LASER GYROSCOPES

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Calculating the parameters of a synchronisation zone of the frequencies of counterpropagating waves of a laser gyro

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Abstract. **Based on the analysis of a well-known system of equations describing the dynamics of a two-isotope laser gyro with an equal-*Q* resonator under conditions of its fine-tuning to the centre of the emission line and balanced currents in the discharge arms, we have derived the formulas for calculating the parameters of the synchronisation zone for the frequencies of counterpropagating electromagnetic waves generated in the device. The formulas make it possible to calculate the coordinates on the axis of the angular velocity of the left and right boundaries of the synchronisation zone, the coordinate of its centre and half-width. It follows from the analysis that, in the general case of the asymmetric linear coupling between the counterpropagating waves via backscattering, absorption, and transmission of radiation from the mirrors of the gyro, the left and right boundaries of the synchronisation zone are located at different distances with respect to the origin of coordinates, so that the centre of the region is displaced along the axis of the angular velocity. The analysis of the formulas also implies that the shift of the centre of the synchronisation zone and its half-width decrease with increasing medium gain.**

Keywords: *laser gyroscope, ring gas laser, frequency locking of counterpropagating waves.*

1. Introduction

1.1. Basic relations

Among main types of laser gyros widely used in practice, we can single out the device based on a ring gas He–Ne laser (the ratio of the isotope concentrations, ^{20}Ne : ^{22}Ne = 1 : 1) with a flat N-mirror (N = 3, 4) resonator ensuring generation of linearly polarised radiation in the sagittal plane. The laser, usually operating at 0.6328 μm, is pumped by a dc parallel discharge obtained by a common cathode and two anodes [1–3].

According to relations (5.55)–(5.57) from [3] and to expressions (6.45)–(6.47) from [4], when the currents are balanced in the discharge arms, the resonator is fine tuned to the centre of the emission line and the losses are identical, the system of equations describing the dynamics of the dimensionless intensities I_j (j = 1, 2) and the phase difference ψ of counterpropagating waves of such a laser gyro can be written as

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$$\dot{I}_1 = (\alpha - \beta I_1 - \theta I_2)I_1 - 2r_2(I_1I_2)^{1/2}\cos(\psi + \varepsilon_2),$$

$$\dot{I}_2 = (\alpha - \beta I_2 - \theta I_1)I_2 - 2r_1(I_1I_2)^{1/2}\cos(\psi - \varepsilon_1), \quad (1)$$

$$\dot{\psi} = M\Omega + r_2(I_2/I_1)^{1/2}\sin(\psi + \varepsilon_2) + r_1(I_1/I_2)^{1/2}\sin(\psi - \varepsilon_1).$$

In deriving these equations it was taken into account that the wave with j = 1 propagates in the direction of the laser gyro rotation. In system (1) α, β, θ are the Lamb coefficients that characterise the properties of the active medium; $M = (1 + K_{\rm a})M_{\rm g}$ is the scale factor of the laser gyro, primarily determined by the geometrical component $M_{\rm g} = 8\pi S/(\lambda L)$ and also taking into account the properties of the medium through a small parameter $K_{\rm a}$ (L is the perimeter of the axial contour; S is the covered area); Ω is the angular velocity of the device rotation in the inertial space; r_j and ε_j are the moduli and arguments of complex integral coefficients $r_j\exp\{{\rm i}\varepsilon_j\}$ of the linear coupling of counterpropagating waves, characterising their interaction through backscattering, absorption and transmission of radiation on the mirrors. (The relations for calculating the parameters α, β, θ of system (1) can be found, for example, in [5], the parameter $K_{\rm a}$ – in [6]. An empirical formula for calculating $K_{\rm a}$ is derived in [3]. In addition, a set of expressions to estimate the parameters $\alpha, \beta, \theta, K_{\rm a}, r_j, \varepsilon_j$ is given in [7].)

