ACTIVE MEDIA

PACS numbers: 42.55.Vc; 32.10.Bi; 76.80.+y
DOI: 10.1070/QE2011v041n06ABEH014342

Mössbauer medium with a hidden nuclear population inversion and negative absorption of gamma quanta

L.A. Rivlin

Abstract. **We consider physical foundations of an eventual experiment aimed at observing stimulated gamma-photon emission of long-lived Mössbauer isomers through selective frequency modulation of gamma-resonances establishing hidden population inversion without exceeding the number of excited nuclei over unexcited ones and without additional pumping. The examples of suitable nuclei and numerical estimates of the parameters are presented.**

Keywords: *quantum nucleonics, kinetics of accumulation of isomeric nuclei in a Mössbauer crystal, hidden inversion under selective frequency modulation, vibration of crystal atoms in an alternating magnetic field, single-crystal gamma-ray cavity, threshold conditions for the process of stimulated emission.*

1. Introduction

Various schemes of a Mössbauer gamma laser were repeatedly discussed even in the initial studies half a century ago. All the variants of stimulated gamma emission were based on the use of zero-phonon nuclear gamma transitions with the natural radiation linewidth in combination with different pump modes to invert the states populations of isomeric nuclei. However, despite the great sophistication of some of the proposed approaches, none of them envisaged a possibility of producing a medium with the population inversion of nuclei, adequate for this purpose (see, for example, [1, 2]).

Meanwhile, the concentration of excited nuclei in a standard Mössbauer source can reach by no means low values, even without any pumping. Therefore, there arises a problem about the possibility to observe stimulated gamma emission when so-called hidden inversion (discussed earlier in relation to a gamma-ray laser on free nuclei [3, 4]) is established in a Mössbauer medium, i.e., when resonant absorption of gamma quanta by nuclei in the ground state is eliminated, their number may even exceed the number of excited nuclei. The analysis of the resulting constraints and prospects of this approach has motivated the present research.

L.A. Rivlin Applied Physics Laboratory, Moscow State Institute of Radioengineering, Electronics and Automation (Technical University), prosp. Vernadskogo 78, 119454 Moscow, Russia;
e-mail: lev.rivlin@gmail.com

Received 16 April 2010; revision received 11 April 2011
Kvantovaya Elektronika **41** (6) 495 – 500 (2011)
Translated by I.A. Ulitkin

2. Kinetics of accumulation of isomeric nuclei in a Mössbauer gamma-photon source

In a conventional Mössbauer experiment, mother isotopes of the initial concentration n_0, decaying along different channels with a characteristic time τ_0, give rise to the excited Mössbauer isomers, which, in turn, spontaneously emit gamma quanta for a characteristic time τ and produce nuclei in the ground state [hereafter $\tau = T_{1/2}(\ln 2)^{-1}$)].

The kinetics of this process, resulting from integration of rate equations with zero initial conditions, is as follows: the relative concentrations of the Mössbauer isomers in the excited and ground states change with time as

$$\frac{n_{\mathrm{e}}}{n_0} \equiv \alpha_{\mathrm{e}} = \frac{a}{1-a}\left[\exp(-\theta) - \exp\left(-\frac{\theta}{a}\right)\right]$$

$$\approx a\left[\exp(-\theta) - \exp\left(-\frac{\theta}{a}\right)\right], \tag{1}$$

$$\frac{n_{\mathrm{g}}}{n_0} \equiv \alpha_{\mathrm{g}} = 1 - (1-a)^{-1}\left[\exp(-\theta) - a\exp\left(-\frac{\theta}{a}\right)\right]$$

$$\approx 1 - \exp(-\theta) + a\exp\left(-\frac{\theta}{a}\right), \tag{2}$$

where $\theta \equiv t/\tau_0$ is the normalised time. Approximate equalities refer to the case $a \equiv \tau/\tau_0 \ll 1$, which is caused, on the one hand, by the maximum lifetime of the Mössbauer isomer, consistent with observing a zero-phonon line of natural width, and, on the other hand, – by technological constraints of the time of the experiment from below.

