

SOLITONS

PACS numbers: 42.81.Dp; 42.65.Tg
DOI: 10.1070/QE2011v041n06ABEH014437

Symbolic computation of solitons in the normal dispersion regime of inhomogeneous optical fibres

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Abstract. **A nonlinear Schrödinger equation with varying dispersion, nonlinearity and gain (or absorption) is studied for ultrashort optical pulses propagating in inhomogeneous optical fibres in the case of normal dispersion. Using the modified Hirota method and symbolic computation, the bilinear form and analytic soliton solution are derived. Stable bright and dark solitons are observed in the normal dispersion regime. A periodically varying soliton and compressed soliton without any fluctuation are obtained. Combined and kink-shaped solitons are observed. Possibly applicable soliton control techniques, which are used to design dispersion-managed systems, are proposed. The proposed techniques may find applications in soliton management communication links, soliton compression and soliton control.**

Keywords: *symbolic computation, nonlinear Schrödinger equation, solitons.*

1. Introduction

Since the first theoretical [1] paper and experimental [2] work on solitons in the optical fibres, studies of the solitons have been attractive and active for their potential applications in the long-distance communication systems and all-optical ultrafast switching devices [3–6]. Solitons arising as a result of balance between the focusing-type nonlinearity and anomalous group velocity dispersion (GVD) of the pulses are called bright solitons [7] as they are in essence a kind of the localised nonlinear light. In turn, dark solitons resulting from the mutual compensation between the defocusing-type nonlinearity and normal GVD of the pulses appear as the localiased intensity dips against a finite carrier wave background [8].

Solitons can be used to encode digital optical data for high-speed and long-distance communication systems [9]. However, when applying the soliton techniques to these systems, there arises a problem associated with the GVD distribution in the already installed optical fibres [10]. In this case, the dispersion management (DM) technique has been used to overcome the GVD distribution, thereby ensuring better transmission quality than a uniform GVD line [11]. However, this technique has its limitations, for example, the Gordon–Haus jitter [10], which is caused by the GVD [12]. Besides, it seems difficult to transmit a single-channel high-speed signal over transoceanic distances because of the large nonlinear interaction between the adjacent solitons, which limits the transmission distance [13]. Hence, soliton control techniques have been proposed to extend the transmission capacity and distance [9, 14].

By using the soliton control technique, Nakazawa et al. [15] have succeeded in transmitting solitons over unlimited distances. Next, the active soliton control through synchronous regeneration by such modulators as the $LiNbO_3$ Mach–Zehnder [16], Kerr fibre [17] and electroabsorption modulators [18] has been demonstrated [19], and an inline synchronous modulation technique for the DM transmission line composed of the dispersion shifted fibre (DSF) with the dispersion compensation (DC) has been proposed [20, 21]. With the inline synchronous modulation technique, a 40-Gbit s^{-1} soliton transmission experiment covering 70 000 km has been reported [20]. Besides, a similar technique has been used to stabilise the soliton energy in the wavelength-division multiplexing soliton transmission system with the DSF [22], and a modified soliton control method has been proposed and extended to a strong DM line composed of the standard (non-dispersion shifted) fibre with the DC [23]. Using that modified soliton control method, the 40-Gbit s^{-1} return-to-zero pulses have been transmitted over more than 20 000 km through a DM line [23]. Moreover, a linear stability analysis of the DM solitons controlled by the inline narrow-band filters has been presented [24], and the spectral filtering technique has been employed for soliton control [25]. Furthermore, the authors of paper [26] have introduced a novel approach to achieve completely nonlinear control of the process, and the predicted phenomena in Ref. [27] have offered different opportunities for soliton control. Recently, the stable control of the pulse time delay has been achieved by means of the resonance soliton solutions [28].

The above techniques are aimed at a search for a possible control of the bright soliton motion [29]. However, the bright soliton has a drawback: it fully utilises the line capacity because it is necessary to keep relatively large separation between the solitons in order to avoid the accumulation of bit errors [30]. In addition, the optical losses decrease the bright soliton intensity with increasing duration [30]. The influence of this effect on the dark soliton is much less than that on the bright soliton [30]. For the dark soliton, it appears as an intensity dip against a finite carrier wave background [29]. Compared with

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Received 20 September 2010
Kvantovaya Elektronika **41** (6) 545–551 (2011)
Submitted in English

the bright soliton, the time jitter in the dark soliton is lower than in the corresponding bright one [31]. In view of its potential applications, the dark soliton has been the object of growing attention [32] and a review on the dark soliton has been given [3]. Besides, the dark soliton propagation in the nonlinear optical fibres has been analysed [29]. Furthermore, a soliton control system has been considered and the main features of the dark soliton have been presented in Ref. [33].

