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Filamentation of femtosecond Gaussian pulses with close-to-linear or -circular elliptical polarisation

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Abstract. **A numerical investigation was made of the formation and development of filaments in the propagation of high-power femtosecond Gaussian laser pulses in argon, whose polarisation is close to the linear or circular one. Filaments produced by close-to-circularly polarised pulses were found to be more uniform, greater in diameter, and higher in intensity than the filaments produced by close-to-linearly polarised pulses. For incident pulses with a close-to-linear (circular) polarisation, the degree of ellipticity of the radiation on the axis of the resultant filament becomes equal to zero (unity) at the instant of the peak of the local intensity.**

Keywords: *femtosecond pulse, filament, Kerr nonlinearity, gas ionisation, elliptic polarisation.*

1. Introduction

The interest in the filamentation of femtosecond pulses is due not only to the problems of atmospheric optics, but also due to diverse nonlinear-optical effects which accompany the propagation of high-power laser radiation [1, 2]. Numerous experiments and theoretical investigations were carried out primarily with linearly polarised femtosecond pulses [1, 2], although the formation and development of filaments under the action of an elliptically polarised pulse exhibits a number of special features. For instance, a filament produced by a circularly polarised pulse exhibited a higher radiation intensity than the filament produced by a linearly polarised initial pulse [3], while the resultant supercontinuum energy rises in going from linearly to circularly polarised pulses [4]. These effects are caused both by the higher critical self-focusing power of the latter [5] and by the lower probability of nonlinear laser-induced ionisation of the medium [6, 7]. In the view of the authors of Ref. [8], the self-focusing of linearly and circularly polarised beams, which prevails at the initial stage of filament's development, is immune to perturbations of its polarisation. However, the model employed in their work did not take into account the ionisation of the medium and the complex spatio-temporal structure of the filament.

Numerical investigations of the filamentation of a pulse with an arbitrary initial degree of ellipticity [9] showed a lower number of free electrons and a smaller high-frequency spectrum broadening for a pulse with an initial circular polarisation in comparison with the case of linear polarisation. In the description of the intensity dependence of the free-electron production rate, however, the ionisation probability in a single radiation – gas molecule interaction event was calculated using the same multiphoton ionisation coefficient irrespective of the initial degree of ellipticity. Ammosov et al. [7] experimentally determined the ratio between the probability $W_c(\tilde{I})$ of ionisation of an argon atom by a circularly polarised pulse and the probability $W_{lin}(\tilde{I})$ of its ionisation by a linearly polarised pulse in the case when their intensities $\tilde{I} = cn_0 I/(8\pi)$ were equal. Here, $I(r,z,\tau) = (|E_+|^2 + |E_-|^2)/2$; $E_\pm(r,z,\tau) = E_x \pm \mathrm{i}E_y$ are the slowly varying complex amplitudes of the circularly polarised waves which make up the elliptically polarised wave propagating along the z axis of cylindrical coordinate system. A least squares approximation of the dependence of W_c/W_{lin} on $\tilde{I}$ yields

$$W_c(\tilde{I}) = 2.96 \times 10^{-3}(\tilde{I} \times 10^{-13})^{1.337} W_{lin}(\tilde{I}),$$

where $\tilde{I}$ is expressed in units of W cm^{-2}.

The aim of our work is to numerically simulate the formation and development of filaments emerging in argon in the self-action of high-power elliptically polarised femtosecond Gaussian pulses whose polarisations are close-to-linear or -circular.