The concentration α_{e} of nuclei in the upper isomeric state, which reaches a maximum

$$\alpha_{\mathrm{e}}^{\max} = a^{1/(1-a)} \approx a \tag{3}$$

at

$$\theta = \theta(\alpha_{\mathrm{e}}^{\max}) = \frac{a}{1-a}\ln a^{-1} \approx a\ln a^{-1}, \tag{4}$$

can be maintained at a level $\alpha_{\mathrm{e}} \geqslant \alpha_{\mathrm{e}}^{\max}/2 \approx a/2$ in the time interval

$$a/2 \leqslant \theta \leqslant \ln 2 = 0.7. \tag{5}$$

The concentration of nuclei in the ground state reaches

$$\alpha_g(\theta(\alpha_e^{\max})) = 1 - (1+a)a^{a/(1-a)} \approx a \ln a^{-1} > \alpha_e^{\max} \quad (6)$$

by the time $\theta = \theta(\alpha_e^{\max})$.

3. Hidden population inversion of states under selective frequency modulation of emitters

Resonant absorption by nuclei in the ground state can be eliminated at least by two ways: establishment of hidden population inversion, especially through selective frequency modulation of the emitters [5], and build-up of inversionless gain through purposeful creation of quantum coherence of states [6, 7]. (The third known method for producing hidden inversion that occurs due to the recoil of free nuclei in radiative transitions [3, 4] is inapplicable because of the sense itself of the zero-phonon gamma transition of nuclei in the crystal.) Below we analyse the prospects of the first method [5].

If the frequency of the oscillator with the fundamental frequency ω_0 is modulated harmonically

$$\omega = \omega_0[1 + \xi \cos(\Omega t)] \quad (7)$$

with the frequency $\Omega \ll \omega_0$ and modulation depth ξ, then its spectrum exhibits series of doublet resonances with frequencies

$$\omega_m = \omega_0 \pm m\Omega \quad (m = 0, 1, 2 \ldots) \quad (8)$$

and amplitudes, proportional to the squares of the mth-order Bessel function of the first kind $J_m^2(\eta)$. The argument of the Bessel function (modulation index)

$$\eta = \xi\omega_0/\Omega \quad (9)$$

can be not small because of the large ratio $\omega_0/\Omega \gg 1$, even at very low modulation depth $\xi \ll 1$ and the lower bound $\Omega > 2\pi/\tau$, required for sufficient resolution of the terms of series (8).

It is important to bear in mind that the spectrum of resonances of form (8) arises in the case of a simple initial spectrum consisting of a single line with the frequency ω_0, i.e., on assumption that splitting of any nature is absent. In particular, the Zeeman splitting in the field of terrestrial magnetism should be eliminated by imposing a compensating field of such a device as Helmholtz coils. In the presence of splitting the problem becomes more complicated, without losing the fundamental solvability. The validity of the assumption about the absence of splitting should be considered in relation to a particular isomer.

The existence of zeros of Bessel functions stipulates an important fact: if the argument η is equal to the rth root of the mth-order Bessel function ($\eta = \eta_{mr}$), then at the modulation frequency $\Omega_r = \xi\omega_0/\eta_{mr}$, the doublet with the frequency

$$\omega_m = \omega_0(1 \pm m\xi/\eta_{mr}) \quad (10)$$

does fall out of the spectrum. Therefore, if the absorption resonance spectrum completely lacks the mth-order term of the series with the arguments $\eta_g = \eta_{mr}$ and frequency ω_m (10), and the emission spectrum, on the contrary, has the same term with the argument η_e, for which the condition $\eta_e = \eta_{mr}$ is not fulfilled, the hidden inversion will be established for radiation with the frequency ω_m without actually exceeding the number of excited oscillators over unexcited ones. The necessary difference in the modulation indices ($\eta_e \neq \eta_g$) is realised when the modulations depths ($\xi_e \neq \xi_g$) are not equal due to differences in the properties of excited and unexcited oscillators.

Here are some specific types of such elimination of absorption resonances of oscillators.

(i) The most attractive (but hardly feasible) case $\xi_e = 0$, when there is no modulation of the excited oscillators and the modulation index of the absorbing oscillators is, for example, equal to the first root $\eta_g = \eta_{01} \approx 2.4$ and $\xi_g = 2.4(\Omega/\omega_0)$. In this case, the absorption at the fundamental frequency ω_0 is completely suppressed and hidden inversion occurs at the fundamental frequency ω_0 with the relative amplitude of the resonance $J_0^2 = 1$ and in complete absence of higher resonances in the emission spectrum.

