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NONLINEAR OPTICAL PHENOMENA

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Determining the parameters of the laser radiation intensity limiter based on a time-dependent equation of radiative transfer in a nonlinear medium

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Abstract. **The effect of the radiation intensity limitation is described using the time-dependent radiative transfer equation taking into account the nonlinearity of the working material of the limiter, which makes it possible to abstract from specific microscopic mechanisms of interaction of radiation with matter. An expression is presented which describes both the deformation of the shape of the laser pulse propagating through a nonlinear medium and the output characteristic (dependence of the output energy on the energy of incident radiation) of the limiter at a known dependence of the absorption coefficient on the laser pulsed radiation intensity with a given pulse shape. A functional equation is derived to determine the dependence of the absorption coefficient on the intensity from the experimental output characteristic, which allows one to predict the limiter properties for different thicknesses of the working medium, as well as to effectively compare limiters of different types.**

Keywords: *laser radiation intensity limitation, nonlinear medium, time-dependent radiative transfer equation, pulse shape, absorption coefficient.*

The development of the laser technology, which penetrated into various spheres of human activity, has revealed the importance of the problems of protection of eyes and optical sensors subjected to high-power radiation. The urgency of this problem is caused by a noticeable increase in the radiation intensity of laser devices operating in a wide spectral range [1].

The problem of designing laser radiation intensity limiters, capable of triggering during the time much shorter than the laser pulse duration, has long attracted the attention of researchers and engineers [2 – 5]. The main attention is now on finding, developing and studying new optical materials of the limiters based on theoretical and experimental consideration of the features of the physical processes of interaction of light with a nonlinear absorbing and scattering media [5 – 8].

The description of the output characteristic of the limiters (dependence of the output energy on the energy of incident radiation), based on consideration of specific microscopic mechanisms limiting the radiation intensity does not provide a sufficient agreement between theory and experiment. This is explained by the fact that in a real limiter several radiation – matter interaction mechanisms, whose degree of influence is not known, take place simultaneously; the description of each mechanism is based on many numerical parameters whose values may not correspond to the real limiter; and, finally, the calculation of the macroscopic characteristics of the limiter is a complex problem that is not completely solved yet. In addition, the problem of development of effective laser radiation intensity limiters requires a set of experiments at different values of the limiter parameters and radiation characteristics.

As a rule, in various computational models the experimental data on the functioning of a particular limiter are used only for comparison with the results of preliminary calculations [9, 10]. At the same time, more promising is apparently the approach in which experimental data are used already at the stage of preliminary calculations. This approach, in particular, allows one to calculate the dependence of the nonlinear absorption on the radiation intensity using the measured output characteristic of the given limiter, and then using the calculated dependence to predict the output characteristic of the limiter with the same working medium, but, for example, with another thickness of its layer or with another pulse duration.

In [7], we proposed an approach to the description of the radiation intensity limitation by using the steady-state radiative transfer equation, which allows one to determine the dependence of the absorption coefficient of the limiter's working medium on the laser intensity directly from the experimental output characteristic of the given limiter. Obtaining this dependence makes it possible to predict the properties of the limiters at different thicknesses of the working medium, and to compare different types of limiters with each other. However, because the steady-state radiative transfer equation was used in consideration, the temporal characteristics of radiation could be taken into account only indirectly.

In this paper, we developed an approach to describe the interaction of laser pulses with a nonlinear medium with the help of the time-dependent radiative transfer equation. This approach allows one to explicitly take into account the initial pulse shape, to describe the deformation of the shape of the pulses propagating through the nonlinear medium, and to predict the properties of the limiter not only for different thicknesses of the working medium but also for

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other shapes of laser pulses. Note that in this paper, unlike [7], the scattering properties of the limiter's working medium are not considered.

Let us consider the time-dependent radiative transfer equation (RTE) for a purely absorbing medium (PAM) [11 – 13]. In this case, we neglect all the radiation – matter interaction processes except absorption. The medium parameters (the absorption coefficient μ) are included in the RTE. We assume that the nonlinear properties of the medium are determined by the dependence of the absorption coefficient on the radiation intensity. At the same time, because the working medium of the limiter represents a homogeneous layer, there will be no explicit dependence of the characteristics of the medium on the coordinates, i.e., $\mu = \mu(\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t))$, where $\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t)$ is the radiation flux density at point $\boldsymbol{r}$ at an instant of time t in the direction $\boldsymbol{\Omega}$. Then the RTE can be written in the form:

$$\frac{1}{\upsilon}\frac{\partial}{\partial t}\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t) + \boldsymbol{\Omega}\,\mathrm{grad}[\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t)] + \mu[\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t)]\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t) = S(\boldsymbol{r}, \boldsymbol{\Omega}, t), \quad (1)$$

where$S(\boldsymbol{r}, \boldsymbol{\Omega}, t)$ is the density of the radiation sources at point $\boldsymbol{r}$ at an instant of time t in the direction $\boldsymbol{\Omega}$ and υ is the modulus of the radiation propagation velocity in the medium.

