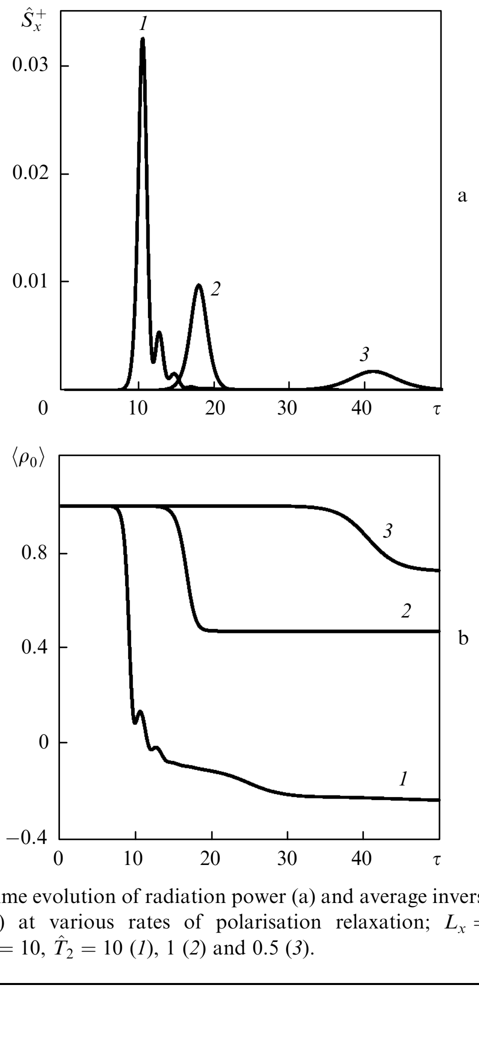

Figure 2. Time evolution of radiation power (a) and average inversion of medium (b) at various rates of polarisation relaxation; $L_x = 0.25$, $L_z = 0.5$, $\alpha = 10$, $\hat{T}_2 = 10$ (*1*), 1 (*2*) and 0.5 (*3*).

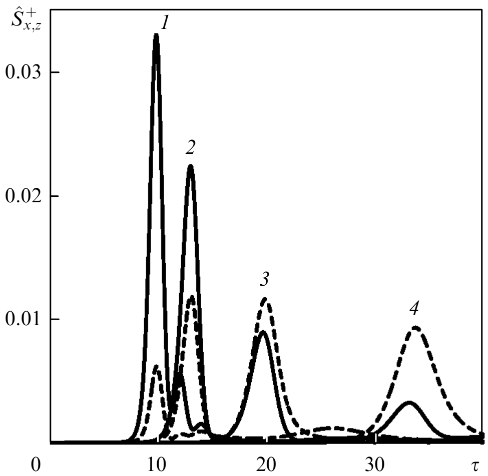

Figure 3. Emitted power versus time and various values of the wave coupling parameter. Solid line – power emitted in the x direction, dashed line – in the z direction; $L_x = 0.25$, $L_z = 0.5$, $\hat{T}_2 = 10$, $\alpha = 10$ (*1*), 5 (*2*), 3 (*3*) and 2 (*4*).

It is also interesting to juxtapose the radiation intensities in the orthogonal directions Z and X for the Bragg structures with various values of the coupling parameter α. At sufficiently small α, the greater part of energy is emitted in the direction $\pm Z$, i.e., from the 'narrow' faces of the active sample. However, with an increase in the wave coupling coefficient, the fluxes in those directions become equal and near the optimal value of α actually all the energy is emitted in the directions $\pm X$, i.e., from the wide faces.

