Quantum Electronics **41** (1) 55 – 57 (2011)

PACS numbers: 42.60.Jf; 42.25.Bs
DOI: 10.1070/QE2011v041n01ABEH014475

Modified Frantz – Nodvik equation for calculating the gain of divergent laser beams

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***Abstract*. We present the modified Frantz – Nodvik equation for calculating the gain of the divergent laser beams with allowance for saturation. We present the results of calculations by the proposed equation and compare their accuracy with the numerical calculations by the 'Fresnel' software.**

Keywords: *diverging laser beams, gain saturation, Frantz – Nodvik equation.*

1. Introduction

Kuznetsova and Mikheev [1] analysed theoretically the system of Franz – Nodvik equations taking into account the specific character of amplification of laser beams with a spherical wavefront. In formulating the problem of the applicability conditions of the truncated equations to describe the propagation of spherical waves, they considered the possibility of using the active gaseous media to amplify femtosecond pulses [1]. A similar problem was solved previously for solid-state amplifiers (see [1] and references therein).

However, although Kuznetsova and Mikheev [1] describe the features of the gain saturation for the beams with divergence angles $0 \leqslant 2\theta_0 \leqslant 60^\circ$ for the unsaturated gain levels up to exp5, simple estimates can be useful in the practical realisation of the gain of divergent laser beams under different experimental conditions. The results of [1] provide such an opportunity only in the limit of very weak or very strong gain saturation {see expressions (34) and (35) in [1]}.

2. Frantz – Nodvik equation for diverging laser beams

The Frantz – Nodvik equation is well known and has the form [2]

$$Q_{\text{out}} = Q_{\text{s}} \ln\{1 + \exp(N_0 \sigma L)[\exp(Q_{\text{in}}/Q_{\text{s}}) - 1]\}, \qquad (1)$$

where Q_{in} and Q_{out} are the energy densities at the input and output of the amplifier; Q_{s} is the saturation energy density; $N_0 \sigma L$ is the unsaturated gain; N_0 is the concentration of active particles; σ is the cross section for stimulated emission; L is the length of the amplifier. We will show that equation (1), used to calculate the gain of collimated laser beams, can be modified for the case of the beams with a spherical wavefront.

For a freely propagating laser beam (having the divergence angle of $2\theta_0$ and the initial radius r_0) with a spherical wavefront, we can give the relation, which describes the decrease in the beam energy density as a function of the coordinate z:

$$Q(z) = \frac{Q_{\text{in}}}{[1 + (z/r_0)\tan\theta_0]^2}. \qquad (2)$$

In an amplifying medium, the energy density in the beam can be conveniently represented in terms of the saturation energy density Q_{s}:

$$\frac{Q(z)}{Q_{\text{s}}} = \frac{Q_{\text{in}}}{Q_{\text{s}}[1 + (z/r_0)\tan\theta_0]^2}. \qquad (3)$$

Formally, the saturation density in the left-hand side of equality (3) can be considered dependent on z, i.e., take $Q_{\text{s}}(z) = Q_{\text{s}}[1 + (z/r_0)\tan\theta_0]^2$ and consider (3) as a relation corresponding to the propagation of a collimated beam in a medium with a variable saturation energy density. In this case, the average saturation energy density can take the value

$$\overline{Q_{\text{s}}(z)} = \frac{Q_{\text{s}}(0) + Q_{\text{s}}(L)}{2} = \frac{Q_{\text{s}}\{1 + [1 + (L/r_0)\tan\theta_0]^2\}}{2}. \qquad (4)$$

After substituting (4) into (1), we obtain

$$Q'_{\text{out}} = \overline{Q_{\text{s}}(z)} \ln\{1 + \exp(N_0 \sigma L)[\exp[Q_{\text{in}}/\overline{Q_{\text{s}}(z)}] - 1]\}. \qquad (5)$$

Because the area of the divergent beam at the amplifier output is $[1 + (L/r_0)\tan\theta_0]^2$ times greater than at the input, the energy density at the amplifier output has the from

$$Q_{\text{out}} = \frac{Q_{\text{s}}\{1 + [1 + (L/r_0)\tan\theta_0]^2\}}{2[1 + (L/r_0)\tan\theta_0]^2} \times \ln\{1 + \exp(N_0 \sigma L)[\exp[Q_{\text{in}}/\overline{Q_{\text{s}}(z)}] - 1]\}. \qquad (6)$$