Before analysing the problem, we introduce into consideration the quantities $r_{\rm p}$ and $r_{\rm m}$, which are convenient combinations of the parameters r_j, ε_j of the linear coupling of the counterpropagating waves:

$$r_{\rm p} = (r_1^2 + r_2^2 + 2r_1r_2\cos\varepsilon_{12})^{1/2},$$
$$r_{\rm m} = (r_1^2 + r_2^2 - 2r_1r_2\cos\varepsilon_{12})^{1/2}, \quad \varepsilon_{12} = \varepsilon_1 + \varepsilon_2. \quad (2)$$

Let us also introduce the parameter $\alpha_{\rm m}$ characterising the inverse relaxation time of the difference between the counterpropagating wave intensities, whose value is calculated by the formulas

$$\alpha_{\rm m} = \alpha_{\rm p}\frac{1-h}{1+h}, \quad \alpha_{\rm p} = \alpha = \frac{c}{L}(g - \Gamma), \quad h = \frac{\theta}{\beta}, \quad (3)$$

where $\alpha_{\rm p}$ is the inverse relaxation time of the sum of the counterpropagating wave intensities; g is the unsaturated linear gain of the active medium; Γ is the resonator losses per trip; h is the parameter depending on the total pressure of the He–Ne mixture [8].

1.2. Analysis of the problem

It is known that one of the most significant factors affecting the laser gyro accuracy is the frequency locking of the counterpropagating electromagnetic waves generated in its resonator. Frequency locking manifests itself in the fact that the output characteristic of the device in the region of low values of the angular velocity has a band of insensitivity to the rotation, which is called the synchronisation zone in the literature.

The synchronisation zone of the counterpropagating waves of the laser gyro can be characterised by four parameters, which are as follows: the coordinates $\Omega_{(-)}$ and $\Omega_{(+)}$ on the axis of the angular velocity Ω of its left and right boundaries, respectively; the coordinate $\Omega_{(0)}$ of the region centre; and, finally, the half-width Ω_s. These quantities are related by [9]

$$\Omega_{(+)} = \Omega_{(0)} + \Omega_s, \quad \Omega_{(-)} = \Omega_{(0)} - \Omega_s. \tag{4}$$

The system of equations (1) analysed in the approximation of weak coupling of counterpropagating waves (when the conditions r_p/α_p, $r_m/\alpha_m \ll 1$ are fulfilled) implies (see, for example, [3, 9–16]) that the synchronisation zone of these waves is located symmetrically with respect to the coordinate origin, i.e.,

$$\Omega_{(+)} = \Omega_s, \quad \Omega_{(-)} = -\Omega_s, \quad \Omega_{(0)} = 0, \tag{5a}$$

its half-width, calculated with

$$\Omega_s = \frac{r_p}{M} = \frac{(r_1^2 + r_2^2 + 2r_1 r_2 \cos\varepsilon_{12})^{1/2}}{M}, \tag{5b}$$

being virtually independent on the active medium gain level (due to the smallness of the K_a parameter in the denominator of this formula, it is acceptable to set $M \approx M_g$).

However, the experiments show that in a real laser gyro the left and right boundaries of the synchronisation zone are located at different distances from the coordinate origin [17], and when the active medium gain increases, the half-width of the region decreases (approximately according to the hyperbolic law), approaching asymptotically the established finite value [18–20].

These circumstances have not found a definitive explanation in the literature. From papers [3, 9–16] we can single out four works [3, 11–13], in which, along with original formula (5b) used to calculate Ω_s, two additional formulas were proposed to estimate this parameter.

The first additional formula for calculating Ω_s, given in [11] (see equation (8) in [11]), has the form:

$$\Omega_s = \frac{2r_1 r_2 |\sin\varepsilon_{12}|}{4(2^{1/2}) r_m M}\left[3 + \left(1 + \frac{2}{B^2}\right)^{1/2}\right]$$

$$\times\left\{1 + B^2\left[\left(1 + \frac{2}{B^2}\right)^{1/2} - 1\right]\right\}^{1/2}, \tag{6}$$

where $B = -2\alpha_m r_1 r_2 \sin\varepsilon_{12}/r_m^3$. Formula (6) implies that with increasing active medium gain g, the quantity Ω_s approaches asymptotically the established finite value $\Omega_s^{\text{asymp}} = 2r_1 r_2 |\sin\varepsilon_{12}|/(2^{1/2} r_m M)$.

Formula (6) qualitatively explains the known experimental dependence $\Omega_s = \Omega_s(g)$. However, this formula is not quite complete, because the asymmetry ($r_1 \neq r_2$) of the linear coupling between the counterpropagating waves was not taken into account while deriving it. In addition, and this is important, formula (6) is not related with expression (5b).