(ii) The case, opposite to the previous one, when at $\xi_g = 0$ and in the absence of modulation of the absorbing oscillators it is sufficient to modulate the frequency of the emitting oscillators with the modulation index $\eta_e > 0$ so that to establish hidden inversion for the resonances with $m > 0$.

(iii) A less attractive (but perhaps more realistic) case is simultaneous modulation of both absorbing and emitting oscillators with different modulation indices. For example, complete suppression of absorption at the fundamental frequency ω_0 in the case of deep modulation of the absorbing oscillators with $\eta_g = \eta_{01} \approx 2.4$ and simultaneous weaker modulation with $\eta_e < \eta_{01} \approx 2.4$ of the emitting oscillators make the hidden inversion at the fundamental frequency ω_0 possible, or when $\eta_e \approx 1.84$ and $\eta_g \ll 1.84$, the hidden inversion of resonances with $m = \pm 1$ is established.

Since the emitting and absorbing oscillators are subjected to extraneous modulation effects of the same nature, as was already mentioned above, the desired difference in the values of the arguments η can only be achieved if the modulation depths ξ in the excited and ground states differ because of different properties of the oscillators in these states, i.e., if the same modulation parameter, subjected to the same extraneous modulation effects, in the oscillators in the excited and ground states is different. An example of such a nuclear modulation parameter is the magnetic moment μ, which is usually different in different states of the nucleus ($\mu_g \neq \mu_e$). In this case to establish hidden inversion it is necessary choose the modulation index η_g of nuclei in the intervals where the derivative modulus $|\mathrm{d}J_m/\mathrm{d}\eta_g| > 0$ for $\xi_e/\xi_g = \mu_e/\mu_g > 1$ and $|\mathrm{d}J_m/\mathrm{d}\eta_g| < 0$ for $\xi_e/\xi_g = \mu_e/\mu_g < 1$.

4. Selective Doppler frequency modulation

Frequency modulation can be constructed on a different physical basis. One example is the Doppler frequency modulation

$$\omega = \omega_0[1 + (v/c)\cos\phi \cos(\Omega t)], \quad (11)$$

arising by the vibrational motion of the oscillators with the frequency Ω and the amplitude of the velocity modulus v, and producing a modulation with a depth

$$\xi = (v/c)\cos\phi \quad (12)$$

(where ϕ is the angle between the velocity vector υ and the wave vector, c is the speed of light).

We should emphasise that the Doppler modulation of the oscillator frequency is a purely kinematic phenomenon, not affecting the internal degrees of freedom of the oscillator (in this case – the nucleus). In fact, Mössbauer methodically managed to make his discovery for this reason: he used the Doppler modulation of the resonances of nuclei in a crystal, moving as a whole, for measuring the width of the gamma lines with reliable resolution of the resonances of both emission and absorption. The Doppler modulation of the resonances of nuclei due to vibrations of atoms inside the crystal was observed more than half a century ago [8] by exciting the acoustic waves in the crystal.

If an oscillator is an atom in the crystal lattice, its vibration amplitude should be much smaller than the interatomic distance d in the lattice, which sets the limit on the ratio of the velocity to the modulation frequency: $\upsilon/\Omega \ll d \approx 0.1$ nm. Therefore, in the case of the Doppler modulation the allowable value of the modulation index is also limited:

$$\eta = \xi(\omega_0/\Omega) = (\upsilon/c)(\omega_0/\Omega)\cos\phi \ll 2\pi(d/\lambda)\cos\phi \quad (13)$$

($\lambda = 2\pi c/\omega_0$), i.e., although η cannot exceed a few units for the gamma quanta with the energy of the order of tens of keV, this value is sufficient for selective frequency modulation discussed in Section 3.

The desired value of the modulation index η is defined by two parameters – the amplitude and direction of the velocity vector υ with the wave vector of gamma waves lying on the surface of the cone around the vector υ and the angle ϕ at the top. Varying the values of υ and ϕ, this allows for the required $\eta = \text{const}$ to optimise (taking into account properties of the crystal lattice) the quantity $\phi = \phi_c$.