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Received 29 October 2010
Kvantovaya Elektronika **41** (1) 55 – 57 (2011)
Translated by I.A. Ulitkin

Accordingly, for the beam energy gain we obtain the expression

$$K = \frac{1 + [1 + (L/r_0)\tan\theta_0]^2}{2Y_0} \ln\left\{1 + \exp(N_0\sigma L) \times \left[\exp\left(\frac{2Y_0}{1 + [1 + (L/r_0)\tan\theta_0]^2}\right) - 1\right]\right\}, \quad (7)$$

where, as in [1], $Y_0 = Q_{\rm in}/Q_{\rm s}$ is the energy density of radiation at the amplifier input in terms of the energy saturation. Similarly, we can consider the amplification of the laser beam with different divergences along the x and y axes.

We will assess the accuracy of calculations by expression (7) by comparing them with the numerical calculations for divergent beams, obtained with the 'Fresnel' software [3]. According to [3], amplification of the laser beams is calculated numerically on a spatial grid consisting of N^2 cells when sectioning the amplifier of length L into m independent sections and performing sequential calculations within each section:

(i) changes in the spatial structure of the beam due to the divergence and diffraction on the length $L/(2m)$;

(ii) changes in the beam energy within each section of the amplifier length L/m;

(iii) changes in the spatial structure of the beam due to the divergence and diffraction on the length $L/(2m)$ after amplification.

In this paper, we performed calculations for $Y_0 = 0.04$, 0.1, 0.25; the beam radius $r_0 = 1$ cm; $N_0\sigma = 0.05$ cm^{-1}; and the amplifier length $L \leqslant 100$ cm. For the adequate comparison of the calculations by expression (6) with the results of [1] and the numerical calculations by the 'Fresnel' software ($N = 512$, $L/m = 2$ cm), the diffraction effects in the latter case were excluded from consideration*.

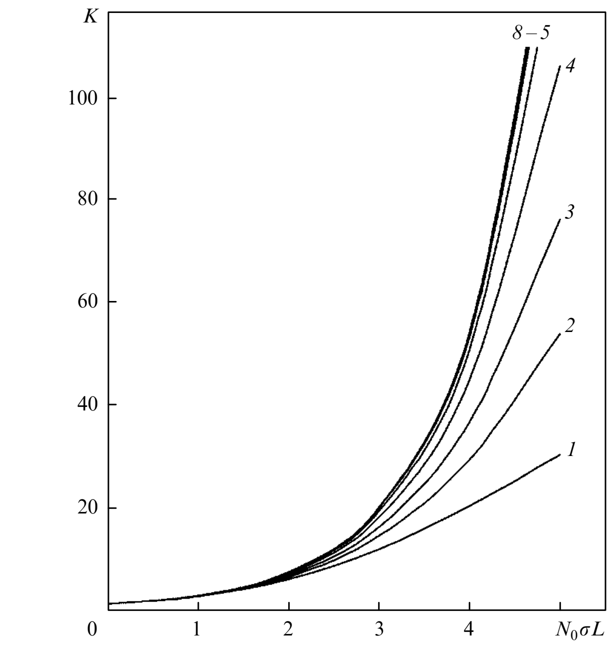

Figure 1. Dependence of the gain of the laser beam energy on the dimensionless parameter $N_0\sigma L$ at divergence angles of $2\theta_0 = 0$ (*1*), 1.25° (*2*), 2.5° (*3*), 5° (*4*), 10° (*5*), 20° (*6*), 30° (*7*) and 60° (*8*). The energy density of the beam at the amplifier input is $Y_0 = 0.1$.

