

PACS numbers: 42.65.Sf; 42.70.Nq
DOI: 10.1070/QE2011v041n01ABEH014463

Autowaves in two-wave mixing in photorefractive media

P.A. Prudkovskii

***Abstract*. The phase part of the system of equations describing two-wave mixing in a photorefractive strongly inertial medium is studied analytically and numerically. It is shown that the solution of the system of equations evolves through a series of quasi-stationary states, and the system switches between them due to a nonlinear wave. The velocity and profile of such a 'switching wave' are completely determined by these states, which is an indication of an autowave process. The results show that the development of four-wave mixing in a strongly illuminated photorefractive medium is inevitably accompanied by intensity fluctuations.**

Keywords: *photorefraction, wave mixing, autowaves.*

1. Introduction

We call the medium photorefractive if its refractive index varies under the action of light. Conventionally, this term is used if the refractive index varies slowly enough and it is inappropriate to describe light propagation in the medium by means of a phenomenological tensor of cubic susceptibility neglecting the medium lagging. A reason for so slow variations is usually an electron density redistribution in a crystal, which results in arising a static electric field modulating the refractive index. In the scale of nonlinear optics, a photorefractive nonlinearity may have a giant response time: in lithium niobate crystals doped with copper or iron atoms, the characteristic Maxwellian time τ_M may be seconds, hours, and even months [1, 2].

An analytical description of wave mixing in such media is similar to that in a media with a conventional cubic nonlinearity only in a stationary case. Nevertheless, even in stationary conditions, an exact solution for an interaction of three and more modes of electromagnetic radiation in a nonlinear medium can only be obtained in specific cases [3, 4]; hence, the solution is often limited to a numerical calculation of the model behaviour.

A description of the dynamics of wave mixing necessitates taking into account the non-instantaneity of the medium response to the electromagnetic field, which considerably complicates the problem. In this case, the system of ordinary differential equations for slowly varying amplitudes of field modes transfers to a system of partial differential equations. From the mathematical point of view, a similar problem arises in considering propagation of limiting short light pulses in a nonlinear medium [5].

For the last 30 years, specific features of two-wave mixing in photorefractive media were repeatedly studied in various experimental conditions [4, 6 – 11]. In a series of experiments with photoinduced light scattering (PILS) it was shown that at moderate intensity of light fields the efficiency of energy transfer from one light mode to another increases gradually and monotonically [7]. However, at increasing the light intensity, the efficiency of energy exchange between modes fluctuates in a sufficiently complicated and unpredictable manner. For example, in [9], two light beams with the intensities differing by three orders in magnitude were mixed in a lithium niobate crystal doped with copper atoms (it was placed in an electrolyte to suppress surface fields) and the intensity of the weak beam behaved as shown in Fig. 1a. Numerous attempts were taken to explain such fluctuations by electric discharges arising inside a photorefractive crystal or by other parasitic effects. However, a numerical investigation of two-wave mixing in a photorefractive medium shows [12] that such fluctuations arise even if noise sources are absent (see Fig. 1b). This study is aimed at analytical investigation of the mechanism of arising instabilities in such a system.

2. Phase subsystem dynamics

In the framework of the classical model for two-wave mixing in a photorefractive medium the equations for mode amplitudes in the mentioned experimental conditions have the form [4, 10, 13, 14]:

$$\frac{\partial E_1}{\partial x} = \mathrm{i}g\mathscr{E}E_2,$$

$$\frac{\partial E_2}{\partial x} = \mathrm{i}g\mathscr{E}^*E_1, \qquad (1)$$

$$\tau_M \frac{\partial E}{\partial t} + \mathscr{E} = \gamma E_1 E_2^*.$$

An interference of the two modes of the electromagnetic field E_1 and E_2 results in that the refractive index is modulated by an electrostatic field $\mathscr{E}$, i.e., the volume diffraction grating, on which the energy redistributes over

P.A. Prudkovskii Department of Physics, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119991 Moscow, Russia; e-mail: vysogota@gmail.com

