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Nonorthogonally magnetised permanent-magnet Faraday isolators

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Abstract. This paper describes a novel configuration of permanent-magnet magnetic systems for high-power Faraday isolators that are used in high-power lasers. An increase in magnetic field is ensured by magnets with a magnetisation vector inclined to the isolator axis. Numerical simulation results agree well with experimentally determined magnetic field distributions.

Keywords: permanent-magnet Faraday rotator, magnetic field calculations, isolation ratio, high average power lasers.

1. Introduction

A major problem limiting the use of Faraday isolators (FIs) in high average power lasers is the unavoidable heat absorption in magneto-optical elements (MOEs) [1, 2], which results in a variety of undesirable thermo-optic effects. The nonuniform cross-sectional temperature distribution caused by light absorption in MOEs leads to a nonuniform polarisation rotation distribution due to the temperature variation of the Verdet constant and gives rise to linear birefringence (photoelastic effect) and transmitted beam wavefront distortions (thermal lensing) [3].

The isolation ratio, a key parameter of FIs, is determined for the most part by polarisation distortions, i.e. by depolarisation in the MOE. Depolarisation due to light absorption (so-called thermal depolarisation) depends significantly on the radiation power and is a major negative factor.

There are several approaches to reducing the thermal depolarisation in FIs. One possibility is to divide an MOE into several elements in the form of thin discs cooled through the optical surface [4], which considerably reduces the transverse temperature gradient and, hence, the thermal distortion in the discs, or to use two elements, with a reciprocal polarisation rotator in between. The distortion produced when the beam passes through the first element is partially compensated in the second element [3, 5, 6]. Another approach is to cool FI elements to liquid-nitrogen

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Received 27 October 2010 *Kvantovaya Elektronika* **41** (1) 71–74 (2011) Translated by O.M. Tsarev temperature, which also ensures significant improvement in the performance of the isolator [7-9]. Yet another approach is to reduce the length of the MOE through increasing the magnetic field strength in the FI magnetic system (MS). In contrast to Mukhin et al. [10], who proposed using magnetic conductors, here we propose novel MS designs that take advantage of nonorthogonal magnetisation configurations (with a component inclined to the FI axis) in order to increase the magnetic field.

2. Magnetic system optimisation

The operating principle of FIs is based on the Faraday effect: the rotation of the plane of polarisation of a linearly polarised electromagnetic wave propagating in a magnetically active medium of length L in a magnetic field H. The Faraday rotation angle is given by

$$\varphi = VHL,\tag{1}$$

where V is the Verdet constant. According to (1), increasing the magnetic field strength in the magnetic system of FIs allows one to reduce the length of the MOE and, hence, to suppress the adverse thermal effects.

The magnetic system of Faraday isolators is a set of permanent-magnet rings [11] with an MOE inside (as a rule in the centre). Conventional MS designs employ axially and radially magnetised rings (Fig. 1a). The magnetic field can be raised by increasing the magnet volume, but this would increase the dimensions and weight of the FI [8]. Moreover, increasing the magnet volume is ineffective because of the increase in the mutual demagnetising action of neighbouring magnets and the necessity to place extra magnets farther away from the centre of the MS. At the same time, the magnetic field can be increased by using rings with a different magnetisation direction (Fig. 1b), without increasing the amount of magnetised material.



Figure 1. (a) Conventional and (b) novel FI magnetic systems, which employ (1) radially, (2) axially and (3) obliquely magnetised rings.

Consider the magnetic field H of an axially magnetised ring (Fig. 1a, ring 2). The longitudinal component of H is given by [12]

$$H_{z}(z,\rho) = M(z+a) \int_{0}^{2\pi} \int_{R_{1}}^{R_{2}} \frac{r dr d\varphi}{\left[\rho^{2} + r^{2} + (z+a)^{2} - 2\rho r \cos\varphi\right]^{3/2}} - M(z-a) \int_{0}^{2\pi} \int_{R_{1}}^{R_{2}} \frac{r dr d\varphi}{\left[\rho^{2} + r^{2} + (z-a)^{2} - 2\rho r \cos\varphi\right]^{3/2}}, \quad (2)$$

where *M* is the magnetisation; R_1 and R_2 are the inner and outer radii of the ring, respectively; 2a is its thickness; and ρ , φ and *z* are cylindrical coordinates (the *z* coordinate is measured from the centre of the ring).

