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On focusing of laser radiation with an axicon

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***Abstract*. The influence of axially symmetric perturbations of the intensity and phase of the laser beam on its focusing by means of an axicon is considered. It is shown that such perturbations give rise to variations in the radiation energy density on the axicon axis with two periods, Λ/γ and Λ^2/λ, where Λ is the period of perturbation of the laser beam intensity, and γ is the angle of convergence of the focused beam.**

Keywords: *axicon, laser beam focusing, laser-induced spark.*

1. Introduction

Focusing a laser beam by an axicon aimed at formation of high-intensity Bessel beams was first discussed in Ref. [1]. Later the work was carried out [2] aimed at producing a continuous laser gas breakdown with the longitudinal linear dimension much greater than the transverse one. However, it was found that instead of homogeneous plasma along the focon axis 'a sequence of point-dashed breakdowns is formed.' Observations of similar quasi-periodic plasma structures, obtained using axicons, were reported also by authors of other papers. A detailed review of these papers is presented in Ref. [3].

At least two theoretical models were proposed to explain this quasi-periodic structure of the plasma. In the first of them a nonlinear process of radiation self-modulation in diffraction-free laser beams in the plasma is considered [4], while in the second one the spatial modulation of the heating radiation due to the interference of the incident light and the one reflected from the plasma boundary is taken into account [5]. In the present paper we propose a simpler explanation, mainly associated with the difference of focusing properties of axicons and common lenses.

2. Focusing the laser radiation with an axicon

In the geometrical optics approximation, assuming the angle of refraction α to be small, a certain ring element of the beam with the radius r at the axicon entrance (Fig. 1) may be associated with a point on the axicon axis with the coordinate $L = r/[\alpha(n-1)] = r/\gamma$, where n is the refractive index of the axicon material; α is the angle between the generating lines of its surfaces; γ is the angle of convergence of the focused rays. Similarly, neglecting the diffraction, the width $\mathrm{d}r$ of the ring zone may be associated with the element $\mathrm{d}L = \mathrm{d}r/\gamma$ of the focal line at the axicon axis. The amount of energy, falling on the element $\mathrm{d}L$, is determined by the beam energy density $Q(r)$ in the corresponding ring element: $\mathrm{d}E = 2\pi Q(r) r \mathrm{d}r$. It follows that if $Q(r) = \mathrm{const}$ and the laser beam diameter is limited ($r \leqslant w$), then the axicon on-axis energy density will be maximal at the point $z_F = w/\gamma$, which, therefore, may be conventionally regarded as the axicon focus. In the shadow zone, i.e., at $z > z_F$, the maximum of the radiation energy density is reached off the axicon axis. For a Gaussian beam with $Q(r) = A \times \exp(-2r^2/w_{\mathrm{G}}^2)$, after performing elementary calculations, we find that the axicon focus lies at the point $z_{\mathrm{G}} = w_{\mathrm{G}}/(2\gamma)$.

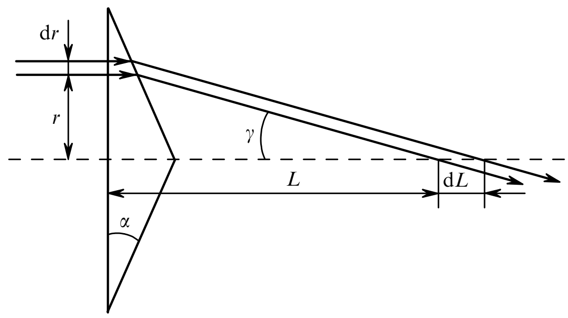

Figure 1. To the calculation of the radiation energy density on the axicon axis.

The length of the focal spot along the axicon axis can also be estimated within a purely geometrical approach. Assuming that the boundaries of the focal spot correspond to the decrease in the radiation energy density by 10 % of the maximal value, for a Gaussian beam we obtain $\Delta z_{\mathrm{G}} = 0.32 w_{\mathrm{G}}/\gamma$. For an axicon with $\alpha = 1^{\circ}$, $n = 1.5$ in the case of a Gaussian beam with the radius $w_{\mathrm{G}} = 1.1$ cm this yields $\Delta z_{\mathrm{G}} \approx 40$ cm at $z_{\mathrm{G}} \approx 62$ cm. For a lens with the focal length $F = 62$ cm, the beam waist length (Rayleigh length) will equal only 0.2 cm in this case.