The production of a filament in argon is described by the following system of equations [9] for $E_\pm(r,z,\tau)$:

$$2\mathrm{i}k_0 \frac{\partial E_\pm}{\partial z} = \hat{T}^{-1} \Delta_\perp E_\pm - k_0 \sum_{n=2}^{\infty} \frac{k^{(n)}}{(\mathrm{i}^n n!)} \frac{\partial^{(n)} E_\pm}{\partial \tau^n}$$
$$+ 2k_0^2 \left(\hat{T}\Delta n_{k\pm} + \hat{T}^{-1}\Delta n_{p\pm}\right) E_\pm - \mathrm{i}k_0 \alpha E_\pm. \qquad (1)$$

Here, $\tau = T - z/v_g$; t is the time; v_g is the group velocity; $\Delta_\perp = r^{-1}\partial/\partial r(r\partial/\partial r)$ is the transverse Laplacian; k_0 is the wavenumber corresponding to a wavelength of 800 nm; and α is the multiphoton absorption coefficient. The second and third terms in the right-hand side of Eqn (1) describe the linear frequency dispersion and the nonlinearity of argon, respectively. In this instance, $k^{(n)}$ is the nth-order dispersion coefficient and $\hat{T} = 1 + \mathrm{i}/(\omega_0 \partial/\partial\tau)$. In the spectral range under consideration, one may neglect the frequency

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dispersion of the cubic nonlinear susceptibility tensor responsible for self-action effects and write the intensity-dependent additions to the refractive index in the following form: $\Delta n_{k\pm} = n_2(|E_\pm|^2 + 2|E_\mp|^2)/6$ [9, 10], where $n_2 = 10^{-19}$ cm^2 W^{-1}. The additions $\Delta n_{\mathrm{p}\pm}$ to the refractive index n_0, which emerge due to the laser plasma produced in the propagation of a femtosecond pulse, are determined proceeding from the Drude model [1, 2]: $\Delta n_{\mathrm{p}\pm}$. The variation of the electron plasma density $\Delta n_{\mathrm{p}\pm} \sim N_{\mathrm{e}}$ is described by the equation $\partial N_{\mathrm{e}}/\partial\tau = W(\tilde{I})\,(N_0 - N_{\mathrm{e}})$, where N_0 is the density of neutral atoms, $W(\tilde{I}) = W_{\mathrm{lin}}$ when the initial pulse is close-to-linearly polarised, and $W(\tilde{I}) = W_{\mathrm{c}}$ when it is close-to-circularly polarised. In this case, the probability W_{lin} was defined according to Ref. [6].

We assume that a laser pulse with a duration τ_0 and a beam radius r_0 is incident on the medium boundary ($z = 0$), the electric field intensity being defined by the formula

$$E_\pm(r, z = 0, \tau) = \sqrt{I_0(1 \pm M_0)}\exp\left(-\frac{r^2}{2r_0^2} - \frac{\tau^2}{2\tau_0^2}\right). \quad (2)$$

Here, I_0 and M_0 are the values of the normalised intensity and the degree of ellipticity $M(r, z, \tau) = (|E_+|^2 - |E_-|^2)/(2I)$ of the polarisation ellipse for $r = z = 0$ at the point in time $\tau = 0$. When the incident pulse is linearly (circularly) polarised, $M_0 = 0$ ($M_0 = \pm1$).

In the numerical integration of Eqns (1), advantage was taken of the method of splitting in physical factors at each step Δz of the computational grid. We sequentially took into account diffraction (using the sweep method), dispersion, nonlinear phase incursion and nonlinear absorption in the self-induced plasma. The computational grid was uniform in variables r and τ with grid steps of 4.8 μm and 0.033 fs (the numbers of points were 2001 and 2^{16}, respectively). The value of Δz was so selected that the phase incursion between adjacent points did not exceed 0.1 rad. For $M_0 = 0$, the intensity and polarisation of radiation coincided to within 0.1 % with those determined using the algorithm employed in Ref. [11] for calculating the characteristics of a linearly polarised pulse, whose polarisation remained invariable in the course of propagation. We also note that the application of the algorithm employed in our work to the system of equations [12], which is more complex than Eqns (1) and describes the interaction between the first and second harmonic pulses of a Ti:sapphire laser, yielded data in good agreement with experimental ones.