In Fig. 4, evolution of the amplitude spatial distribution for the partial waves $\hat{C}^+_{x,z}$ is shown at the optimal value of the coupling parameter α. At the initial stage ($\tau < 3$), the mentioned distributions are random in character, in which case the scale of field inhomogeneity corresponds to the prescribed initial fluctuations of polarisation. After several wave passes across the Bragg cavity ($\tau = 4$) on the linear interaction stage, the spatial distribution of partial wave fields is formed, whose structure is close to the fundamental mode of the two-dimensional Bragg cavity [2, 12]. The spatial scale of field inhomogeneity in this case becomes comparable with the characteristic dimensions of the active medium, i.e., a spatial synchronisation of radiation occurs. In transferring to the nonlinear stage ($\tau > 8$), at which a maximum intensity is reached, the field distribution is somewhat distorted, but the characteristic inhomogeneity scale remains the same. It worth noting that the field

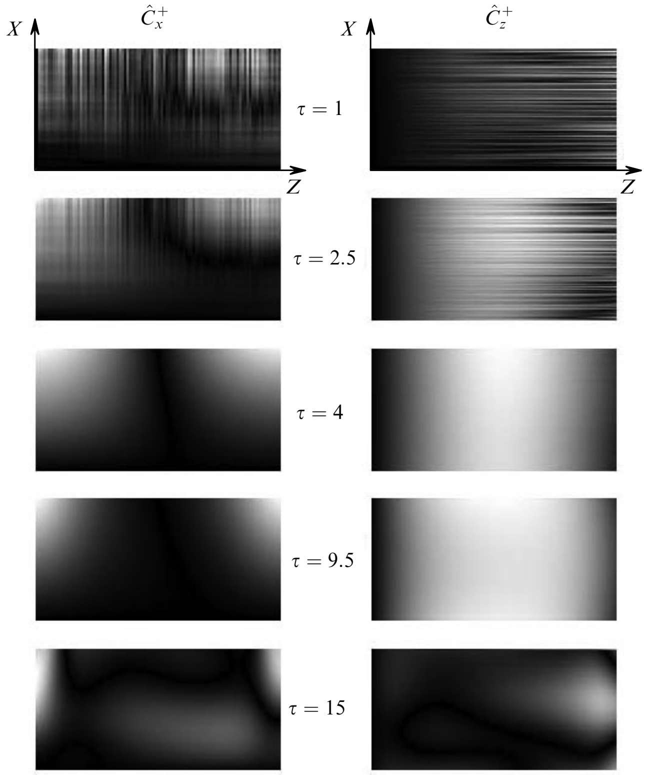

Figure 4. Evolution of spatial amplitude distribution for the partial waves $\hat{C}_x^+$ and $\hat{C}_z^+$ in the active volume at $L_x = 0.25$, $L_z = 0.5$, $\hat{T}_2 = 10$, $\alpha = 10$.

distribution in Fig. 4 is normalised to the maximum value at the current moment. Hence, a sufficiently intricate profile at the instant $\tau = 15$ is actually observed after the main pulse has been emitted and the absolute values of fields have become small.

One should also pay attention to a considerable influence of secondary nonlinear lattices induced due to the modulation of inversion. If the lattices are excluded [the terms $\hat{\rho}_{2x,2z,z\pm x}$ in (6) are neglected], the peak intensity of SR pulses increases several times as compared to that presented in Figs 2 and 3.

In Fig. 5, the phase fronts of wave fluxes $\hat{C}_{x,z}^+$ are shown at the instant corresponding to the maximal intensity of the SR pulse, which, obviously, exhibit deterministic character. A phase correlation of partial wave fluxes emitted in the $\pm x$ and $\pm z$ directions from the corresponding sample faces also occurs in this case.

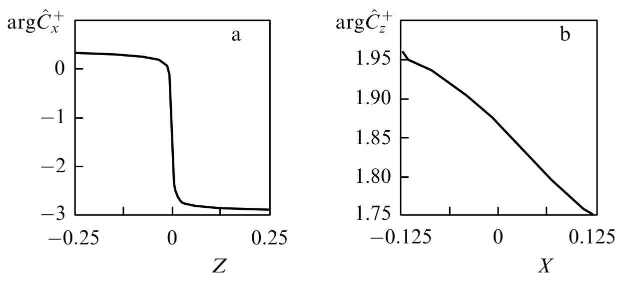

Figure 5. Phase front of SR pulse at $L_x = 0.25$, $L_z = 0.5$, $\hat{T}_2 = 10$ for the partial waves $\hat{C}_x^+$ (a) and $\hat{C}_z^+$ (b).