Here an important reservation must be done. The adopted concept of the Doppler modulation is based on the assumption of preservation of selective excitation of vibrations of atoms with different modulation parameters in a real crystal. However, each of these atoms, which vibrates under the influence of an external force, also serves in a crystal as a source of phonons with a frequency Ω, which increases their content in the phonon spectrum in comparison with the existing thermal background. Therefore, there is a danger that the presence of excess phonons can lead to levelling of the selective effect of the external modulation force on the atoms with different modulation parameters. Assessment of the extent of the threat, depending on the relaxation rate of excess phonons and other factors, requires a separate analysis, which is beyond the present discussion; the final conclusion can be seemingly drawn only using the results of the experiment. With this caveat in mind, further consideration here is based on the assumption that hypothesis about the preservation of the selective character of atomic vibrations is valid.

5. Doppler modulation in an alternating magnetic field

The effect of selective modulation on the nuclei can be implemented, in particular, by applying an alternating magnetic field to the nuclei with unequal magnetic moments in the ground and excited states ($\mu_g \neq \mu_e$). In an alternating magnetic field with the frequency Ω_B, the modulus of the magnetic induction vector $B(z,t) = B_0(z)\cos\Omega_B t$ that is parallel to the axis z, and with the gradient $\mathrm{d}B_0/\mathrm{d}z \neq 0$, the force $F = \mathrm{grad}(\mu B) = \mu(\mathrm{d}B_0/\mathrm{d}z)\cos\Omega_B t$ is applied on a nucleus ($\mu = \mu_0\mu_{g,e}$ and $\mu_0 = 5\times10^{-24}$ erg G^{-1} is the nuclear magneton).

When placing the nuclei at a point of the magnetic field, where $B = 0$ and the gradient $\mathrm{d}B_0/\mathrm{d}z$ is maximal, the zero value of B helps to prevent unwanted additional Zeeman splitting of the spectrum of gamma-resonances (this point is, for example, the centre of the structure of the type of Helmholtz coils with alternating currents of opposite signs in coils).

If in accordance with the above assumptions we leave aside the strict phonon description of dynamics of atoms in the crystal and consider the motion of a single externally modulated atom as forced oscillations of an isolated classical linear vibrator, the maximum steady-state oscillation amplitudes and velocities

$$A \approx \frac{\mu B_{max}\tau_a}{2Mc}\frac{\lambda_B}{\Lambda} \quad \text{and} \quad \upsilon = A\Omega_B \quad (14)$$

are established in the resonance with a small relative detuning of three frequencies, i.e., when the frequency of the alternating magnetic field, $\Omega_B = 2\pi c/\lambda_B$, the eigenfrequency Ω_a of the vibrator with a sufficiently high Q factor $[(\Omega_a\tau_a)^2 \gg 1]$, and the required modulation frequency Ω (7) practically coincide ($\Omega_B = \Omega_a = \Omega$) ($M$ is the atomic mass; τ_a is the vibrator relaxation time; Λ is the spatial period of the magnetic field, which at low frequencies can be much smaller than λ_B, i.e., $\Lambda \ll \lambda_B$).

In view of the practical equality of the masses, vibration eigenfrequencies and Q factors of the atomic vibrators with the nuclei in the ground and excited states, expressions (14) are valid for the atoms of both species. Therefore, the ratios of their vibration amplitudes and velocities are practically equal to the ratios of the corresponding magnetic moments of the nuclei:

$$\frac{\upsilon_g}{\upsilon_e} = \frac{A_g}{A_e} = \frac{\mu_g}{\mu_e}. \quad (15)$$

The amplitude of the alternative magnetic induction B_{max} (in Gauss) required to reach the desired value of the modulation index $\eta = \eta_{g,e}$, is calculated for any of the states of the nucleus:

$$B_{max} \approx \frac{2Mc^2(\Lambda/\lambda_B)}{\mu_0\mu_{g,e}\tau_a\omega_0\cos\phi}\,\eta_{g,e}$$

$$\approx 5\times10^5\,\frac{M_0(\Lambda/\lambda_B)}{(\hbar\omega_0)\mu_{g,e}\tau_a\cos\phi}\,\eta_{g,e}, \quad (16)$$

where M_0 is the mass number of the atom; $\mu_{g,e}$ is the magnetic moment of the nuclear normalised to the nuclear magneton (in the numerical formula $\hbar\omega_0$ is taken in keV, τ_a – in seconds). The value of B_{max} can be substantially reduced at $\Lambda/\lambda_B \ll 1$.