An analysis of the numerical solutions of the system of Eqns (1) subject to boundary conditions (2) revealed the following: for a close-to-circular polarisation of the incident pulse, the origin of the filament is further from the medium boundary than for a close-to-linearly polarised pulse. In this case, the peak intensity I_{m} in the filament is higher. This is due to the rise of self-focusing threshold and the lowering of ionisation probability with increasing $|M_0|$ [5–7]. This statement is illustrated by Fig. 1a plotted for $r_0 = 1.2$ mm, $\tau_0 = 27$ fs, and a pulse energy of 3.2 mJ. For a circular polarisation of the incident pulse (the solid curve), the peak intensity in the filament is as high as 170 TW cm^{-2}, while for a linear one (the dashed curve) it amount to only 120 TW cm^{-2}, which corresponds to the data of Ref. [11]. We emphasise that these values are primarily determined by the degree of ellipticity of the polarisation ellipse of the initial pulse and depend only slightly on its shape, energy, duration, geometrical dimensions, and wavefront curvature.

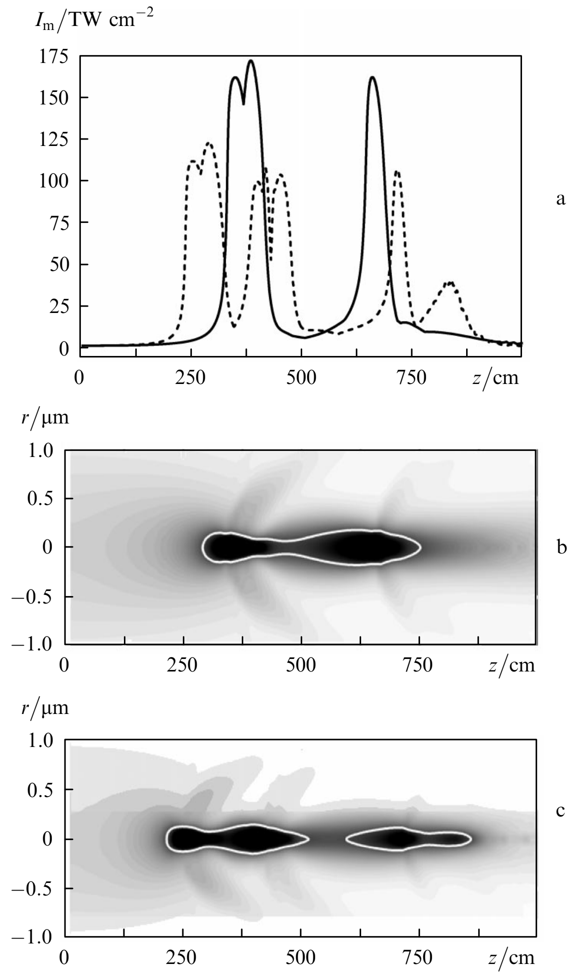

Figure 1. Peak intensity on the filament axis as a function of propagation coordinate for a pulse with a circular (solid curve) and linear (dashed curve) polarisation (a) and spatial distribution of energy density for circularly (b) and linearly (c) polarised incident pulses. The contour corresponds to a level of 0.3 J cm^{-2}.

Shown in shades of grey in Figs 1b and 1c are the spatial energy density distributions

$$J(r, z) = \frac{cn_0}{8\pi}\int_{-\infty}^{\infty} I(r, z, \tau)\mathrm{d}\tau$$

of laser radiation in the filamentation of circularly and linearly polarised pulses (the contour indicates the level of 0.3 J cm^{-2}). As is easy to see, for $M_0 = \pm1$ the channel with the high density of radiation energy in the filament is more uniform (both in the longitudinal and transverse coordinates) than for $M_0 = 0$. For a circular polarisation of the initial pulse, the average channel diameter is equal to 240 μm and for a linear polarisation it is equal to only 180 μm.

In the general case ($M_0 \neq 0$, $M_0 \neq \pm1$), the degree of ellipticity of the propagating femtosecond pulse changes significantly in space and time. On the beam axis for a fixed value of z, the function $M(\tau)$ varies nonmonotonically with increasing τ. At the points of local peaking of $I(\tau)$, the values of $M(\tau)$ are close to zero (or to ±1) in the filamentation of a close-to-linearly (-circularly) polarised pulse.