4. Conclusion

Thus, the possibility of efficient employment of two-dimensional Bragg structures for generating short-duration high-intensity super-radiance pulses in a super-radiant regime through a spatial synchronisation of radiation in the two-dimensional active media characterised by large Fresnel parameter values. Note that we investigated here the simplest regimel, in which the two-dimensional Bragg structure is a dielectric with a double-periodic modulation of the real part of permittivity. Similarly to the case of one-dimensional Bragg structures, such a modulation can be realised in a medium with reactive nonlinearity by using crossed laser beams propagating in the orthogonal directions along the vectors of a reciprocal lattice of the two-dimensional Bragg structure. In this case, the orthogonality of polarisation of the beams gives a chance to exclude the additional induced lattice caused by a mutual influence of the beams. In addition, two-dimensional Bragg structures can be realised by means of thin (in the wavelength scale) dielectric plates, in which one or two surfaces are corrugated according to formula (1); a sinusoidal crimping can be changed to staggered one [2, 3]. By using similar planar two-dimensional Bragg structures, it is possible to couple waves of the TM – TM or TM – TE polarisations and at the corresponding relationships between the relaxation times and gains obtain the super-radiance from various types of active media with different polarisations of amplified waves.

Acknowledgements. Authors are grateful to Vl.V. Kocharovskii and E.P. Kocharovskaya for fruitful discussions. The study was supported by the Russian Foundation for Basic Research (Grant No. 08-08-00966) and the Dynasty Foundation.

References

1. Baryshev V.R., Ginzburg N.S., Sergeev A.S. *Pis'ma Zh. Tekh. Fiz.*, **34**, 47 (2008).
2. Baryshev V.R., Ginzburg N.S., Zaslavskii V.Yu., Malkin A.M., Sergeev A.S., Tumm M. *Kvantovaya Elektron.*, **39**, 463 (2009) [*Quantum Electron.*, **39**, 463 (2009)].
3. Baryshev V.R., Ginzburg N.S., Malkin A.M., Sergeev A.S. *Kvantovaya Elektron.*, **39**, 1159 (2009) [*Quantum Electron.*, **39**, 1159 (2009)].
4. Ammerlahn D., Kuhl J., Grote B., Koch S.W., Khitrova G., Gibbs H. *Phys. Rev. B*, **62**, 7350 (2000).
5. Pilozzi L., D'Andrea A., Cho K. *Phys. Rev. B*, **69**, 205311 (2004).
6. Belyanin A.A., Kocharovskii V.V., Kocharovskii Vl.V., Pestov D.S. *Izv. Vyssh. Uchebn. Zaved. Ser. Radiofiz.*, **44**, 184 (2001).
7. Belyanin A.A., Kocharovsky V.V., Kocharovsky Vl.V., Pestov, D.S. *Laser Phys.*, **13**, 161 (2003).
8. Vasil'ev P.P. *Kvantovaya Elektron.*, **29**, 4 (1999) [*Quantum Electron.*, **29**, 842 (1999)].
9. Ginzburg N.S., Kocharovskaya E.P., Sergeev A.S. *Izv. Ros. Akad. Nauk, Ser. Fiz.*, **72**, 226 (2008).
10. Ginzburg N.S., Peskov N.Yu., Sergeev A.S. *Opt. Commun.*, **96**, 254 (1993).
11. Yariv A. *Quantum Electronics* (New York: Wiley, 1975).
12. Andreev A.V. *Usp. Fiz. Nauk.*, **160**, 1 (1990) [*Sov. Phys. Usp.*, **33**, 997 (1990)].