For a directed source of pulsed radiation, which is a laser,

$$S(\boldsymbol{r}, \boldsymbol{\Omega}, t) = \left[U_0 f(t) \Big/ \int_0^{\infty} f(t)\mathrm{d}t\right]\delta(\boldsymbol{r})\delta_2(\boldsymbol{\Omega}\boldsymbol{\Omega}_0),$$

where U_0 is the pulse energy; $f(t)$ is the laser pulse shape; $\boldsymbol{\Omega}_0$ is the direction of the radiation source perpendicular to the layer of the working medium of the limiter; $\delta(\boldsymbol{r})$ and $\delta_2(\boldsymbol{\Omega}\boldsymbol{\Omega}_0)$ is the three-dimensional and surface delta functions, respectively. We choose the Cartesian coordinates (x, y, z) with z axis along the vector $\boldsymbol{\Omega}_0$. In this case, the solution of equation (1) should have the form: $\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t) = I(z, t)\delta(x)\times\ \delta(y)\delta_2(\boldsymbol{\Omega}\boldsymbol{\Omega}_0)$, where $I(z, t)$ is the radiation intensity at point z at an instant of time t; $\delta(x)$ is a one-dimensional delta function. Accordingly, $\mu(\Phi(\boldsymbol{r}, \boldsymbol{\Omega}, t)) = \mu(I(z, t))$. Then, the RTE can be written as

$$\frac{1}{\upsilon}\frac{\partial}{\partial t}I(z, t) + \frac{\partial}{\partial z}I(z, t) + \mu[I(z, t)]I(z, t) = 0, \quad (2)$$

and the point source can be written as a boundary condition:

$$I(0, t) = I_0 f(t) = \left[U_0 \Big/ \int_0^{\infty} f(t)\mathrm{d}t\right] f(t),$$

where $U_0 = \int_0^{\infty} I_0 f(t)\mathrm{d}t$ is the total pulse energy.

Since in a nonlinear medium the dependence on the pulse energy U_0 is nonlinear, we will introduce U_0 in the list of arguments of the solution $I(U_0, d, t)$, where d is the thickness of the working medium of the limiter. In the implicit form, the solution $I(U_0, d, t)$ of equation (2) will have the form

$$\int_{U_0 f(t-d/\upsilon)/\int_0^{\infty} f(\tau)\mathrm{d}\tau}^{I(U_0,d,t)} \frac{\mathrm{d}I}{I\mu(I)} = -d. \quad (3)$$

Using expression (3), we can determine both the change in pulse shape $F(t) = I(U_0, d, t)/I_0$ as compared with the original shape $[f(t)]$ and the limiter's output characteristic, i.e., the dependence of the total pulse energy

$$U = \varphi(U_0, d) = \int_0^{\infty} I(U_0, t, t)\mathrm{d}t$$

after its passage through the limiter on the total energy of the initial pulse U_0.

The dependence of the absorption coefficient on the radiation intensity can be expanded in a Taylor series:

$$\mu(I) = \sum_{n=0}^{\infty}\frac{1}{n!}\frac{\mathrm{d}^n\mu(I)}{\mathrm{d}I^n}\bigg|_{I=0}(I)^n = \sum_{n=0}^{\infty}\frac{1}{n!}\,\mu^{(n)}I^n. \quad (4)$$