6. Acting value of stimulated emission cross section in the case of frequency modulation

Under conditions of hidden inversion, the cross sections of stimulated emission

$$\sigma_{\rm st} = \frac{\lambda^2}{2\pi}\,\beta f_{\rm DW} J_m^2(\eta_{\rm e}) \quad (17)$$

and resonant absorption

$$\sigma_{\rm res} = \frac{\lambda^2}{2\pi}\,\beta f_{\rm DW} J_m^2(\eta_{\rm g}) \quad (18)$$

are proportional to the square of the corresponding Bessel functions $J_m^2(\eta_{\rm e})$ and $J_m^2(\eta_{\rm g})$. Here, $f_{\rm DW} < 1$ is the Debye – Waller factor; β is the ratio of natural radiation linewidth to its value, taking into account both the homogeneous and inhomogeneous broadenings [for the Mössbauer transition $\beta \to (1+\alpha)^{-1}$, where α is the coefficient of internal electronic conversion] [2].

7. Threshold conditions

The threshold condition $w_{\rm st} - w_{\rm res} \geqslant w_{\rm q}$ of the stimulated gamma emission consists in exceeding the probability of stimulated emission of photons $w_{\rm st} = c\sigma_{\rm st} n_{\rm e}$ over the sum of the probabilities of their resonant absorption $w_{\rm res} = c\sigma_{\rm res} n_{\rm g}(2I_{\rm e}+1)(2I_{\rm g}+1)^{-1}$ and nnonresonant losses $w_{\rm q}$:

$$\alpha_{\rm e} J_m^2(\eta_{\rm e}) - \alpha_{\rm g} J_m^2(\eta_{\rm g}) \frac{2I_{\rm e}+1}{2I_{\rm g}+1} \geqslant 2\pi w_{\rm q}[c\lambda^2\beta f_{\rm DW}]^{-1}, \quad (19)$$

where $I_{\rm e}$ and $I_{\rm g}$ are the spins of the nuclei in the excited and ground states. The value of $w_{\rm q}$ consists of photon losses inside the crystal and on its surface.

Upon successful combination of the parameters, the Mössbauer crystal itself can serve as a gamma-ray cavity. The resonance condition in a single-crystal cavity [9] is the intersection of the Ewald sphere in the reciprocal lattice space by at least two nodes, not accounting for the node at the origin of the coordinates, except for the limiting case of intersection with a single node if the latter and the origin of the coordinates lie at the ends of the sphere diameter. The simple meaning of this condition is that the multiple Bragg reflections of photons, belonging to the resonance mode, prevent their escape from the entire volume V of the crystal through its surface with the area S. The possibility to escape is inherent only in photons from a thin layer (adjacent to the surface) with an effective thickness κd in which almost complete Bragg reflection occurs on κ atomic planes. In this case, the photons escape per unit volume of the crystal at a rate $s_S = c(N/V)\kappa d(S/V)^2$ that is proportional to the number of photons per unit volume N/V.

The rate of loss of photons per unit volume of the crystal is $s_V = c(N/V)\sigma_0 bn$, where $n \approx 3\times10^{22}$ cm^{-3} is the total concentration of atoms of all types; σ_0 is the average acting value of the cross section of photon losses; b is the attenuation loss coefficient due to Borrmann effect. The total loss probability, proportional to the sum of the rates $s_S + s_V$, is

$$w_{\rm q} = (s_S + s_V)(N/V)^{-1} = c\sigma_0 bn\big[1 + (\kappa d/\sigma_0 bn)(S/V)^2\big], \quad (20)$$