On the other hand, soliton control in the nonlinear inhomogeneous media is a subject of interest [26]. The nonlinear inhomogeneous media include the discrete and bulk media with such periodically varying coefficients as dispersion and nonlinearity [34], as well as the systems with the localised defects [35]. In the inhomogeneous optical fibres, the propagation of the optical solitons in the normal dispersion regime can be described by the nonlinear Schrödinger (NLS) equation [36]:

$$\mathrm{i}u_\xi - \mathrm{i}\frac{g(\xi)}{2}u - \frac{1}{2}\beta(\xi)u_{\tau\tau} + \gamma(\xi)|u|^2u = 0, \tag{1}$$

where $g(\xi)$, $\beta(\xi)$ and $\gamma(\xi)$ are the functions of the normalised propagation distance ξ, which are related to amplification (or absorption), GVD and Kerr nonlinearity, respectively; $u(\xi,\tau)$ is the complex envelope of the electric field (τ is the normalised delay time, subscripts ξ and τ denote partial derivatives). Equation (1) is said in Ref. [36] to be able to describe physical systems in nonlinear optics and condensed matter physics. In the optical context, it describes the evolution of the slowly varying envelope $u(\xi,\tau)$ of an optical pulse propagating along the ξ axis in a fibre amplifier or compressor.

In the practical applications, Eqn (1) is of interest not only for the amplification and compression of optical solitons in inhomogeneous systems, but also for the stable transmission of the DM solitons [37]. However, we should note that analytical discussions of Eqn (1) about soliton control in inhomogeneous optical fibres in the normal dispersion regime (except Ref. [33]) are absent.

In this paper, we will study the soliton control in the inhomogeneous optical fibres and pursue two goals. First, based on symbolic computation, we will obtain a family of the analytic soliton solutions for Eqn (1) solved by the modified Hirota method; then, we will study the dynamics of the solitons in the presence of the nonuniform dispersion, gain and nonlinearity analytically. Second, we will consider the GVD coefficients of Eqn (1) based on the dispersion profiles of the dispersion-decreasing fibre (DDF) earlier developed for the adiabatic soliton compression in the anomalous GVD regime (see Ref. [38]), which has not been studied in details yet, and discuss how to control the dark soliton in the inhomogeneous optical fibres. The structure of the present paper will be as follows. In Sec. 2: with the aid of symbolic computation, the bilinear form for Eqn (1) will be derived using the modified Hirota method. In Sec. 3: the fundamental soliton solution will be presented based on its bilinear form, and the analysis of the solitons via the obtained soliton solutions will performed. Finally, our conclusions will be given in Sec. 4.

2. Bilinear form for Eqn (1)

With symbolic computation [6, 39, 40], in the normal dispersion regime, we will present the bilinear form for Eqn (1) using the modified Hirota method.

Let us introduce the dependent variable transformation [5]

$$u(\xi,\tau) = A(\xi)\frac{h(\xi,\tau)}{f(\xi,\tau)}, \tag{2}$$

where $h(\xi,\tau)$ is a complex differentiable function; $A(\xi)$ and $f(\xi,\tau)$ are both real parts. After some symbolic manipulations, we obtain the bilinear form for Eqn (1):

$$\left[\mathrm{i}D_\xi - \frac{1}{2}\beta(\xi)D_\tau^2 + \lambda\right]h\cdot f = 0, \tag{3}$$

$$[\beta(\xi)D_\tau^2 - 2\lambda]f\cdot f = -2\gamma(\xi)\exp\left[\int g(\xi)\mathrm{d}\xi\right]|h|^2. \tag{4}$$

Here, λ is a parameter to be determined and Hirota's bilinear operators D_ξ and D_τ [41, 42] are defined by

$$D_\xi^m D_\tau^n(G\cdot F) = \left(\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\xi'}\right)^m\left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\tau'}\right)^n G(\xi,\tau)F(\xi',\tau')\Big|_{\xi'=\xi,\,\tau'=\tau}. \tag{5}$$

Equations (3), (4) can be solved by introducing the following power series expansions for h and f:

$$h = h_0(1 + \varepsilon h_1 + \varepsilon^2 h_2 + \ldots), \tag{6}$$

$$f = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \ldots, \tag{7}$$

where ε is a formal expansion parameter. It is assumed that $f_1, f_2, \ldots, h_1, h_2, \ldots$ tend to zero at $\xi \to -\infty$. Substituting expressions (6), (7) into Eqns (3), (4) and equating coefficients of the same powers of ε to zero we can obtain the recursion relations for $f_n(\xi,\tau)$ and $h_n(\xi,\tau)$ $(n = 1, 2, \ldots)$.