The second additional formula for calculating Ω_s, given in [3, 12, 13] {see expression (5.51) in [3], (6.17) in [12] and (7.3.16) in [13]}, has the form:

$$\Omega_s = \frac{2r^2}{\alpha_m M}. \tag{7}$$

This formula was derived for the special case when the parameters of the linear coupling between the counterpropagating waves in system (1) take the values $r_1 = r_2 = r$, $\varepsilon_{12} = \pi$. Formula (7) also qualitatively explains, although somewhat differently, the known experimental dependence $\Omega_s = \Omega_s(g)$. In particular, it follows from this formula that with increasing active medium gain g the parameter Ω_s asymptotically tends to zero.

Thus, the laser gyro developers, who should estimate the expected value of Ω_s at the design phase of the gyro, face the problem of selecting one of the working computational models from above relations (5)–(7).

This raises the question: is it possible to develop a more complete [more than (5)] mathematical model of the parameters of the synchronisation zone of the counterpropagating waves of the laser gyro under study, which, on the one hand, could serve as a basis for interpreting known experimental data on a qualitative level and, on the other hand, would include (5) as a constituent part?

The development of such a model is the subject of this article.

2. Formulation of the problem and its solution

Based on the analysis of the system of equations (1), it is required to obtain in the weak coupling approximation of counterpropagating waves such estimates for $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$ and Ω_s, which would allow one to model mathematically experimental dependences specified in [17–20]. These formulas should yield, as a special case, the relations of initial model (5).

Consider the third equation of system (1). Using the identity transformations we can reduce it to the form

$$\dot{\psi} = M\Omega + \omega_p \sin(\psi + \phi_p), \tag{8}$$

where

$$\omega_p = [(I_1/I_2) r_1^2 + (I_2/I_1) r_2^2 + 2r_1 r_2 \cos\varepsilon_{12}]^{1/2}, \tag{9}$$

$$\phi_p = \arctan\frac{(I_2/I_1)^{1/2} r_2 \sin\varepsilon_2 - (I_1/I_2)^{1/2} r_1 \sin\varepsilon_1}{(I_2/I_1)^{1/2} r_2 \cos\varepsilon_2 + (I_1/I_2)^{1/2} r_1 \cos\varepsilon_1}. \tag{10}$$

Differential relation (8) in the laser gyro theory is well known and called the phase equation. The parameter ω_p in it describes the half-width of the synchronisation zone of counterpropagating waves of the laser gyro in circular frequencies, and the parameter ϕ_p is the phase angle. The problem is to calculate the quantity ω_p for the left [$\omega_{p(-)}$] and right [$\omega_{p(+)}$] boundaries of the synchronisation zone, and then using the formulas

$$\Omega_{(-)} = -\frac{\omega_{p(-)}}{M}, \quad \Omega_{(+)} = \frac{\omega_{p(+)}}{M} \tag{11}$$

to find the coordinates $\Omega_{(-)}$ and $\Omega_{(+)}$ of these boundaries at the axis of the angular velocity Ω. It is important to emphasise that $\omega_{p(-)}$ and $\omega_{p(+)}$ in (11) are positive, i.e., $\omega_{p(-)} > 0$, $\omega_{p(+)} > 0$.

To solve the problem, we will use the method of successive approximations. In the zero approximation, we set $I_1 = I_2$ in expression (10). Then, equation (8) takes the form

$$\dot{\psi} = M\Omega + \omega_p \sin(\psi + \varphi_p), \tag{12}$$

where

$$\varphi_p = \arctan\frac{r_2\sin\varepsilon_2 - r_1\sin\varepsilon_1}{r_2\cos\varepsilon_2 + r_1\cos\varepsilon_1} \tag{13}$$

is the phase angle, which, unlike ϕ_p, is independent of the intensities I_1, I_2.