and the threshold condition (19) is

$$\alpha_{\rm e}^{\rm thr} = \alpha_{\rm g}\left[\frac{J_m(\eta_{\rm g})}{J_m(\eta_{\rm e})}\right]^2 \frac{2I_{\rm e}+1}{2I_{\rm g}+1} + \frac{2\pi\sigma_0 b(n/n_0)}{\lambda^2\beta f_{\rm DW} J_m^2(\eta_{\rm e})} \times \left[1 + \frac{\kappa d}{\sigma_0 bn}\left(\frac{S}{V}\right)^2\right] \approx \alpha_{\rm g}\left[\frac{J_m(\eta_{\rm g})}{J_m(\eta_{\rm e})}\right]^2 \frac{2I_{\rm e}+1}{2I_{\rm g}+1} + \frac{2\pi\sigma_0 b(n/n_0)}{\lambda^2\beta f_{\rm DW} J_m^2(\eta_{\rm e})}, \quad (21)$$

where the approximate equality applies to a macroscopic crystal ($V/S \geqslant 10^{-3}$ cm) with predominant volume photon losses.

Due to Borrmann effect [10], when the nodes of the electric component of the standing wave of the cavity mode coincide with the lattice knots, we can expect a noticeable decrease in the nonresonant losses without a fall of the interaction efficiency of the gamma-wave with a nuclear transition, whose multipolarity is higher than the electric dipole one [11 – 13]. According to the estimates of [13], the Borrmann effect can reduce the loss by approximately two orders of magnitude, which is taken into account above in the coefficient b. In this case, the modulation angle $\phi = \phi_c$ should be equal to the Bragg sliding angle of the highest-Q-factor 'Borrmann' mode of the cavity.

The orders of magnitude and their desirable values can be inferred from the estimates resulting from the threshold condition (21). If we assume the loss cross section σ_0 of the photon with energies of the order of 100 keV to be $\sim 10^{-20}$ cm^2 [14], even for optimistic values of other factors ($\beta \sim 1$, $f_{\rm DW} \sim 1$, $n_0/n \sim 1$), the right hand side of (21) is ~ 0.1, and taking into account the possible Borrmann effect it is $\sim 10^{-3}$. If $J^2(\eta_{\rm g}) \ll J^2(\eta_{\rm e}) \approx 1$, then $\alpha_{\rm e}^{\rm thr} \approx a = \tau/\tau_0 > 10^{-3}$. Under less optimistic view, the latter inequality should be increased: $a = \tau/\tau_0 \gg 10^{-4} - 10^{-3}$. These estimated values, despite all their uncertainties, indicate the direction of a search for a nucleus for a possible experiment.

8. Search for a suitable isotope

If we assume that the experiment measurements require at least about 10 minutes, then the inequality $a = \tau/\tau_0 > 10^{-3}$ leads to the need to use a nucleus with a lifetime of $\tau \sim 10$ s for the Mössbauer experiment.

At first glance at the table of the Mössbauer nuclei [15], it becomes apparent that out of 135 nuclides included in it in 1991, the only one of them – an isotope of silver $^{107m}_{47}$Ag ($\tau = 63.3$ s) – could claim to the role of the working nuclei; the second possible candidate, which was added after 1991, is $^{109m}_{47}$Ag ($\tau = 57$ s). Just this observation of gamma emission with the zero-phonon Mössbauer line of natural width by such long-lived isomers, performed in a series of unique experiments at the Institute for Theoretical and Experimental Physics ([16, 17], etc.), motivated the present analysis. Unfortunately, both isotopes are not optimal for a solution of the problem under study, in particular due to a high rate of the internal electronic conversion (~ 20).

Although, as far as is known, successful Mössbauer experiments were carried out only for the above two isotopes of silver, they are not the only possible candidates. The short list of the candidates can include, for example, $^{79m}_{36}$Kr, $^{107m}_{46}$Pd, $^{125m}_{54}$Xe, and $^{135m}_{58}$Ce. Despite the paucity and incompleteness of the available experimental data, it is useful for purely illustrative purposes only to make calculations for the isomer $^{79m}_{36}$Kr, which looks most attractive. The calculation results are presented below; some unavailable reliable data are substituted by interpolating values from [14, 15, 18] or postulated plausible values marked by (?).