3. Soliton solution for Eqn (1)

To obtain the fundamental soliton solution for Eqn (1), we assume that

$$h_0 = \mu e^\theta, \quad h_1 = -e^\vartheta, \quad f_1 = me^\vartheta, \tag{8}$$

where μ is an arbitrary complex parameter; $\theta = \mathrm{i}a(\xi) + \mathrm{i}b\tau$ and $\vartheta = k(\xi) + \omega\tau + \delta$ with functions $a(\xi)$ and $k(\xi)$ to be determined; b, ω, δ and m are real constants. Substituting $h(\xi,\tau)$ into the resulting set of linear partial differential equations, and after some calculations, we can get

$$A(\xi) = \exp\left[\int\frac{g(\xi)}{2}d\xi\right],$$

$$a(\xi) = \frac{1}{2}\int\left\{\beta(\xi)b^2 + 2\exp\left[\int g(\xi)\mathrm{d}\xi\right]\mu\bar{\mu}\gamma(\xi)\right\}\mathrm{d}\xi,$$

$$k(\xi) = b\omega\int\beta(\xi)\mathrm{d}\xi,$$

$$\gamma(\xi) = \exp\left[-\int g(\xi)\mathrm{d}\xi\right]\omega^2\beta(\xi)\Big/4\mu\bar{\mu},$$

$$m = 1, \quad \lambda = \exp\left[-\int g(\xi)\mathrm{d}\xi\right]\mu\bar{\mu}\gamma(\xi),$$

and

$$h_n(\xi,\tau) = 0, \quad f_n(\xi,\tau) = 0 \quad (n = 2,3,4,\ldots).$$

Here, $\bar{\mu}$ is the complex conjugate of μ. Without loss of generality, we set $\varepsilon = 1$. Thus, the one-soliton solution can be expressed as

$$u(\xi,\tau) = \exp\left[\int \frac{g(\xi)}{2}\mathrm{d}\xi\right]\frac{h_0(1+h_1)}{1+f_1}$$

$$= \mu\exp\left[\theta + \int \frac{g(\xi)}{2}\mathrm{d}\xi\right]\frac{1-\exp\theta}{1+\exp\theta}$$

$$= \mu\exp\left[\theta + \int \frac{g(\xi)}{2}\mathrm{d}\xi\right]\tanh\left(-\frac{\vartheta}{2}\right). \tag{9}$$

Expression (9) is the solution for a fundamental dark soliton, shown in Fig. 1a at $g(\xi) = 0$. The dark soliton is remarkably stable during its propagation. If we use the following fixed parameters: $\mu = 1$, $\delta = 4$, $\beta(\xi) = 1$, $\omega = 2$ and $b = 0.5$, than $u(\xi,\tau) = \tanh(b\xi + \tau + 2)$ and the soliton velocity depends on $-1/b$. Thus, the propagation velocity of the soliton can be affected through changing the value of b in Fig. 1a. The higher the value of b, the faster the soliton velocity. The nonlinear effects can cancel perfectly the GVD effects and neither the pulse shape nor the pulse spectrum changes along the optical fibre. Relevant issues can be found in Ref. [43].

The aforementioned is valid if $g(\xi) = 0$, i.e., the results shown in Fig. 1a describe the fundamental dark soliton in the absence of absorption/amplification. Consider now the behaviour of the soliton evolution when $g(\xi) \neq 0$. Firstly, we assume that $b = 0$. When $g(\xi) = -0.01\xi$ and all other parameters are identical to those used in Fig. 1a, we obtain the bright soliton shown in Fig. 1b. The pulse width and the wave number of the bright soliton both remain constant during its propagation along the optical fibres. The bright soliton can only be observed in the anomalous GVD regime [4]. To our knowledge, the bright soliton for Eqn (1) in the normal GVD regime is first reported in this paper.

At $g(\xi) = -0.1\cos(2\xi)$, the soliton shown in Fig. 1b periodically oscillates with ξ (Fig. 2a). The rapidly moving solitons result in multiple intensity oscillations (Fig. 2b). These oscillations become more pronounced, and the number and the amplitude of oscillations increase with increasing parameter $g(\xi)$. Therefore, the amplitude and period of the soliton can be controlled by changing $g(\xi)$, namely: the larger the coefficient and period of $g(\xi)$, the stronger the soliton amplitude and period. Besides, the soliton amplitude increases with the propagation distance. This may find applications in the soliton management communication links where the fibre absorption is compensated periodically by an amplification system. In fact, the form of $g(\xi)$ can be used to generate other types of solitons (see Figs 3–5).

Figure 3 shows the intensity profile for the bright soliton in the normal GVD regime. As can be deduced, a clean sech-shaped soliton is generated in that regime. The two solitons (Figs 1b and 3a) have almost the same power but different full