The presented values of the position and the length of the focal region of the axicon, calculated within the approximation of geometrical optics, agree well with the results of diffraction numerical calculation using the pro-

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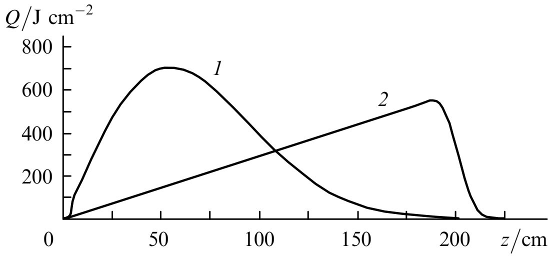

Figure 2. The radiation energy density on the axicon axis for a Gaussian beam (*1*) and a flat-top beam (*2*). The axicon and beam parameters are given in the text.

gramme 'Fresnel' (Fig. 2) for a Gaussian beam with the radius $w_G = 1.1$ cm and a flat-top beam with the diameter $2w = 4$ cm (the scale of smoothing at the edges is 0.1 cm).

The only aim of all above statements is to emphasise that, while for an ordinary lens the field in the focal region is determined by the field in the entire entrance aperture, for an axicon the radiation energy density at each point along the optical axis depends mainly on the field magnitude in the corresponding ring zone. This fact allows one to make use of the geometrical optics for calculating the vector field behind the axicon as well [6]. Therefore, axially symmetric perturbations of the intensity or phase of the laser beam, introduced at the axicon entrance, can nothing to do but affect the distribution of the radiation energy density along its axis.

3. Axially symmetric perturbations of the radiation intensity and phase

The diffraction calculations, the results of which are presented in this Section, were performed for a laser beam with flat top, having the diameter $2w = 4$ cm, the smoothing at the edges with the scale 0.1 cm and the pulse energy 1 J at the radiation wavelength 1.06 μm. The calculations were carried out for a glass axicon with $n = 1.5$ and $\alpha = 1°$.

3.1 The influence of intensity modulations

The energy density at the axicon entrance was given by the function

$$Q(r) = F(r)\left[1 + m\cos\left(\frac{2\pi}{\Lambda(r)}r\right)\right], \tag{1}$$

where $F(r)$ is the intensity distribution (assumed standard in the 'Fresnel' programme) for a flat-top laser beam; m is the intensity modulation depth; $\Lambda(r)$ is the period of radial perturbations, which in the case of diffraction by apertures is usually variable. The calculation was carried out both for the constant $\Lambda(r) = \Lambda_1 = 0.1$ cm and for $\Lambda(r) = \Lambda_2 = 0.5(1 + r)$ (in cm). The value of m was chosen within the interval 0.01 – 0.33.

The radiation energy density distribution along the z axis of the axicon $Q(z)$ at the period Λ_1 and $m = 0.08$ is shown in Fig. 3a [curve (*1*)]. The distortions, caused by the input beam intensity modulation, are more clearly reflected by the quantity

$$\frac{\mathrm{d}Q}{Q_0} = \frac{Q(z) - Q_0(z)}{Q_0(z)}, \tag{2}$$

where $Q_0(z)$ is the radiation energy density distribution along the axicon axis for the unperturbed beam. The dependence $\mathrm{d}Q(z)/Q_0$ at constant Λ_1 and $m = 0.08$ is shown in Fig. 3b, and for the variable period Λ_2 and $m = 0.08$ in Fig. 3c. In both cases the maximal relative amplitude of oscillations with the period $\Lambda(z) \approx \Lambda_{1,2}/\gamma$ is almost exactly equal to the modulation depth m. This relation holds in the whole range of m values (0.01 – 0.33).