In the regime of frequency locking between the counterpropagating waves

$$\dot{I}_1 = 0, \quad \dot{I}_2 = 0, \quad \dot{\psi} = 0, \tag{14}$$

$$I_1 = \text{const} > 0, \quad I_2 = \text{const} > 0, \quad \psi = \text{const} \in [0, 2\pi]. \tag{15}$$

For the left boundary of the synchronisation zone, when $\Omega = \Omega_{(-)}$, at $\Omega_{(-)} < 0$, using (12), (14) we have

$$0 = M\Omega_{(-)} + \omega_{p(-)}\sin(\psi_{(-)} + \varphi_p), \tag{16}$$

which, taking into account $\omega_{p(-)} > 0$, yields

$$\sin(\psi_{(-)} + \varphi_p) = 1. \tag{17}$$

In this connection, the phase difference of the counterpropagating waves of the laser gyro for the left boundary of the synchronisation zone has the form

$$\psi_{(-)} = \pi/2 - \varphi_p. \tag{18}$$

Similarly, for the right boundary of the synchronisation zone, when $\Omega = \Omega_{(+)}$, at $\Omega_{(+)} > 0$ we have

$$0 = M\Omega_{(+)} + \omega_{p(+)}\sin(\psi_{(+)} + \varphi_p), \tag{19}$$

which, taking into account $\omega_{p(+)} > 0$, yields

$$\sin(\psi_{(+)} + \varphi_p) = -1. \tag{20}$$

Thus, the phase difference of the counterpropagating waves of the laser gyro for the right boundary of the synchronisation zone has the form

$$\psi_{(+)} = -\pi/2 - \varphi_p. \tag{21}$$

Consider now the first two equations of system (1), which describe the dynamics of the intensities I_1, I_2. Taking into account (14), (15) and dividing these equations by $I_{1,2}$, after renormalisation we obtain

$$\begin{aligned} \beta I_1 + \theta I_2 &= \alpha - 2r_2(I_2/I_1)^{1/2}\cos(\psi + \varepsilon_2), \\ \theta I_1 + \beta I_2 &= \alpha - 2r_1(I_1/I_2)^{1/2}\cos(\psi - \varepsilon_1). \end{aligned} \tag{22}$$

Relations (22) represent a system of two nonlinear algebraic equations with respect to the unknown I_1, I_2 under the condition that ψ is given.

In the zero approximation we set $I_1 = I_2$ in the right-hand sides of this system; as a result, the system transforms into a system of two linear equations

$$\begin{aligned} \beta I_1 + \theta I_2 &= \alpha - 2r_2\cos(\psi + \varepsilon_2), \\ \theta I_1 + \beta I_2 &= \alpha - 2r_1\cos(\psi - \varepsilon_1), \end{aligned} \tag{23}$$

which has the solution

$$I_1 = U\left\{1 - \frac{2}{\alpha(\beta - \theta)}[\beta r_2\cos(\psi + \varepsilon_2) - \theta r_1\cos(\psi - \varepsilon_1)]\right\}, \tag{24}$$

$$I_2 = U\left\{1 + \frac{2}{\alpha(\beta - \theta)}[\theta r_2\cos(\psi + \varepsilon_2) - \beta r_1\cos(\psi - \varepsilon_1)]\right\}. \tag{25}$$

Here $U = \alpha/(\beta + \theta))$ is the constant component of the counterpropagating wave intensities, calculated at $r_j = 0$.

Now we have everything we need to find formulas to estimate the coordinates of the left [$\Omega_{(-)}$], and right [$\Omega_{(+)}$] boundaries of the synchronisation zone of counterpropagating waves of the laser gyro on the axis of the angular velocity Ω. To do this, we should substitute expression (18) for $\psi_{(-)}$, and then expression (21) for $\psi_{(+)}$ into (24), (25).

Let us estimate the parameter $\Omega_{(-)}$. Taking into account (18), and the relations

$$\begin{aligned} r_p\sin\varphi_p &= r_2\sin\varepsilon_2 - r_1\sin\varepsilon_1, \\ r_p\cos\varphi_p &= r_2\cos\varepsilon_2 + r_1\cos\varepsilon_1, \end{aligned} \tag{26}$$

we can show that

$$\begin{aligned} \cos(\psi_{(-)} + \varepsilon_2) &= -r_1\sin\varepsilon_{12}/r_p, \\ \cos(\psi_{(-)} - \varepsilon_1) &= r_2\sin\varepsilon_{12}/r_p. \end{aligned} \tag{27}$$

Then, after substituting (27) into (24), (25), we obtain

$$I_{1(-)} = (1 + \mu)U, \quad I_{2(-)} = (1 - \mu)U, \tag{28}$$

where

$$\mu = 2r_1r_2\sin\varepsilon_{12}/(\alpha_m r_p) \quad (|\mu| < 1) \tag{29}$$

is a small dimensionless parameter depending on r_j, ε_j and the parameter α_m.