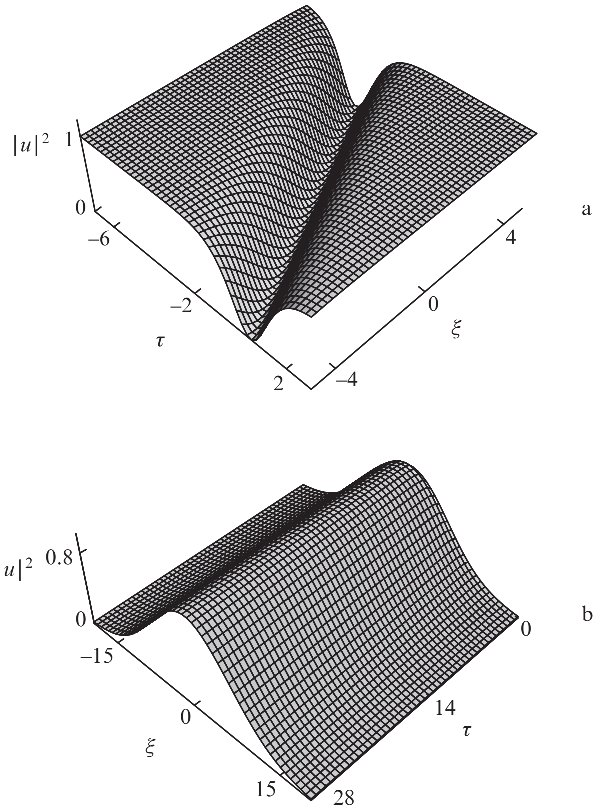

Figure 1. (a) Stable propagation of the dark solitons with $b = 0.5$ and $g(\xi) = 0$ and (b) the bright soliton with $b = 0$ and $g(\xi) = -0.01\xi$ in the nonlinear optical fibres with $\mu = 1$, $\delta = 4$, $\omega = 2$ and $\beta(\xi) = 1$.

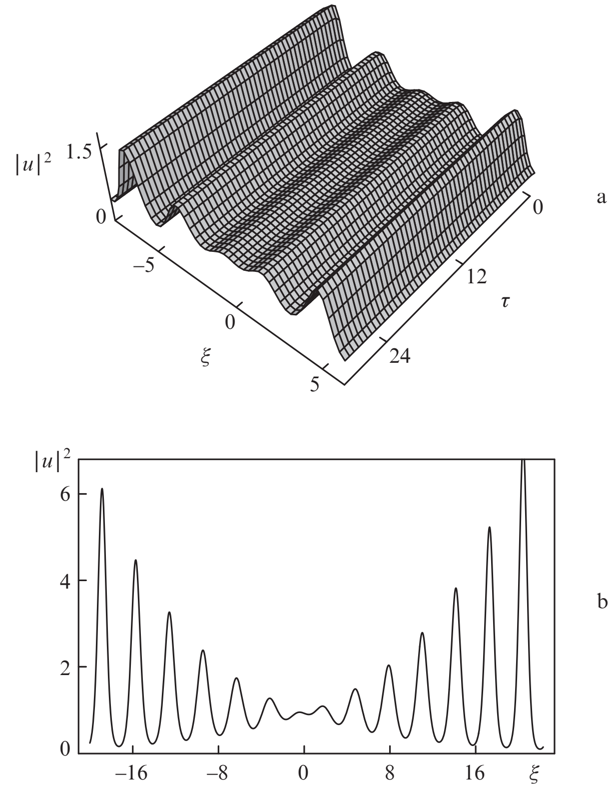

Figure 2. (a) Periodic oscillations of the soliton intensity as a function of the fibre absorption loss modulation $g(\xi)$. The parameters adopted here are $\mu = 1$, $\delta = 4$, $\beta(\xi) = 1$, $\omega = 2$, $b = 0$ and $g(\xi) = -0.1\cos(2\xi)$. (b) Cross section plot of (a).

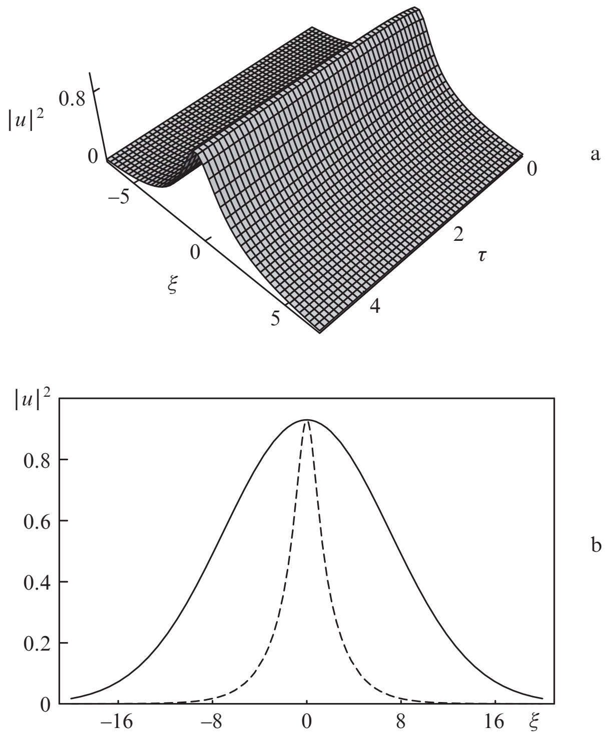

Figure 3. (a) Stable propagation of the bright soliton in the nonlinear optical fibres with $\mu = 1, \delta = 4, \beta(\xi) = 1, \omega = 2, b = 0$ and $g(\xi) = -0.5\tanh(\xi)$, (b) comparison of the intensity profile of the solitons at $g(\xi) = -0.01\xi$ (solid curve) and $g(\xi) = -0.5\tanh(\xi)$ (dashed curve).