The axial $(\rho = 0)$ field is

$$H_{z}(z,\rho=0) = 2\pi M \left\{ \frac{z+a}{\left[R_{1}^{2}+\left(z+a\right)^{2}\right]^{1/2}} - \frac{z+a}{\left[R_{2}^{2}+\left(z+a\right)^{2}\right]^{1/2}} - \frac{z-a}{\left[R_{1}^{2}+\left(z-a\right)^{2}\right]^{1/2}} + \frac{z-a}{\left[R_{2}^{2}+\left(z-a\right)^{2}\right]^{1/2}} \right\}.$$
 (3)

The longitudinal component of H in a radially magnetised ring with its magnetisation directed towards its centre (Fig. 1a, ring 1) has the form

$$H_{z}(z,\rho) = -MR_{2} \int_{0}^{2\pi} \int_{-a}^{a} \frac{(z-z')d\varphi dz'}{\left[\rho^{2} + R_{2}^{2} + (z-z')^{2} - 2\rho R_{2}\cos\varphi\right]^{3/2}} \\ + MR_{1} \int_{0}^{2\pi} \int_{-a}^{a} \frac{(z-z')d\varphi dz'}{\left[\rho^{2} + R_{1}^{2} + (z-z')^{2} - 2\rho R_{1}\cos\varphi\right]^{3/2}} \\ + M \int_{R_{1}}^{R_{2}} \int_{0}^{2\pi} \int_{-a}^{a} \frac{(z-z')d\varphi dz'}{\left[\rho^{2} + r^{2} + (z-z')^{2} - 2\rho r\cos\varphi\right]^{3/2}}, \quad (4)$$

and that on the axis is given by

$$H_{z}(z,\rho=0) = 2\pi M$$

$$\times \ln \left\{ \frac{R_{2} + \left[R_{2}^{2} + (z-a)^{2}\right]^{1/2}}{R_{2} + \left[R_{2}^{2} + (z+a)^{2}\right]^{1/2}} \frac{R_{1} + \left[R_{1}^{2} + (z+a)^{2}\right]^{1/2}}{R_{1} + \left[R_{1}^{2} + (z-a)^{2}\right]^{1/2}} \right\}$$

$$+ 2\pi M R_{1} \left\{ \frac{1}{\left[R_{1}^{2} + (z-a)^{2}\right]^{1/2}} - \frac{1}{\left[R_{1}^{2} + (z+a)^{2}\right]^{1/2}} \right\}$$

$$- 2\pi M R_{2} \left\{ \frac{1}{\left[R_{2}^{2} + (z-a)^{2}\right]^{1/2}} - \frac{1}{\left[R_{2}^{2} + (z+a)^{2}\right]^{1/2}} \right\}. (5)$$

The field of a ring whose magnetisation makes an angle α with the isolator axis can be considered the superposition of the fields of an axially magnetised ring with a magnetisation $M_{ax} = M \cos \alpha$ and a radially magnetised ring with a magnetisation $M_{rad} = M \sin \alpha$. Therefore, Eqns (3) and (5) can be used to calculate the field of MS's made up of an arbitrary number of various rings.

At constant outer dimensions, the field inside the MS shown in Fig. 1b depends on three parameters: the position of the inner boundaries of the obliquely magnetised rings, the position of their outer boundaries and the angle between their magnetisation and the isolator axis. Varying these parameters, one can control the field inside the system. Our task is to maximise the field in the region of the MOE, i.e. in the centre of the MS, so a criterion for an MS to be considered optimal is that the integral of the magnetic field over the length of the MOE (or over half of its length because the magnetic field distribution is symmetric with respect to the centre of the magnetic system) be maximal:

$$\int_{0}^{0.5L} H_z(z) \mathrm{d}z \to \max.$$
 (6)

Thus, the problem reduces to finding the maximum of a multidimensional function. We wrote a program to find optimal parameters of rings for condition (6) to be fulfilled.

Note that optimisation of the magnetisation distribution can be used in two ways. First, one can increase the magnetic field in the region of the MOE, while maintaining the weight and dimensions of the system unchanged, which would allow one to reduce the length of the MOE and, hence, to suppress the adverse thermal effects. (The latter would enable an increase in the maximum permissible average laser power when a Faraday isolator is used.) Second, one can reduce the MS length without changing the integral of the magnetic field over the crystal length (i.e. with no change in the length of the MOE).