Received 12 October 2010
Kvantovaya Elektronika **41** (1) 30 – 33 (2011)
Translated by N.A. Raspopov

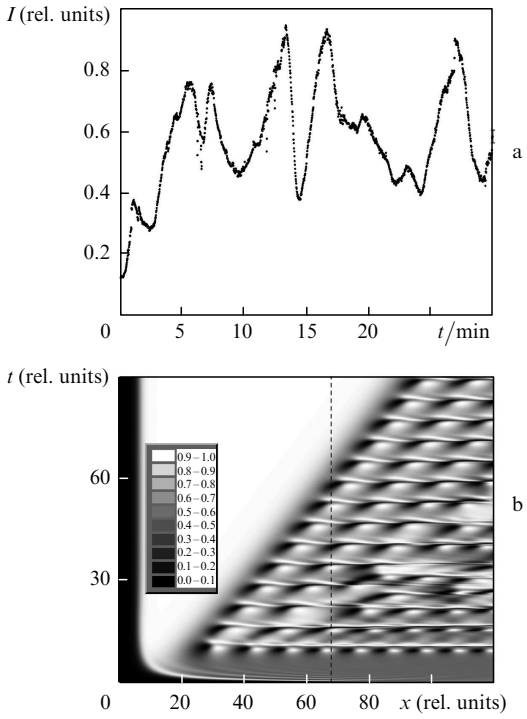

Figure 1. Two-wave mixing in a photorefractive medium: the experimental intensity of a weak light beam versus time upon two-wave mixing in a photorefractive crystal $LiNbO_3$:Cu [9] (the intensity of the strong light beam is $I = 20$ mW) (a) and numerical solution of system (1) – the intensity of a signal light beam versus time and crystal length (the dashed curve corresponds to Fig. 1a) [12] (b).

the field modes, gradually increases. The factors g and τ_M only determine the interaction scale and we will escape those by making the substitution $\tilde{t} = t/\tau_M$ and $\tilde{x} = gx$. In (1), $\gamma = \beta/\sigma$, where $\gamma = \beta/\sigma$ is the medium conductance and β is a component of the photovoltaic tensor [15, 16] that is just responsible for arising photorefraction. Taking into account that in the general case a photovoltaic tensor has both real and imaginary parts we can write $\gamma = be^{i\delta}$.

These combined equations neglect the dependence of the photoconductance on coordinates (which is justified if the intensity of one light beam is much greater than of the other) and possible influence of the fields of surface charges, which arise if the crystal is in the 'open' state [17].

In view of the energy conservation law $|E_1|^2 + |E_2|^2 = I_0 = \text{const}$ and indefiniteness of the total phase of the electromagnetic field, combined equations (1) present four real equations, for which we have no method of solution. The direction of energy transfer is determined by the phase relationship for the electromagnetic field $E_k = \sqrt{I_k}e^{i\varphi_k}$ and electrostatic field $\mathscr{E} = Fe^{i\chi}$. In what follows, we will focus on just the phase part of system (1). Let us separate the two equations describing evolution of the phases $\varphi = \varphi_2 - \varphi_1$ and $\psi = \chi - \delta$:

$$\psi_t = -A\sin(\psi + \varphi),$$
$$\varphi_x = -B\sin(\psi + \varphi - \alpha), \tag{2}$$

where $A = b\sqrt{I_1 I_2}/F$; $B = F(I_1 - I_2)/\sqrt{I_1 I_2}$; and the subscripts are responsible for a partial derivative with respect to the corresponding variable. The parameter $\alpha = \pi/2 - \delta$, which determines the relation between the imaginary and real parts of photovoltaic tensor, is a substantial characteristics of the photorefractive response. It is well known [18, 19] that a stationary energy exchange is impossible on a non-shifted diffraction grating, which arises in the case of a real photovoltaic tensor ($\delta = 0$). On the other hand, in the case of a purely imaginary photovoltaic tensor ($\delta = \pi/2$) or diffusion mechanism of photorefraction, i.e., in the case of the grating shifted by a quarter wavelength relative to the interference pattern, combined equations (1) reduce to the sin-Gordon equation with attenuation [12, 14]. We, however, will consider the general case of an arbitrary shift of the diffraction grating $0 \leqslant \alpha \leqslant \pi/2$.