As a result, after substituting (28) into (9) and taking into account the first expression in (11), we derive

$$\Omega_{(-)} = -M^{-1}\left(\frac{1+\mu}{1-\mu}r_1^2 + \frac{1-\mu}{1+\mu}r_2^2 + 2r_1r_2\cos\varepsilon_{12}\right)^{1/2}. \tag{30}$$

Let us estimate now the parameter $\Omega_{(+)}$. Taking into account (21) and (26) we can show that

$$\begin{aligned} \cos(\psi_{(+)} + \varepsilon_2) &= r_1\sin\varepsilon_{12}/r_p, \\ \cos(\psi_{(+)} - \varepsilon_1) &= -r_2\sin\varepsilon_{12}/r_p. \end{aligned} \tag{31}$$

After substituting these expressions into (24), (25), we obtain

$$I_{1(+)} = (1-\mu)U, \quad I_{2(+)} = (1+\mu)U. \tag{32}$$

As a result, taking into account (32), (9) and second expression in (11), we derive

$$\Omega_{(+)} = M^{-1}\left(\frac{1-\mu}{1+\mu}r_1^2 + \frac{1+\mu}{1-\mu}r_2^2 + 2r_1r_2\cos\varepsilon_{12}\right)^{1/2}. \tag{33}$$

Expressions (30) and (33) yield the desired solution of the problem with respect to the coordinates of the left [$\Omega_{(-)}$] and right [$\Omega_{(+)}$] boundaries of the synchronisation zone of the counterpropagating waves of the laser gyro on the axis of the angular velocity Ω. However, these expressions do not yield in an explicit form the dependence of $\Omega_{(-)}$, $\Omega_{(+)}$ on the asymmetry ($r_1 \neq r_2$) of the linear coupling of the counterpropagating waves. Therefore, using identity transformations we will reduce the above relations to another, more informative form

$$\Omega_{(\pm)} = \pm\frac{[r_p^2 + \mu^2 r_m^2 \pm 2\mu(r_2^2 - r_1^2)]^{1/2}}{(1-\mu^2)^{1/2}M}, \tag{34a}$$

which, using (4), yields

$$\Omega_{(0)} = \tag{34b}$$

$$\frac{[r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)]^{1/2} - [r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)]^{1/2}}{2(1-\mu^2)^{1/2}M},$$

$$\Omega_s = \tag{34c}$$

$$\frac{[r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)]^{1/2} + [r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)]^{1/2}}{2(1-\mu^2)^{1/2}M}.$$

Under the condition of smallness of the difference $r_1 - r_2$ of the moduli of complex linear coupling coefficients of the counterpropagating waves (see, for example, [3]), expressions (34) can be approximately represented in the form more convenient for the analysis:

$$\Omega_{(\pm)} = \Omega_{(0)} \pm \Omega_s,$$

$$\Omega_{(0)} = \frac{\mu(r_2^2 - r_1^2)}{[(1-\mu^2)(r_p^2 + \mu^2 r_m^2)]^{1/2}M}, \tag{35}$$

$$\Omega_s = \frac{(r_p^2 + \mu^2 r_m^2)^{1/2}}{(1-\mu^2)^{1/2}M}.$$

Relations (34), (35) [together with expressions (2), (3), (29)] are the result of the solution of a formulated problem and form the desired mathematical model of the parameters of the synchronisation zone of the counterpropagating waves of the laser gyro. The latter includes initial model (5) as a constituent part.

Analysis of expressions (34), (35) makes it possible to draw the conclusions.

(i) In the general case of the asymmetric ($r_1 \neq r_2$) linear coupling of the counterpropagating waves, the left and right boundaries of the synchronisation zone of the laser gyro are located at different distances from the coordinate origin: $\Omega_{(+)} \neq -\Omega_{(-)}$. As a result, the centre of the region is shifted along the axis of the angular velocity Ω by the finite quantity $\Omega_{(0)} \neq 0$.