3. Numerical modelling

3.1 Parameter optimisation

We modelled a widespread FI with a clear aperture of 12 mm and TGG crystal length of 18 mm. The magnets were of a Nd-Fe-B alloy (remanence of 12 kG and coercivity of 13 kOe). Figure 2 presents numerical simulation results for the optimal dimensions of rings in three MS's. The MS in Fig. 2a consists of only radially and axially magnetised rings, whereas the MS's in Figs 2b and 2c comprise not only radially and axially magnetised rings but also pairs of obliquely magnetised rings. In the system in Fig. 2b, the optimal angle between the magnetisation direction in the obliquely magnetised rings and the FI axis is $\alpha = 49^{\circ}$. It follows from our calculations that such changes in the geometry of the system make it possible either to reduce its length by about a factor of 1.5 (Fig. 2b), while maintaining the average field in the region of the MOE unchanged, or to reduce the length of the MOE by 12% (by increasing the average magnetic field in the region of interest) without changing the dimensions of the system.

The system can be further optimised. An even stronger field can be obtained at the same dimensions of the system by adding obliquely magnetised rings in the peripheral part of the system at the expense of the radially magnetised rings (Fig. 2c). The optimal angle between the magnetisation direction in the outer rings and the FI axis is then $\beta = 119^{\circ}$. The angle for the pair of inner rings remains unchanged ($\alpha = 49^{\circ}$) because there are radially magnetised regions between the obliquely magnetised pairs.

Our calculations indicate that this magnetisation distribution increases the magnetic field on average by 15% compared to the standard configuration. The magnetic field in this system only slightly exceeds that in the system containing only two obliquely magnetised rings (near its centre) because the field generated by the outer rings, which are far away from the magneto-optical element, is considerably weaker.

In the limiting case, the MS should consist of an infinite number of rings, the rings should be infinitely thin, and the



Figure 2. Optimal dimensions of the magnetic rings in the MS's of FIs with a 12-mm clear aperture: (a) radially and axially magnetised rings; (b, c) radially, axially and obliquely magnetised rings. All dimensions are given in millimetres.

angle between the magnetisation vector in each ring and the FI axis should be optimised. Figure 3 plots the angle between the magnetisation direction and FI axis against axial coordinate. Calculations show that, without changes in the geometric dimensions of the system, such a magnetisation distribution ensures an about 18 % increase in the integral of the magnetic field. In our opinion, there is no point in further optimisation, with a larger number of rings, because the gain will not exceed 3 %.



Figure 3. Optimal angle between the magnetisation direction and FI axis against z coordinate for an MS made up of thin rings.

Note that, in the absence of thermal effects, the isolation ratio depends not only on the integral of the magnetic field over the length of the MOE but also on the homogeneity of the integral in the cross section of the MOE. In the magnetic system represented in Fig. 2a, the magnetic field inhomogeneity is 3.7%, which corresponds to an isolation ratio of 35 dB. In the MS's containing one pair and two pairs of obliquely magnetised rings (Figs 2a, 2b), the field inhomogeneity is 6.2% and 5.3%, respectively (isolation ratios of 31 and 32 dB). This level is usually sufficient for FIs.

It is worth pointing out that the use of obliquely magnetised rings in FIs is an alternative to increasing the magnetic field with the use of magnetic conductors. Using magnetic conductors (external screen and two internal pole terminals), Mukhin et al. [10] were able to increase the field from 1.7 to 2.1 T. Our calculations suggest that obliquely magnetised rings may ensure an increase in magnetic field to 2.5 T.