Although the coefficients A and B, which are combinations of amplitudes, are far from constant it is reasonable to assume that the phase variations much faster change the character of energy exchange qualitatively than the amplitude variations do (except for a specific case of equal intensities of the field modes). Hence, we may consider a dynamics of system (2) in the approximation of constant coefficients A and B. Below, we will show that the switching autowave arises in such a system, which is partially similar to the overthrow wave considered in 1937 in the pioneer work by Kolmogorov, Petrovskii, and Piskunov [20]: the form and shape of the autowave are only determined by the states, between which the swtching occurs.

3. Quasi-stationary states and a switching wave

3.1 Quasi-stationary solution of combined equations (2)

At $\alpha \neq 0$, the phases φ and ψ cannot be constant simultaneously in time and space. It is yet possible to find a solution for which the sum $\psi + \varphi = d$ is constant. By integrating both equations (2) and comparing them with each other, we obtain the form of such a solution, which we will denote by subscript 'q':

$$\psi_q(x,t) = -At\sin d + Bx\sin(d-\alpha) + d + C,$$
$$\varphi_q(x,t) = At\sin d - Bx\sin(d-\alpha) - C. \tag{3}$$

The values constants d and C can be found from a boundary or initial conditions. The phase difference for modes of the electromagnetic field prior to entering the crystal remains constant, which gives the boundary condition $\varphi(0,t) = \varphi_0 = \text{const}$, from which follows $d_1 = 0; \pi$. On the other hand, the initial phase of the diffraction grating determines the initial condition $\psi(x,0) = \psi_0 = \text{const}$, which entails $d_2 = \alpha; \pi - \alpha$. Thus, initially the solution is formed, which is constant with respect to x and varies in time $\psi_2(x,t) = \psi_0 - At\sin\alpha$, $\varphi_2(x,t) = d_2 - \psi(x,t)$. Then the solution is changed to the stationary solution

$$\varphi_1(x,t) = \varphi_0 - Bx\sin\alpha,$$
$$\psi_1(x,t) = d_1 - \varphi(x,t). \tag{4}$$

One solution is changed to another through a passing switching wave. In Fig. 2, the result of numerical calculation of combined equations (2) is shown, which demonstrates such a transition process.

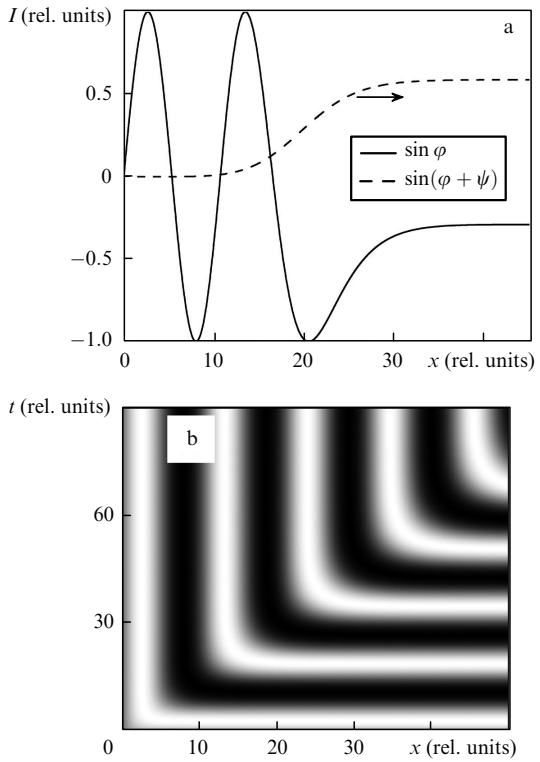

Figure 2. A numerical solution of system (2) at $\alpha = 0.2\pi$ – the switching between the quasi-stationary solutions $\varphi_2 + \psi_2 = d_2 = \alpha$ and $\varphi_1 + \psi_1 = d_1 = 0$: the switching wave at the instant $t = 20$, the arrow shows the direction of switching wave propagation (a) and two-dimensional dependence of $\sin\varphi$ on time and coordinate (b).