The basic laws of the radiation intensity limitation can be established by retaining a few terms. For example, retaining two or three terms in (4), we obtain the linear [Fig. 1, curve (*1*)] and nonlinear [Fig. 1, curve (*2*)] dependences of the absorption coefficient on the radiation intensity, respectively:

$$\mu(I) = \mu^{(0)} + \mu^{(1)}I, \quad (5)$$

$$\mu(I) = \mu^{(0)} + \mu^{(1)}I + \frac{1}{2}\mu^{(2)}I^2. \quad (6)$$

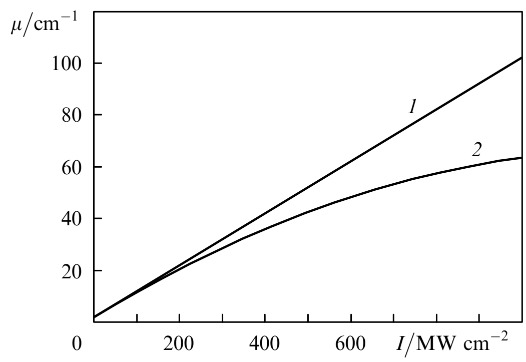

Figure 1. Linear [$\mu^{(0)} = 2$ cm^{-1}, $\mu^{(1)} = 100 \times 10^{-9}$ cm^{-1} J^{-1} s] (*1*) and nonlinear [$\mu^{(0)} = 2$ cm^{-1}, $\mu^{(1)} = 100 \times 10^{-9}$ cm^{-1} J^{-1} s, $\mu^{(2)} = -0.8\times 10^{-18}$ cm^{-1} J^{-1} s^2] (*2*) dependences of the absorption coefficient on the radiation intensity.

Consider the change in the pulse shape in the linear case (5). By substituting (5) into (3), we obtain

$$I(U_0, d, t) = \frac{\mu^{(0)}\exp\left(-\mu^{(0)}d\right)U_0 f(t - d/\upsilon)}{\mu^{(0)}\int_0^{\infty} f(\tau)\mathrm{d}\tau + \mu^{(1)}[1 - \exp\left(-\mu^{(0)}d\right)]U_0 f(t - d/\upsilon)}. \quad (7)$$

Figure 2 shows the change in the shape of the initial triangular pulse in accordance with expression (7).

Knowing the initial pulse shape $f(t)$, we can obtain the output characteristic of the limiter $U = \varphi(U_0, d)$. For example, in the case of a rectangular pulse of duration T, we have $f(t) = T^{-1}[\eta(t) - \eta(t - T)]$, where $\eta()$ is a step Heaviside function. Then,

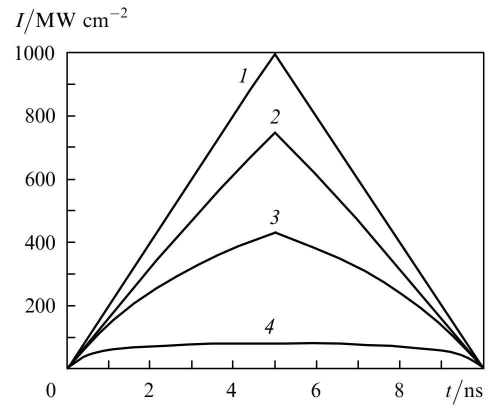

Figure 2. Initial pulse (*1*) and change in its shape during the passage through the layer of the limiter material of thickness $d = 1$ mm with $\mu^{(0)} = 2$ cm^{-1}, $\mu^{(1)} = 1 \times 10^{-9}$ cm^{-1} J^{-1} s (*2*), $\mu^{(1)} = 10 \times 10^{-9}$ cm^{-1} J^{-1} s (*3*), and $\mu^{(1)} = 100 \times 10^{-9}$ cm^{-1} J^{-1} s (*4*).

$$U = \varphi(U_0, d) = \int_0^{\infty} I(U_0, d, t)\mathrm{d}t$$

$$= \frac{U_0 \mu^{(0)} \exp(-\mu^{(0)} d)}{\mu^{(0)} + \mu^{(1)}[1 - \exp(-\mu^{(0)} d)] U_0 / T}. \quad (8)$$

Unlike the steady-state case [5], expression (8) explicitly includes the pulse duration T. Figure 3 shows the dependence of the output characteristic $U = \varphi(U_0)$ on the duration of the rectangular pulse. Similar dependences can be obtained for any specified pulse shape.