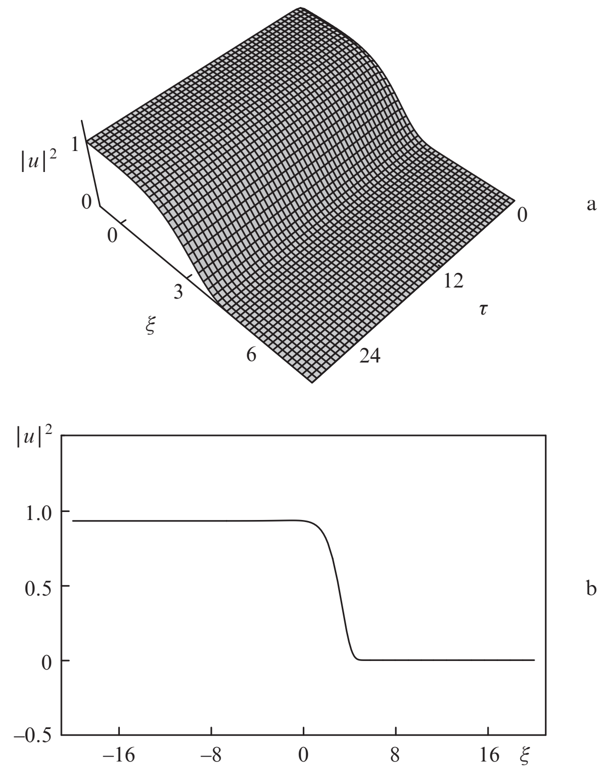

Figure 5. (a) Intensity profile of the kink-shaped soliton with the parameters $\mu = 1$, $\delta = 4$, $\beta(\xi) = 1$, $\omega = 2$, $g(\xi) = -0.01\xi$ and $b = 0$, (b) cross section plot of (a).

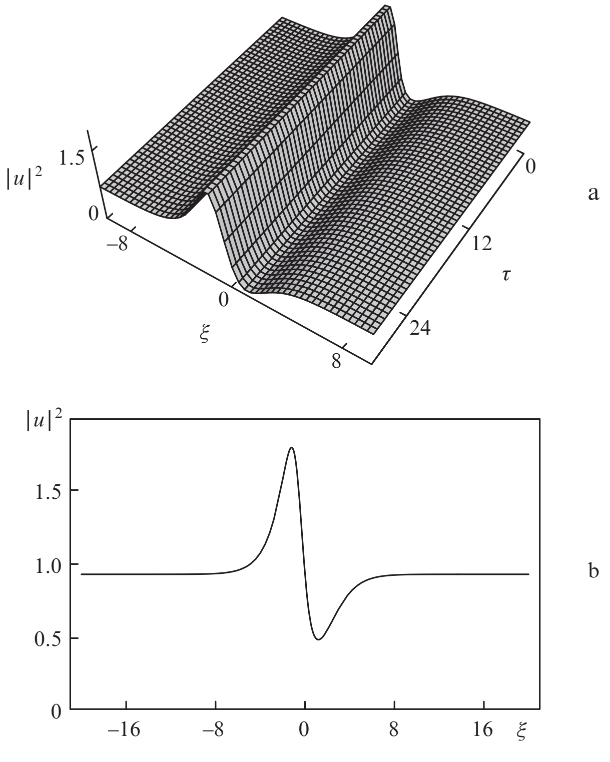

Figure 4. (a) Combined soliton with the parameters similar to those given in Fig. 3, but with $g(\xi) = -\text{sech}(\xi)$, (b) cross section plot of (a).

width at half maximum. Note that the soliton has a sech-shaped profile and is totally smooth under the condition $b = 0$. If $b \neq 0$, the soliton envelope will exhibit fluctuations. This property might be useful for the soliton compression and pulse clean-up application.

If $g(\xi) = -\text{sech}(\xi)$, the absorption drastically affects the soliton properties. The representative example of the intensity profile for the soliton is depicted in Fig. 4, which shows a combined soliton. One can see from Fig. 4 that the corresponding profiles can be considered as a combination of a bright and dark soliton. Besides, the amplitude of the solitons can be controlled by changing the parameters of $g(\xi)$. Interestingly, the combined soliton can be transmitted over unlimited distances without any distortion. This may find potential applications in the optical communication systems which produce bright and dark solitons simultaneously. When $g(\xi) = -0.01\exp(\xi)$, the combined soliton (Fig. 4) turns into the kink-shaped soliton (Fig. 5a). To our knowledge, the kink-shaped solitons for Eqn (1) have not been reported earlier.

Figure 6 shows the evolution of the fundamental dark soliton in an optical fibre with ξ-dependent GVD but constant values of $g(\xi) = 0$ for the specific case $b(\xi) = 0.1\xi$. The figure illustrates the parabolic-type evolution of the soliton and describes the non-travelling-wave features of the soliton with variable propagation velocities. We hope that these phenomena could be observed in the future nonlinear optics experiments.