3.2 Stability analysis

Prior to studying the switching wave, it is necessary to specify which of two possible values for the constants d_1 and d_2 are realised. For this purpose, we will analyse stability of solution (3). Let us introduce small deviations $\psi = \psi_q + \varepsilon$ and $\varphi = \varphi_q + \rho$ and linearise combined equations (2) with respect to them:

$$\varepsilon_t = -A(\varepsilon + \rho)\cos d, \qquad \rho_x = -B(\varepsilon + \rho)\cos(d - \alpha). \tag{5}$$

Assuming a harmonic dependence of small deviations $\varepsilon, \rho \sim \exp(i\omega t - ikx)$, we obtain

$$\omega = \frac{Ak\cos d}{B\cos(d - \alpha) - ik}. \tag{6}$$

From the stability condition we have $\mathrm{Im}\,\omega > 0$, i.e., the stable solutions are $d_1 = 0$ and $d_2 = \alpha$.

3.3 Modification of the initial condition

As was shown, the stationary solution (4) is realised in some time in a system, which corresponds to the boundary condition $\varphi(0, t) = \varphi_0$. But, if we make allowance for the fact that the coefficient B, which plays the role of a spatial scale for the system, is not, generally speaking, constant then this solution cannot be considered complete. In changing the coefficient B, the system leaves the stationary state and a spatial distribution of the phase of an electrostatic field $\psi(x)$ will become a new initial condition $\psi(x, 0) = \psi_0 + B_{\mathrm{old}} x \sin\alpha = \psi_0 + B_{\mathrm{new}} x \sin\gamma$. To this initial condition corresponds the quasi-stationary solution (3) with the parameter $d_3 = \alpha + \gamma$; $\pi + \alpha - \gamma$:

$$\psi_3(x, t) = \psi_0 + Bx\sin\gamma - At\sin(\gamma \pm \alpha), \qquad \varphi_3(x, t) = d_3 - \psi(x, t). \tag{7}$$

It is important that under the condition $\pi/2 - \alpha < \gamma < \pi/2$, the solution is unstable at both values of the parameter d_3.

Unstable character of the solution is revealed in that several sharp oscillations periodically occur in the system, then the system returns again to the previous state for a certain time lapse. Then oscillations arise again and so on until the switching wave reaches this particular place and transfers the system into state (4). In Fig. 3, the results are shown of a numerical solution of combined equations (2) for this case, which reveal both the phase break resulting from lost stability for the solution with $d_3 = \alpha + \gamma$ and the switching wave.