(ii) With increasing active medium gain g, the shift $\Omega_{(0)}$ of the centre of the synchronisation zone and its half-width Ω_s decrease, approaching asymptotically the established finite values

$$\Omega_{(0)}^{\text{asymp}} = 0, \quad \Omega_s^{\text{asymp}} = \frac{r_p}{M} = \frac{(r_1^2 + r_2^2 + 2r_1r_2\cos\varepsilon_{12})^{1/2}}{M}. \tag{36}$$

3. On the validity of using equations (1) to the problem solution

It is known (see, for example, [3]) that the initial system of equations (1) describing the dynamics of the dimensionless intensities I_j ($j = 1, 2$) and the phase difference ψ of counterpropagating waves of the laser gyro under study is valid in the weak field approximation when the gain g of the active medium exceeds the resonator losses Γ by no more than 1.2 times. At the same time, in the present paper we did not impose any restrictions on the value of the parameter g and, moreover, in deriving formulas (36) we used the limiting process $g \to \infty$. In this regard, the reader may question the validity of the usage of system (1) to solve the above problem.

The answer to this question may be as follows. Restriction on the use of system (1) is applied only to its first two energy equations

$$\dot I_1 = (\alpha - \beta I_1 - \theta I_2)I_1 - 2r_2(I_1I_2)^{1/2}\cos(\psi + \varepsilon_2),$$
$$\dot I_2 = (\alpha - \beta I_2 - \theta I_1)I_2 - 2r_1(I_1I_2)^{1/2}\cos(\psi - \varepsilon_1) \tag{37}$$

and manifests itself in the fact that the formula $U = \alpha/(\beta + \theta)$ for estimating the constant components of the intensities I_j of counterpropagating waves can be used only at small g. With a further increase in g, the methodical error of the formula increases dramatically.

However, in solving the problem under study, energy equations (37) of system (1) play a purely supporting role: they are used to determine only small additions to U, caused by the linear coupling between the counterpropagating waves. In particular, for the left and right boundaries of the synchronisation zone the intensities I_j take values that are close to U, namely:

$$I_{1(-)} = (1+\mu)U, \quad I_{2(-)} = (1-\mu)U,$$

$$I_{1(+)} = (1-\mu)U, \quad I_{2(+)} = (1+\mu)U.$$

However, the main role in solving the problem is played by the phase equation of system (1)

$$\dot\psi = M\Omega + r_2(I_2/I_1)^{1/2}\sin(\psi + \varepsilon_2)$$
$$+ r_1(I_1/I_2)^{1/2}\sin(\psi - \varepsilon_1), \tag{38}$$

into which the counterpropagating wave intensities enter as ratios I_1/I_2 and I_2/I_1. It is obvious that by substituting the listed quantities $I_{1(-)}$, $I_{2(-)}$, $I_{1(+)}$, $I_{2(+)}$ into these ratios, the factor U that is common for them is reduced. Therefore, the quantity of the methodical error of estimation of the parameter U has no effect on the final result of the calculation. This suggests that the system of equations (1) can be used to solve the above problem at large values of the gain g as well.

4. Conclusions

Based on the analysis of system (1) of dynamic equations for a laser gyro, we have derived formulas (34), (35) for calculating the parameters of the synchronisation zone for the frequencies generated in the device of counterpropagating electromagnetic waves. The formulas make it possible to calculate the coordinates on the axis of the angular velocity Ω of the left [$\Omega_{(-)}$] and right [$\Omega_{(+)}$] boundaries of the synchronisation zone, the coordinate $\Omega_{(0)}$ of its centre and the half-width Ω_s.

Expressions (34), (35) are valid under the condition of weak coupling of the counterpropagating waves, which suggests that in the entire range of the discharge currents used in the laser gyro, the ratios r_p/α_p and r_m/α_m remain much less than unity. In modern devices operating at sufficiently large excesses of the pump above the threshold [3], the above condition is usually satisfied.

Expressions (34) (35) are the generalisation of the known [3, 9–16] model (5) of the parameters of the synchronisation zone of the counterpropagating wave of the laser gyro under study and allow one to mathematically model the experimental dependences given in [17–20]. The results of this modelling by the example of a square four-mirror resonator laser gyro will be presented in a separate publication.

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