Mössbauer isomer	$^{79m}_{36}$Kr
Mother isotope	$^{79}_{37}$Rb
Type and time of decay of τ_0/min (electron capture)	32.8
Transition energy $\hbar\omega_0$/keV	129.8
Transition time τ/s	71.5
Spins of the states (I_g, I_e), multipolarity of transition	$1/2^-$; $7/2^+$; E3
Magnetic moments of ground (μ_g) and excited (μ_e) states	0.1, 4.0 (?)
Internal electronic conversion coefficient α	2 (?)
Ratio of the lifetimes $a = \tau/\tau_0$	3.65×10^{-3}
Modulation index of excited nuclei η_e	1.84
Modulation index of nuclei in the ground state $\eta_g = \eta_e(\mu_g/\mu_e)$	4.6×10^{-2}
Amplitude of emission resonance $J_1(\eta_e)$	0.582
Amplitude of absorption resonance $J_1(\eta_g)$	0.023
Ratio of the squares of resonance amplitudes $[J_1(\eta_e/J_1(\eta_g)]^2$	640
Maximum concentration of excited nuclei $\alpha_e^{max} \approx a$	3.65×10^{-3}
Time to reach maximum concentration $\theta = \theta(\alpha_e^{max})$	0.0205
Concentration of nuclei in the ground state $\alpha_g(\theta(\alpha_e^{max}))$	0.0205
Averaged cross section of nonresonant losses σ_0/cm^2	10^{-20} (?)
Coefficient of 'Borrmann' attenuation of losses b	0.25×10^{-2} (?)
Width ratio of the emission line $\beta = (1+\alpha)^{-1}$	0.33
Relative content of the mother isotope n_0/n	1
Debye–Waller factor	0.5 (?)
$\alpha_g[J_m(\eta_g)/J_m(\eta_e)]^2(2I_e+1)(2I_g+1)^{-1}$	0.127×10^{-3}
$2\pi\sigma_0 b(n/n_0)[\lambda^2\beta f_{DW} J_m^2]^{-1}$	3.05×10^{-3}
Threshold concentration of excited nuclei α_e^{thr}	3.18×10^{-3}
Modulation frequency $\Omega/2\pi$/s^{-1}	0.1
Ratio of the spatial period to the wavelength of the magnetic field Λ/λ_B	10^{-10}
Amplitude of the alternating magnetic induction B_{max}/G	$1.4 \times 10^{-5}/(\tau_a \cos\phi)$
Ratio $\alpha_e^{max}/\alpha_e^{thr}$	1.13

Thus, under adopted assumptions the threshold conditions are fulfilled and a nuclear negative absorption of gamma quanta is established in the Mössbauer medium with hidden population inversion.

9. Conclusions

The considered concept of a Mössbauer source with hidden population inversion looks suitable for observation of stimulated gamma emission by nuclei and can be treated as an important step towards creation of a nuclear gamma laser. The concept has an important strategic advantage because it does not need a heavy arsenal of such experimental tools as relativistic electron-beam X-ray generators, reactor neutron sources, etc., which makes it attractive in an attempt to implement a demonstration experiment. However, realisation of this concept requires a complex Mössbauer experiment with very long-lived isomers (which, as far as we know, was carried out only in unique experiments [16, 17, etc.]) and selective frequency modulation of nuclear resonances.

Despite the encouraging results of illustrative calculations, their discussion using the above data indicates extreme critical conditions for the fulfilment of the threshold inequality $\alpha_e^{max}/\alpha_e^{thr} > 1$ not only to parameters of the nuclei itself (which, after the selection of the isomer are actually already fixed), but also to other characteristics of the experiment. In fact, even a slight deterioration in values of the parameters relative to their favourable values adopted rather arbitrarily ($b = 0.25 \times 10^{-2}$, $f_{DW} = 0.5$, $n_0/n = 1$), immediately violates the threshold condition. Therefore, the obtained result ($\alpha_e^{max}/\alpha_e^{thr} > 1$) should be considered as an indication of the feasibility of the concept under condition of exclusively fine tuning of 'non-nuclear' factors of the experiment.

At the same time, from the illustrative calculation we can draw the following important conclusions about a very minor contribution of resonant absorption to the threshold condition (upon absorption suppression by the modulation method) and about the loss of gamma photons through the surface of a macroscopic crystal cavity. Thus, the key 'non-nuclear' experimental parameters are the nonresonant photon absorption and its 'Borrmann' attenuation as well as the Debye–Waller factor.