Figure 7 displays another situation of the soliton propagation in the homogeneous optical fibre, when the GVD profile of the inhomogeneous optical fibre is linear ($\beta(\xi) = |18 + 18(18^{-1} - 1)\xi|$), and the soliton undergoes a fascinating change in its velocity. With changing the parameters of $\beta(\xi)$, the influence of the soliton velocity in the inhomogeneous optical fibre can be simultaneously enhanced. This phenomenon can prove important for the control of the evolution and propagation of the solitons in a real optical communication system.

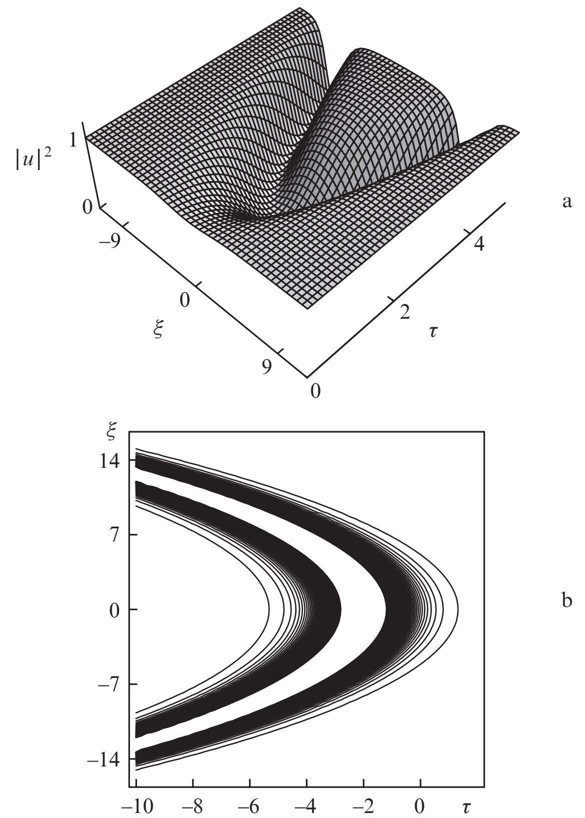

Figure 6. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) at $\beta(\xi) = 0.1\xi$. The parameters adopted are: $\mu = 1$, $\delta = 4$, $g(\xi) = 1$, $\omega = 2$ and $b = 0.5$, (b) contour plot of (a).

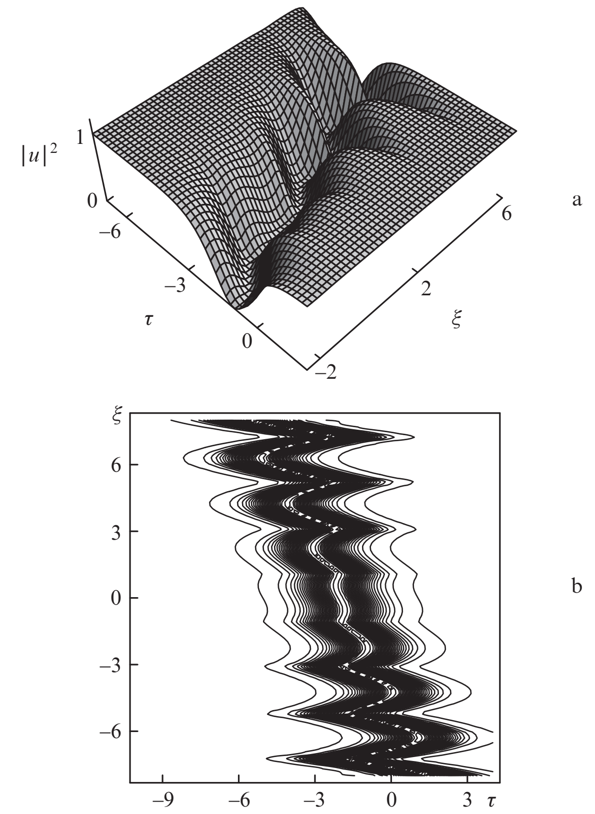

Figure 8. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) with parameters to similar those given in Fig. 6, but with $\beta = |\cos[\cos^{-1}(18^{-1})\xi]|$, (b) contour plot of (a).

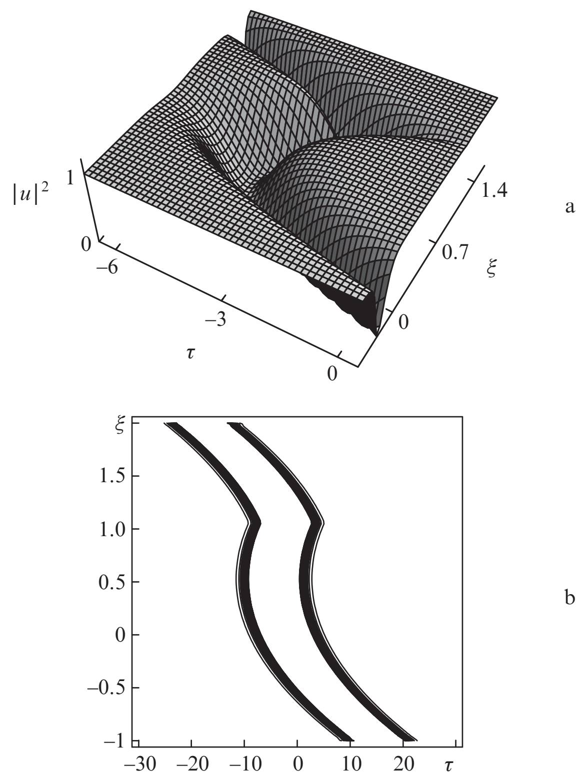

Figure 7. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) with parameters similar to those given in Fig. 6, but with $\beta(\xi) = |18 + 18(18^{-1} - 1)\xi|$, (b) contour plot of (a).