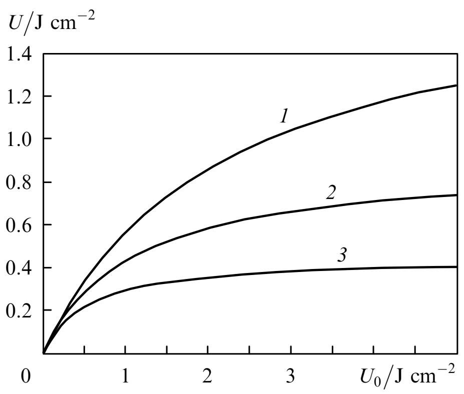

Figure 3. Dependence of the output characteristic $U = \varphi(U_0)$ for the linear dependence of the absorption coefficient on the radiation intensity at a rectangular pulse duration of 20 (*1*), 10 (*2*), and 5 ns (*3*); the thickness of the limiter material is 1.0 mm.

Using expression (3), for the given dependence of the absorption coefficient on the radiation intensity $\mu(I)$ and the initial pulse shape $f(t)$, we can find $I(U_0, d, t)$ and output characteristic

$$\varphi(U_0, d) = \int_0^{\infty} I(U_0, d, t)\mathrm{d}t.$$

Figure 4 presents the output characteristics for the nonlinear dependence of the absorption on the radiation intensity [Fig. 1, curve (*2*)] and rectangular pulses of different durations.

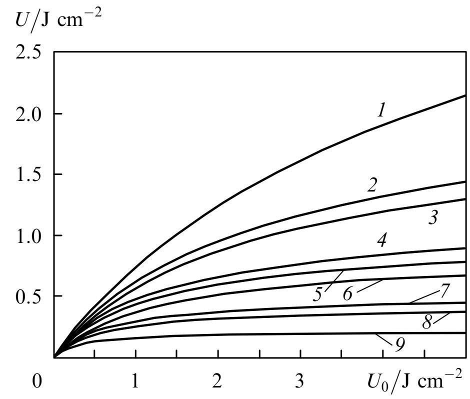

Figure 4. Output characteristics of the limiter at a rectangular pulse duration of 5 (*4*, *7*, *9*), 10 (*2*, *6*, *8*), and 20 ns (*1*, *3*, *6*) for the working layer of thickness 0.5 (*1*, *2*, *4*), 1.0 (*3*, *5*, *7*) and 2.0 mm (*6*, *8*, *9*).

If we know the output characteristic $U = \varphi(U_0, d)$ and the pulse shape $f(t)$, differentiating (3) in U_0, we obtain the functional equation for determining the dependence of the absorption coefficient on the radiation intensity $\mu(I)$:

$$U_0 \mu\left(\frac{U_0 f(t - d/v)}{\int_0^{\infty} f(\tau)\mathrm{d}\tau}\right) \frac{\partial}{\partial U_0} I(U_0, d, t)$$

$$= I(U_0, d, t)\mu[I(U_0, d, t)]. \quad (9)$$

For a rectangular pulse of duration T and total energy U_0, the intensity of propagated radiation is $I(U_0, d, t) = I_1(U_0, d)[\eta(t) - \eta(t - T)]$ and

$$\varphi(U_0, d, T) = \int_0^{\infty} I_1(U_0, d)[\eta(t) - \eta(t - T)]\mathrm{d}t = I_1(U_0, d)T.$$

Then, at $t \in (d/v, d/v + T)$, we have

$$U_0 \mu\left(\frac{U_0}{T}\right) \frac{\partial}{\partial U_0} \varphi(U_0, d, T).$$

$$= \varphi(U_0, d, T)\mu\left(\frac{\varphi(U_0, d, T)}{T}\right). \quad (10)$$

Moreover, the value of $\mu(0)$ can be found by the initial part of the output characteristic at $U \to 0$, assuming that the Bouguer–Lambert–Beer law is fulfilled. The resulting dependence of the absorption coefficient μ on the radiation intensity I can be used to calculate the output characteristic $U = \varphi(U_0, d, T)$ for different thicknesses d of the limiter and rectangular pulse durations T, which will reduce the number of costly full-scale experiments.

Thus, we have proposed an approach to describe the laser radiation intensity limitation, taking into account both the nonlinear properties of the limiter material and temporal characteristics of incident radiation. We present the equations for determining the output characteristic by the known dependence of the absorption coefficient on the pulsed radiation intensity with a given pulse shape and by the dependence of the absorption coefficient on the intensity by the experimental output characteristic. The estimate of the optical characteristic of the limiter material directly from the experimental output characteristic can be used to predict its properties at other thicknesses of the limiter material and at other pulse shape of the incident radiation. In addition, the proposed approach provides an effective comparison of different types of limiters by using the dependence of the absorption coefficient of the limiter material on the radiation intensity.

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