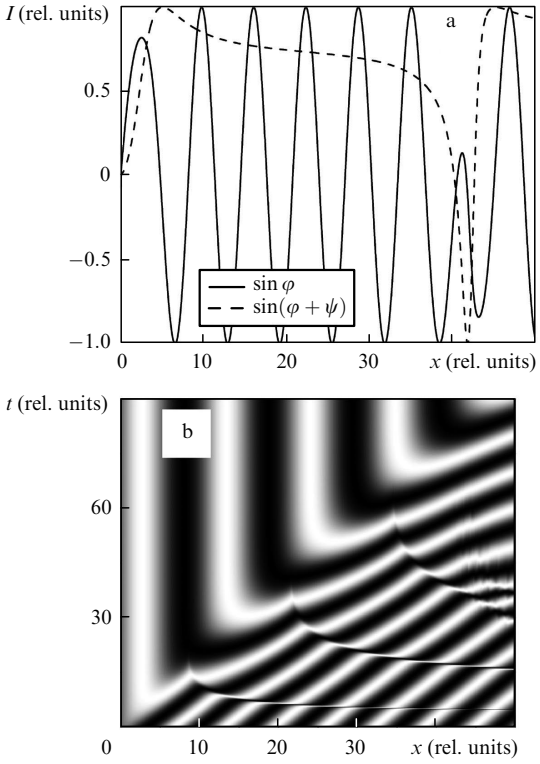

Figure 3. A numerical solution of system (2) at $\alpha = 0.2\pi$, $\gamma = 0.48\pi$ – the switching between the quasi-stationary solutions $\varphi_3 + \psi_3 = d_3 = \alpha + \gamma$ and $\varphi_1 + \psi_1 = d_1 = 0$: the switching wave $(0 < x < 15)$ and oscillation caused by the instability $(30 < x < 50)$ at the instant $t = 3.3$ min (a) and two-dimensional dependence of $\sin\varphi$ on time and coordinate (b).

3.4 Switching wave

Let us search for a switching wave in the form $\psi + \varphi = f(x - Vt)$. Then from (2) follows the equation for the function f:

$$Vf' = A\sin f - BV\sin(f - \alpha) - C. \tag{8}$$

The wave velocity and constant C are determined by the states with the parameters d_1 and d_3; between these states the switching $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to\infty} f(x) = \alpha + \gamma$ occurs:

$$V = A\cos\left(\frac{\alpha+\gamma}{2}\right)\Big/ B\cos\left(\frac{\alpha-\gamma}{2}\right),$$
$$C = BV\sin\alpha. \tag{9}$$

By integrating (8) we obtain the shape of the switching wave

$$f(x) = \frac{d_3}{2} + 2\arctan\left[\tan\frac{d_3}{4}\tanh\left(4Bx\tan\frac{d_3}{2}\sin\alpha\right)\right]. \tag{10}$$

It is shown in Fig. 4 that this expression well describes a shape of the switching waves obtained from a numerical solution of combined equations (2) (see Figs 2a and 3a).

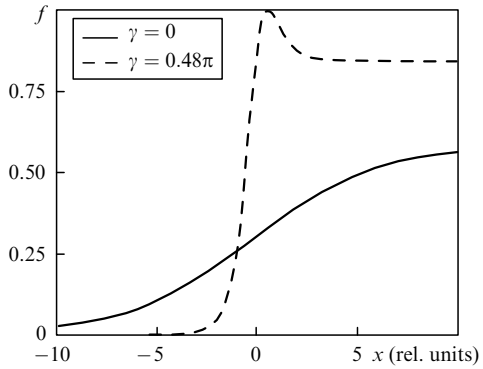

Figure 4. Exact shape of the switching wave (10) with the parameters, corresponding to those of the numerical solution from Fig. 2a (solid curve) and from Fig. 3a (dashed curve).

4. Conclusion

Thus, in the present study we investigated the subsystem of equations (2), which describes evolution of the phase difference of optical fields φ and the phase of electrostatic field ψ in two-wave mixing in a photorefractive crystal. In view of the results obtained, the solution for combined equations (1) found numerically in [12] (see Fig. 1b) can be written in the approximation of alternately changing phases and amplitudes. At the initial stage, quasi-stationary phase distribution (3) is formed with the parameters $d_2 = \alpha$, which, after passing a switching wave, is changed to solution (4). However, changes in the amplitudes of the volume diffraction grating and electromagnetic field modes force the coefficients A and B to change, which results in forming solution (7) in the system. After a recurrent switching wave passes, the latter solution is again changed to solution (4) and so on. Additional fluctuations arise due to instabilities, which develop in the quasi-stationary conditions of type (7). This continues until the amplitude distribution of the electrostatic field and light beams is formed, which corresponds to the stationary solution of combined equations (1) under the condition of constant sum for the phases $(\psi + \varphi = d_1 = 0)$:

$$A = 1,$$
$$B(x) = -I_0 b\tanh(I_0 bx\cos\alpha - \theta_0), \tag{11}$$

where the initial phase θ_0 is determined by the boundary condition for the relation between the intensities of electromagnetic field modes at an input face of the crystal. It is clear now, why fluctuations of the efficiency of energy exchange are only observed at a sufficiently high total intensity of light beams: whereas the coefficient A determining the time scale of the problem is actually independent of I_0, the coefficient B is proportional to the total light intensity. In other words, the spatial scale of the problem reduces at greater intensity I_0, and the effective length of a photorefractive crystal increases.