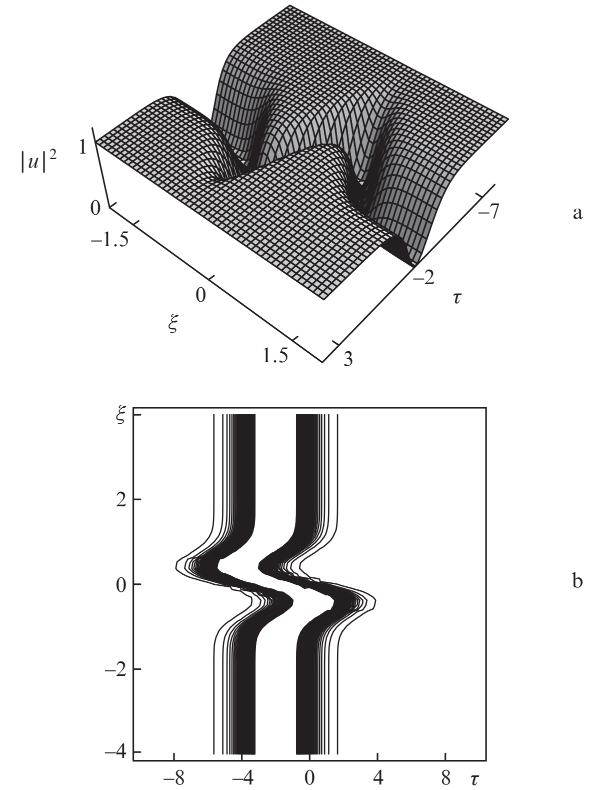

Figure 9. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) with parameters similar to those given in Fig. 6, but with $\beta(\xi) = |-18\exp[-\log(18)\xi^2]|$, (b) contour plot of (a).

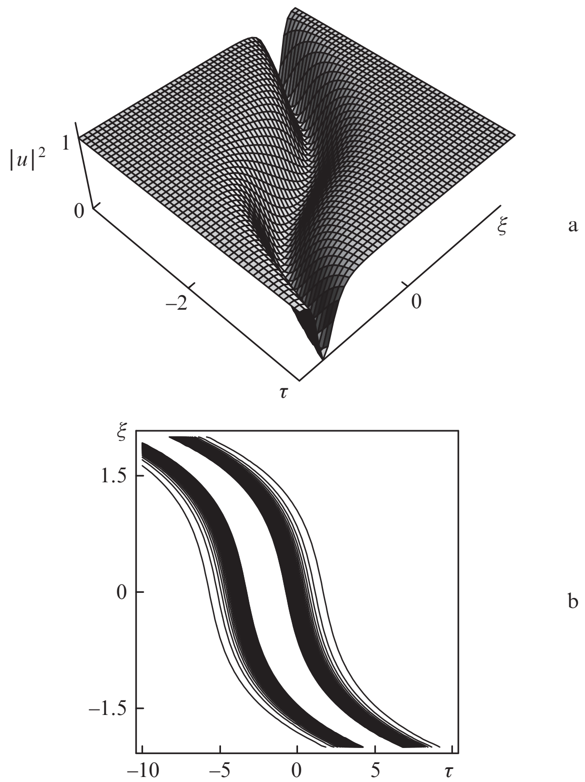

Figure 10. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) with parameters similar to those given in Fig. 6, but with $\beta(\xi) = \exp\xi + \exp(-\xi)$, (b) contour plot of (a).