With all the strategic advantages of the concept, the specific tactics of a possible experiment requires not only a sophisticated laboratory technology, but also a deep and detailed preliminary study of a number of issues, including:

(i) a targeted search for new long-lived Mössbauer isomers and their introduction into the experimental practice;

(ii) theoretical and experimental verification of the assumptions about preserving the selectivity of nuclear modulation in the crystal;

(iii) study of the possibility of suppressing unwanted splitting of the gamma-ray emission line (it is worth noting that in the long-lived isomers $^{109m}_{47}$Ag, the expected dipole–dipole splitting of the line was not observed [17]);

(iv) theoretical and experimental study of the ways to improve 'Borrmann' attenuation of nonresonant absorption of gamma quanta in the crystal;

(v) development of optimal crystalline matrices with a high content of the mother isotope n_0/n, a large Debye–Waller factor f_{DW}, and the minimum value of the 'Borrmann' coefficient b, which are suitable for the formation of a single crystal gamma cavity or waveguide;

(vi) study of the possibility of solving the listed problems by constructing composite structures by the methods of nanophysics as an alternative to natural crystalline matrices.

Acknowledgements. The author is grateful to A.V. Davydov for the friendly attention and the information provided. This work was supported by the program 'Development of Scientific Potential of Higher School' (Project No. 2.1.1/12404).

References

1. Baldwin G.C., Solem J.C. *Rev. Mod. Phys.*, **69**, 1085 (1997).
2. Rivlin L.A. *Kvantovaya Elektron.*, **37**, 723 (2007) [*Quantum Electron.*, **37**, 723 (2007)].
3. Marcuse D. *Proc. IEEE*, **51**, 849 (1963).
4. Rivlin L.A. *Kvantovaya Elektron.*, **19**, 513 (1992) [*Quantum Electron.*, **22**, 471 (1992)].
5. Rivlin L.A. *Kvantovaya Elektron.* **7**, 634 (1980) [*Sov. J. Quantum Electron.*, **10**, 361 (1980)].

6. Kocharovskaya O.A., Khanin Ya.I. *Pis'ma Zh. Eksp. Teor. Fiz.*, **48**, 581 (1988).
7. Kocharovskaya O.A.,Kolesov R., Rostovtsev Yu. *Phys. Rev. Lett.*, **82**, 3593 (1999).
8. Ruby S.L., Bolev D.I. *Phys. Rev. Lett.*, **5**, 5 (1960).
9. Rivlin L.A., in *Voprosy radioelektroniki, Ser. I 'Elektronika'* (Problems of Radio Electronics. Ser. I. 'Electronics') (Moscow: GKSM SSSR po elektronnoi tekhnike, 1962) No. 6, p. 60.
10. Borrmann J. *Phys. Zeit.*, **42**, 157 (1941).
11. Kagan Yu.M. *Pis'ma Zh. Eksp. Teor. Fiz.*, **20**, 27 (1974).
12. Andreev A.V., Il'insky Yu.A. *Zh. Eksp. Teor. Fiz.*, **68**, 811 (1975).
13. Il'insky Yu.A., Khokhlov R.V. *Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz.*, **19**, 792 (1976).
14. Siegbahn K. (Ed) *Beta- and Gamma-ray Spectroscopy* Amsterdam: North-Holland Publ. Co., 1955; Moscow: GIFML, 1959).
15. Grigoriev I.S., Meilikhov E.Z. (Eds) *Handbook of Physical Quantities* (Boca Raton, NY, London: CRC Press, 1996; Moscow: Energoatomizdat, 1991).
16. Bizina G.E., Beda A.G., Burgov N.A., Davydov A.V. *Zh. Eksp. Teor. Fiz.*, **45**, 1408 (1963).
17. Bayukov Yu.D., Davydov A.V., Isaev Yu.N., et al. *Pis'ma Zh. Eksp. Teor. Fiz.*, **90**, 547 (2009).
18. *Table of Isotopes (CD-ROM)*. Ed. by R.B. Firestone (LBNL, 1998).