It worth noting that concurrently with the two-wave mixing, PIRS may develop in a crystal thus absorbing a part of the light intensity and affecting the phase of light beams. However, if the intensity of the weak beam is well above that of seed PIRS radiation [7, 9] then the two-wave mixing develops faster and noisy holographic PIRS gratings cannot compete with the grating, on which the two-wave mixing occurs.

Hence, the investigation of phase subsystem (2) helps explaining important features of the dynamics of two-wave mixing in a photorefractive medium and demonstrates an interesting mechanism of arising autowave solutions.

Acknowledgements. The study was supported by the Russian Foundation for Basic Research (Grant No. 08-02-00555-a).

References

1. Pashkov V.A., Solov'ev N.M., Uyutkin E.M. *Fiz. Tverd. Tela*, **21**, 1879 (1979).
2. Sommerfeldt R., Rupp R.A., Vormann H., Kratzig E. *Phys. Stat. Sol. A*, **99**, K15 (1987).
3. Cronin-Golomb M., White J.O., Fisher B., Yariv A. *Opt. Lett.*, **7,** 313 (1982).
4. Novicov A., Odoulov S., Oleinik O., Sturman B. *Ferroelectrics*, **66**, 1 (1986).
5. Leblond H., Sazonov S.V., Mel'nikov I.V., Mihalache D., Sanchez F. *Phys. Rev. A*, **74**, 063815 (2006).
6. Salamo G.J., Miller M.J., Clark III W.W., Wood G.L., Sharp E.J., Neurgaonkar R. *Appl. Opt.*, **27**, 4356 (1988).
7. Zabrodin K.N., Penin A.N. *Kvantovaya Electron.*, **18**, 622 (1991) [*Sov. J. Quantum Electron.*, **21**, 565 (1991).
8. Pedersen H.C., Johansen P.M. *J. Opt. Soc. Am. B*, **12**, 1065 (1995).
9. Prudkovskii P.A., Penin A.N. *Pis'ma Zh. Eksp. Teor. Fiz.*, **70**, 660 (1999) [*JETP Letters*, **70**, 673 (1999)].
10. Goulkov M., Odoulov S., Woike Th., Imbrock J., Imlau M., Kratzig E., Bäumer C., Hesse H. *Phys.Rev. B*, **65**, 195111 (2002).
11. Schwalenberg S. *Phys. Rev. E*, **71**, 066608 (2005).
12. Prudkovskii P.A. *Pis'ma Zh. Eksp. Teor. Fiz.*, **77**, 421 (2003) [*JETP Letters*, **77**, 353 (2003)].
13. Prudkovskii P.A., Skugarevskii O.V., Penin A.N. *Vestnik Mosk. Univ., Ser. Fiz. Astr.*, **5**, 38 (1998).
14. Bugaychuk S., Kovacs L., Mandula G., Polgar K., Rupp R.A. *Phys. Rev. E*, **67**, 046603 (2003).
15. Glass A.M., von der Linde D., Negran T.J. *Appl. Phys. Lett.*, **25**, 233 (1974).
16. Sturman B.I. *Kvantovaya Electron.*, **7**, 483 (1980) [*Sov. J. Quantum Electron.*, **10**, 276 (1980)].
17. Osipov Yu.M., Sturman B.I. *Opt. Commun.*, **79**, 345 (1990).
18. Zel'dovich Ya.B. *Kr. Soobshch. Fiz. FIAN*, **5**, 20 (1970).
19. Vinetskii V.L., Kukhtarev N.V., Odulov S.G., Soskin M.S. *Usp. Fiz. Nauk*, **129**, 113 (1979).
20. Kolmogorov A.N., Petrovskii I.G., Piskunov N.S. *Byulleten' MGU. Sect. A*, **1**, 1 (1937).