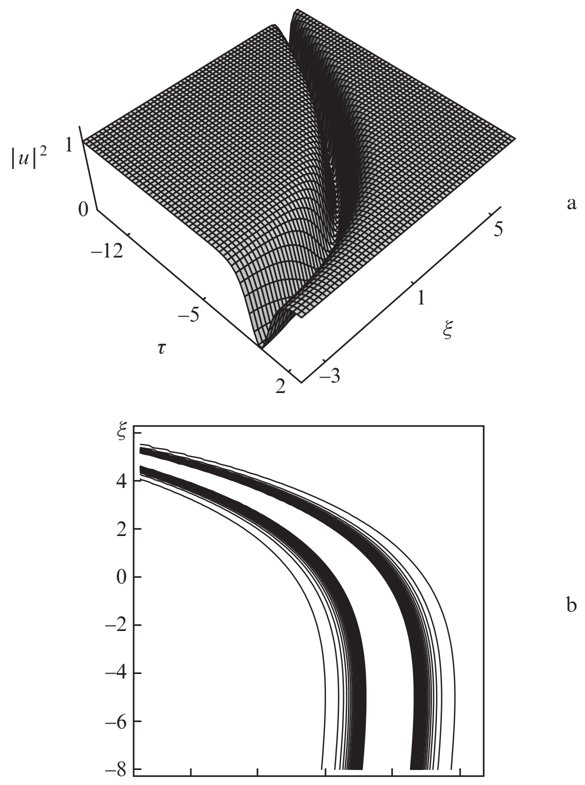

Figure 11. (a) Nonlinear evolution behaviour of the dark soliton given by Eqn (1) with parameters similar to those given in Fig. 6, but with $\beta(\xi) = \exp(0.2\xi)$, (b) contour plot of (a).

If $\beta(\xi)$ is a periodic function, namely, $\beta = |\cos[\cos^{-1}(18^{-1})\xi]|$, i.e., the dispersion profile of the inhomogeneous optical fibre is determined by the cosine, the dark soliton evolves in a periodic one. Figure 8 depicts the soliton whose structure oscillates periodically. One can clearly see from this figure that the soliton velocity changes during its propagation due to the variation in the dispersion profile of the inhomogeneous optical fibre. Using this type of an optical fibre, we may generate the periodic soliton in the soliton management system. Thus, we can conclude that we are able to control the velocity of the solitons by means of the function forms of $\beta(\xi)$ in the optical soliton communication systems.

When the dispersion profile of the optical fibres is Gaussian $\beta(\xi) = |-18\exp[-\log(18)\xi^2]|$, an S-shaped soliton is produced (Fig. 9a). On the contrary, the accelerated soliton structure is achieved (Fig. 10a) when $\beta(\xi) = \exp\xi + \exp(-\xi)$. These results indicate that we can control the soliton velocity using the optical fibres with the different dispersion profiles.

Figure 11 presents the evolution of a stable dark soliton with the accelerated structure when the dispersion profile of the optical fibres is exponential. The position of the fundamental soliton shifts because the variable GVD is imposed on it. The velocity and time shift of the soliton vary with the GVD distribution while the soliton width decreases and the dark soliton keeps its shape. A noteworthy feature is that the trajectory of the pulse centre does not follow a straight line that is the case in Fig. 1a. This can be explained if we turn to the expression $u(\xi,\tau) = \tanh[\int\beta(\xi)\mathrm{d}\xi + \tau + 2]$. This property implies that we can control solitons by controlling the GVD in the soliton management system.

4. Conclusions

To model the propagation of ultrashort optical pulses in inhomogeneous optical fibres in the normal dispersion regime, we have considered Eqn (1) with varying dispersion, nonlinearity and gain (or absorption). With the aid of symbolic computation, we have carried out our study from an analytic viewpoint. Directly applying the modified Hirota method, we have presented the bilinear form for Eqn (1). Based on the bilinear form, the analytic soliton solution has been generated. Of physical and optical interests, relevant properties of the soliton solution have been analysed and graphically discussed in details depending on the values of the parameters. Not only the dark soliton but also the bright has been observed in the normal GVD regime. To our knowledge, the bright soliton for Eqn (1) in the normal GVD regime is reported for the first time in this paper.

Moreover, we have shown that rapidly moving solitons develop the multiple intensity oscillations which become more pronounced with increasing the parameters of $g(\xi)$, which has the potential applications to the soliton management communication links where the fibre absorption loss is compensated periodically by an amplification system. Furthermore, we have derived the compressed soliton without any fluctuation for such a system under the condition $b = 0$. This property might be useful for the soliton compression and pulse clean-up application. Through changing the value of $g(\xi)$, the combined and kink-shaped solitons have been observed. Finally, using different GVD coefficients $\beta(\xi)$ describing the dispersion profiles of the DDF, we have found that the DDF profiles can be used to control the soliton velocity under certain conditions. The obtained results have certain applications in producing the bright and dark solitons simultaneously and may be meaningful and irradiative to manage the evolution and propagation of the solitons in the real optical communication systems.

Acknowledgements. This work was supported by the National Natural Science Foundation of China (Grant No. 60772023); by the Fundamental Funds for the Central Universities of China (Grant No. 2011BUPTYB02); by the Supported Project (SKLSDE-2010ZX-07) and Open Fund (SKLSDE-2011KF-03) of the State Key Laboratory of Software Development Environment, Beijing University of Aeronautics and Astronautics; by the National High Technology Research and Development Program of China (863 Program, Grant No. 2009AA043303); by the National Basic Research Program of China (Grant Nos 2005CB321901 and 2010CB923200); and by the Specialized Research Fund for the Doctoral Program of Higher Education of the Chinese Ministry of Education (Grant No. 200800130006). W.J.L. acknowledges support by the Excellent Ph.D. Students Foundation (Grant No. CX201022) from the Beijing University of Post and Telecommunications.

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