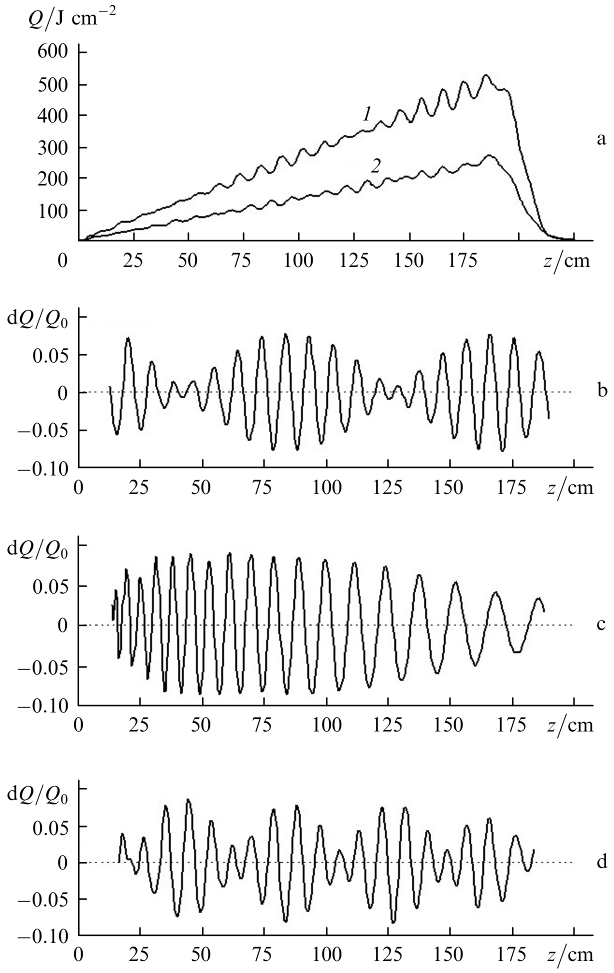

Figure 3. The energy density at the axicon axis $Q(z)$ in the presence of axially symmetric perturbations of the intensity of the flat-top beam (a) and its variation $\mathrm{d}Q(z)/Q_0$ at $m = 0.08$ (b – d) for $\Lambda_1 = 0.1$ cm (b, d), $\Lambda_2 = 0.5(1 + r)$ (c), $\lambda = 1054$ [(*1*); b, c] and 2108 μm [(*2*); d].

Note, that the dependence in Fig. 3b, alongside with the oscillations having the fundamental period Λ_1/γ, exhibits also ancillary modulation of the radiation energy density $Q(z)$ with a somewhat greater period.

The expansion of an ideal Bessel beam in plane waves is represented in the ***k***-space by a ring with the wave vector projection $k_z \equiv (2\pi/\lambda)\cos\gamma$. For the beam having form (1) in the absence of modulation on the axicon axis we have a quasi-Bessel beam, whose expansion contains the harmonics with the spatial frequencies, determined only by the aperture shape. The diameter limitation and the sharpness of the beam boundary in this case result in the broadening of the spectrum k_z, not manifesting itself unless in the character of fall-off of curve (*1*) in Fig. 2 at $z \approx 200$ cm. The small singularity near $z = 0$ is explained by the impossibility to describe the vertex of the conical surface in the calculation with finite discreteness*. The same problem arises in axicon manufacturing [7].

Due to the periodic perturbation of the intensity (or phase), two more beams appear behind the axicon with the projections of the wave vectors $k_z' = (2\pi/\lambda)\cos(\gamma - \lambda/\Lambda)$ and $k_z'' = (2\pi/\lambda)\cos(\gamma + \lambda/\Lambda)$ on the z axis. The interference of the three beams with k_z', k_z'', and k_z gives rise to the modulation on the axicon axis with the fundamental (Λ/γ) and ancillary (Λ^2/λ) periods. At the calculation parameters, corresponding to Fig. 3b, the ratio of these periods is $\lambda/(\gamma\Lambda_1)\approx 0.116$.

The dependence of the ancillary modulation period upon the radiation wavelength is demonstrated by Fig. 3d, which illustrates the result of calculation at the laser radiation wavelength two times greater [$\lambda/(\gamma\Lambda_1)\approx 0.232$] than in Fig. 3b. To a certain extent, the ancillary interference period manifests itself also in Fig. 3c at the variable period Λ_2.