3.2 Magnetic field weakening in sectorial rings

Note that radially magnetised rings are typically fabricated from uniformly magnetised sectors (Fig. 4b). Consider how much the field on the axis of a ring made up of N sectors differs from that of a radially magnetised ring (Fig. 4a). H_z on the axis of a uniformly magnetised sector with a central angle γ is given by

$$H_{z}(z) = 2M \sin \frac{\gamma}{2}$$

$$\times \ln \left\{ \frac{R_{2} + [R_{2}^{2} + (z-a)^{2}]^{1/2}}{R_{2} + [R_{2}^{2} + (z+a)^{2}]^{1/2}} \frac{R_{1} + [R_{1}^{2} + (z+a)^{2}]^{1/2}}{R_{1} + [R_{1}^{2} + (z-a)^{2}]^{1/2}} \right\}$$

$$+ 2MR_{1} \sin \frac{\gamma}{2} \left\{ \frac{1}{[R_{1}^{2} + (z-a)^{2}]^{1/2}} - \frac{1}{[R_{1}^{2} + (z+a)^{2}]^{1/2}} \right\}$$

$$- 2MR_{2} \sin \frac{\gamma}{2} \left\{ \frac{1}{[R_{2}^{2} + (z-a)^{2}]^{1/2}} - \frac{1}{[R_{2}^{2} + (z+a)^{2}]^{1/2}} \right\}. (7)$$



Figure 4. (a) Radially and (b) uniformly magnetised sectors of rings.

Let a disc be made up of N identical sectors ($\gamma = 2\pi/N$). Then we have

$$H_z^{\text{sec d}} = N H_z^{\text{sec}}.$$
 (8)

Using (5) and (7), relation (8) can be written in the form

$$H_z^{\text{sec d}} = \frac{N}{\pi} \sin\left(\frac{\pi}{N}\right) H_z^{\text{rad d}}.$$
(9)

The axial magnetic field loss due to the deviation from radial magnetisation,

$$\delta H_z = H_z^{\text{rad d}} - H_z^{\text{sec d}} = H_z^{\text{rad d}} \left(1 - \frac{N}{\pi} \sin \frac{\pi}{N} \right), \tag{10}$$

decreases with an increase in the number of sectors (Fig. 5).



Figure 5. Magnetic field loss as a function of the number of sectors, N.

Our magnetic systems employed rings made up of eight sectors. It follows from (10) that the field of such rings is just 2.5% weaker than that of radially magnetised rings with the same geometry and magnetic parameters.

4. Experimental results

For experimental verification, we fabricated two MS's (Figs 2a, 2b). Figure 6 shows theoretically calculated (curves) and experimentally determined (data points) longitudinal H_z distributions. The magnetic field was measured by a Mayak-5 teslameter (Saratov, Russia) with a relative uncertainty within 0.5%. The probe diameter was 3 mm. For the standard configuration (Fig. 2a), good agreement between calculation (dashed line) and experiment (circles) was obtained. For the optimised MS (Fig. 2b), the experimental data (Fig. 6, squares) are in qualitative agreement with the numerical simulation results (Fig. 6, dotted line), but there is a significant quantitative discrepancy. The reason for this is that some of the magnets in the rings magnetised at an angle to the isolator axis lost their ferromagnetic properties during the fabrication process. The magnetic field calculated with allowance for the loss (Fig. 6, solid line) is in much better agreement with the experimental data (Fig. 6, squares).

Thus, the use of a pair of obliquely magnetised rings in an FI with a 12-mm clear aperture enabled a marked reduction (by a factor of ~ 1.5) in the magnet volume.

- - Standard system (calculation)
- \cdots Optimised system (calculation with no allowance for the loss)

— Optimised system (calculation with allowance for the loss)



Figure 6. Comparison of experimental and calculation results.

5. Conclusions

We have examined novel configurations of the magnetic system for FIs, which employ rings magnetised at an angle to the isolator axis. Such rings enable optimisation of the magnetisation field, which in turn can be used to increase the magnetic field (with no change in the dimensions of the MS) and, hence, to reduce the length of the MOE and suppress the adverse thermal effects, or to reduce the volume and weight of the ferromagnetic material without changing the magnetic field.

We have simulated and experimentally studied a system obtained by adding a pair of such rings to the standard configuration. The results indicate that the magnetic field in the region of the MOE in such a system is approximately 12% stronger and the length and weight of the MS are a factor of 1.5 smaller. The use of a larger number of such rings enables optimisation of the magnetisation distribution, but the gain rapidly saturates with an increase in the number of rings. There is a limit at an infinite number of rings.

The analytical expression derived for evaluating the magnetic field loss in magnetic systems because of the uniform magnetisation field distribution in radially magnetised sectorial rings is expected to be of great practical importance in designing magnetic systems for Faraday isolators.

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