Although we employ the probabilities of argon ionisation only by a linearly or circularly polarised pulse, this has no appreciable effect on the formation of filaments. This is so, because in high-intensity (of the order of 100 TW cm^{-2}) domains, where the ionisation probability is high, the modulus of the degree of ellipticity is close to zero or unity. When M departs significantly from these values, the radiation intensity at this point turns out to be well below the peak values. The multiphoton ionisation probability depends heavily on the intensity: according to experiments [13], on lowering the intensity from 100 to 50 TW cm^{-2} the ionisation probability drops by two orders of magnitude. That is why the multiphoton ionisation can hardly develop for values of $|M|$ distant from zero and unity.

The latter is indication that the degree of radiation ellipticity in the filament is immune to small-scale perturbations. This is particularly evident at such distances from the medium boundary where the pulse splits into a sequence of subpulses. A typical picture is shown in Fig. 2 plotted for $z = 350$ cm, $r_0 = 1.2$ mm, $\tau_0 = 27$ fs, and a pulse energy of 3.2 mJ. Earlier it was reported [14–16] that the pulse splits into a sequence of 5–10-fs long subpulses under similar parameters. The solid curve depicts the dependence $M(\tau)$ and the dashed curve shows $I(\tau)$ on the beam axis. One can see that the polarisation in the subpulses compressed along the filament becomes circular (linear) when M_0 differs from ± 1 (0) by 10 %–15 %.

Therefore, the formation of filaments of close-to-linearly polarised femtosecond pulses is significantly different from that of close-to-circularly polarised ones. In the latter case, the intensity in the resultant filament is higher and the energy density distribution is more uniform than for $M_0 \approx 0$. The degree of ellipticity of the radiation compressed in the filament tends to linear or circular polarisation for ten-percent departures of input radiation polarisation from the linear or circular ones, respectively. For a strong polarisation ellipticity of the initial pulse ($0.15 < |M_0| < 0.75$), even stronger changes would be expected to occur in the state of field polarisation in the filament. The corresponding investigations will be executed upon generalisation of the formula for gas ionisation probability to the case of elliptically polarised radiation with an arbitrary value of M_0.

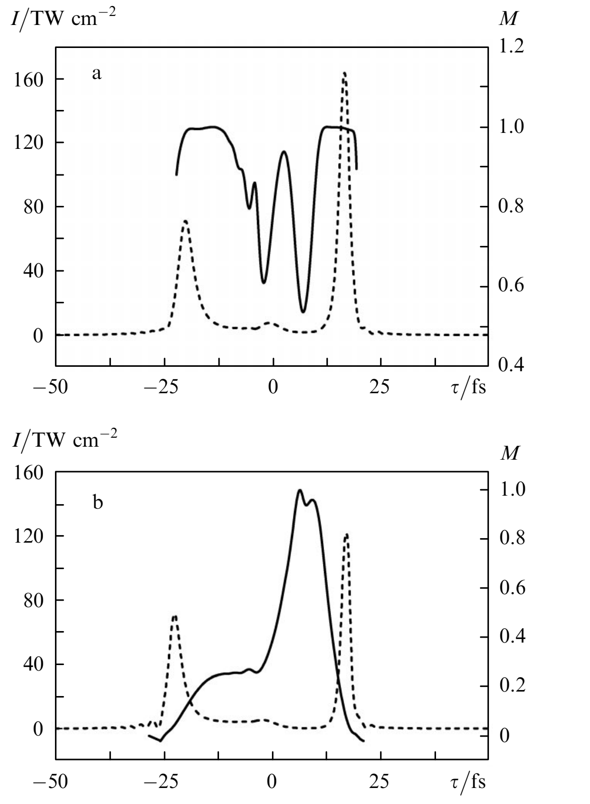

Figure 2. Dependences of $M(\tau)$ (solid curves) and $I(\tau)$ (dashed curves) on the beam axis in the filament produced by an incident pulse with an initial degree of ellipticity $M_0 = 0.9$ (a) and 0.1 (b).

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