The variation in the wavelength allows the demonstration of one more essential difference in focusing radiation using a lens and an axicon. As known, the maximal intensity is proportional to λ^{-2} in the focus of a lens and to λ^{-1} in the focus of an axicon (Fig. 3a). The principal maximum radius in both cases is proportional to the wavelength. Therefore, for a lens the amount of energy in the central maximum is constant and equals $\sim 86\,\%$, while for an axicon it varies proportionally to λ.

3.2 Effect of phase modulations

The modulation of phase in our calculations was described by a phase screen

$$\Phi(r) = \mathrm{i}\,\frac{\psi}{2}\cos\left(\frac{2\pi}{\Lambda(r)}r\right), \tag{3}$$

where ψ is the difference between the maximal and the minimal phase deviations expressed in wavelengths. The quantity ψ was varied from $\lambda/250$ to $\lambda/5$. The dependences $Q(z)$ and $\mathrm{d}Q(z)/Q_0$, obtained in calculations, are completely analogous to those presented in Fig. 3. At the phase modulation with the variable period Λ_2 one can also trace in the dependence $Q(z)$ both the correspondence of the period of the energy density modulation to that of the perturbation and the interference effects (Figs 4a and b). In this case in the chosen range of ψ the approximate equality $\max(\mathrm{d}Q/Q_0)\approx 5.5\psi$ holds, i.e., the phase perturbations $\psi \geqslant \lambda/5.5$ at some z cause almost a 100 % increase or decrease in the energy density on the axicon axis in comparison with the corresponding values of $Q_0(z)$ (Fig. 4a).

The calculation was also carried out at random fluctuations of the phase within the beam aperture. The dependence $\mathrm{d}Q(z)/Q_0$ for phase fluctuations $\pm\lambda/20$ and the correlation radius 0.1 cm is shown in Fig. 4c. It is seen that although the variations in the energy density with respect to $Q_0(z)$ can be also observed in this case, they are almost two orders of magnitude smaller than for analogous axially symmetric perturbations.

Axially symmetric periodical perturbations essentially change the radial distribution of the radiation energy density on the axicon axis. While in the ideal case it is the Bessel function J_0 (Fig. 5a), in the presence of perturbations the distributions $Q(r)$ are not Bessel functions for any z. Moreover, as shown in Figs 5b and c, they appear to be different in the vicinity of maxima and minima of the curve $Q(z)$.

* The calculation was performed on a discrete mesh with 2048 × 2048 points.

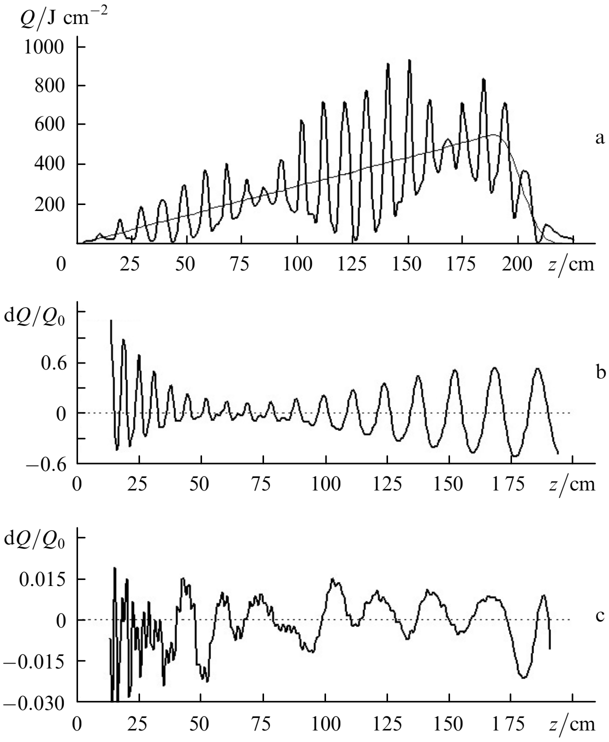

Figure 4. The energy density at the axicon axis $Q(z)$ at perturbations of the phase $\psi = \lambda/10$ of a flat-top beam (a) and its variations $\mathrm{d}Q(z)/Q_0$ (b, c): axially symmetric perturbations with periods $\Lambda_1 = 0.1$ cm (a) and $\Lambda_2 = 0.5(1 + r)$ (b), as well a random perturbations with the correlation radius 0.1 cm (c).

4. Discussion of the results

As follows from the performed calculations, the axially symmetric perturbations of the intensity and phase of the laser beam lead to the appearance of variations in the radiation energy density on the axicon axis with two periods, namely, the fundamental period (Λ/γ) and the ancillary one (Λ^2/λ). In the laser gas-breakdown experiments using the axicons [3] the presence of two periods (differing nearly by an order of magnitude) in the structure of spark plasma is also observed. Moreover, as mentioned in Ref. [8], using the same axicon to focus the radiation in different laser setups keeps the spark structure unchanged. Since, as a rule, the intensity perturbations are associated with the laser beam itself, it is hardly possible that they could be responsible for the observed structure of the spark. Taking the result of Ref. [8] into account, one can say the same about the phase perturbations of the laser beams.

Thus, the observed structure of the laser spark is possibly due to the phase perturbations, related to the deformation of the conical surface of the axicon. The type of this surface itself may give rise to axially symmetric shape perturbations in the course of its manufacturing. Even if these perturbations are nonperiodic, one can always extract the fundamental modulation frequency or its harmonics. As to the calculations, they show that even for

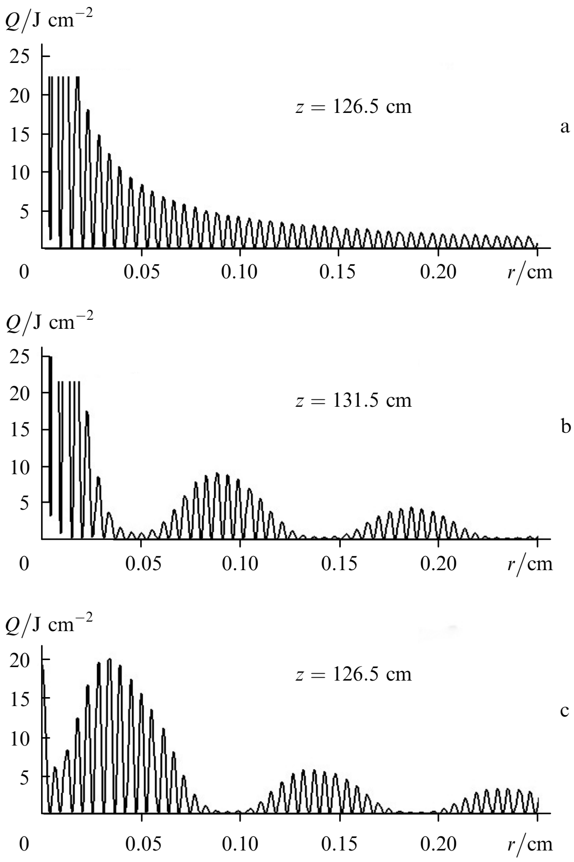

Figure 5. Radiation energy density $Q(r)$ in the vicinity of the axicon axis without (a) and with (b, c) phase perturbations near the minimum (a, c) and the maximum (b) of the function $Q(z)$. In Figs 5a and b the maximal values of $Q(r)$ are not shown.

axially symmetric periodic phase perturbations $\sim\lambda/10$, i.e., for the precision of manufacturing and testing conventional only for flat or spherical surfaces, the modulation of the radiation energy density at the axis of the axicon may exceed 50 %.

5. Conclusions

The validity of the assumption made in the present paper about the role of the axicon surface quality in the formation of the spark structure, when focusing the beam with an axicon, may be checked experimentally either by direct measurements of the radiation energy density distribution $Q(z)$ or by studying the periodic structure arising in the radial distribution $Q(r)$. One may also compare the spark structures, obtained for two wavelengths using the same axicon. According to Refs [4, 5], the period of the spark structure should vary as λ/γ^2. In our case (at least until $\Lambda \gg \lambda$) one of the periods (Λ/γ) does not depend on the radiation wavelength, while the other one is equal to $\sim\Lambda^2/\lambda$.

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