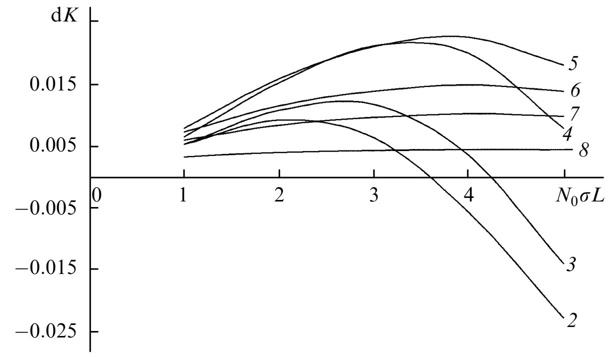

Figure 2. The relative error in computing the gain of the divergent laser beam with an initial energy density $Y_0 = 0.1$ as a function of the gain parameter $N_0\sigma L$ and the divergence angle. Curve numbers correspond to those in Fig. 1.

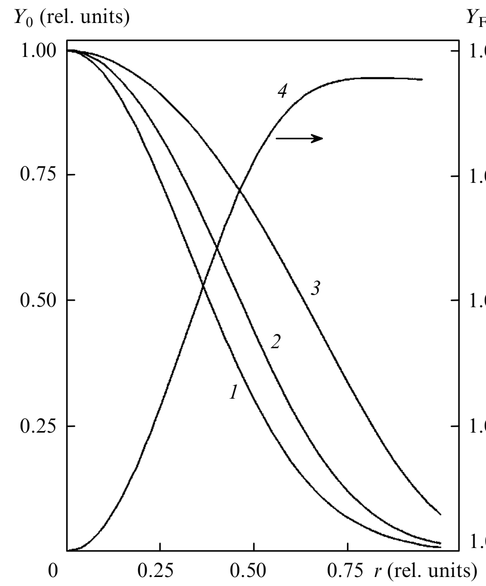

Figure 3. Deformation of the laser beam with the initial Gaussian distribution of the energy density at $Y_0 = 0.25$, $w_0 = 1.3$ cm, $N_0\sigma L = 5$, $2\theta_0 = 5°$ (*2*) and 0 (*3*). Curve (*1*) is the initial energy density distribution in the beam, and curve (*4*) is the ratio of the beam energy density profile calculated by the 'Fresnel' software to the calculated values at $2\theta_0 = 5°$.

Figure 1 shows the results of calculations by expression (7) for $Y_0 = 0.1$ and a set of angles $0 \leqslant 2\theta_0 \leqslant 60°$. The range of deviations of the gains K (Fig. 1) from the coefficients $K_{\rm F}$, calculated by the 'Fresnel' software

$$dK = \frac{K_{\rm F} - K}{K_{\rm F}} \quad (8)$$

amounts to −0.016...0.08. This range, however, can be reduced by using the correction factor p, taking into

*The radiation wavelength was chosen so small that it was possible to neglect the diffraction effects in the used geometry of the amplifier.

account the nonlinearity of the gain saturation. If we assume that

$$\overline{Q_{\mathrm{s}}(z)} = \frac{Q_{\mathrm{s}}\{1 + [1 + (L/r_0)\tan\theta_0]^2\}}{2p}, \quad (9)$$

then the optimised value of $p = 0.893$ leads to a decrease in the range of variation (7) for all used values of Y_0 to $\mathrm{d}K = \pm 0.025$. Figure 2 presents the dependences of $\mathrm{d}K(N_0\sigma L)$ at $Y_0 = 0.1$ and $1.25^\circ \leqslant 2\theta_0 \leqslant 60^\circ$.

Equation (7) can also be used to calculate the spatial deformation of the divergent laser beam in the case of the gain saturation. To do this, Y_0 should be replaced by the dependence $Y_0(r)$ in the input plane of the amplifier. The calculation results for a Gaussian beam of type $Y_0(r) = Y_0 \exp[-2(r/w_0)^2]$ with $Y_0 = 0.25$, $w_0 = 1.3$ cm, $2\theta_0 = 5^\circ$, $N_0\sigma L = 5$ and their comparison with numerical calculations are shown in Fig. 3.

3. Conclusions

Thus, the main parameters of the divergent laser beam amplification can be determined with a high accuracy by the modified Frantz–Nodvik equation, which uses the average value of the saturation energy of the active medium. The presented equation is easily generalised to the case of a beam with different divergence along the axes. Note that the advantage from the use of divergent beams will be obvious if another way to fill the aperture of the amplifier is impossible. This situation takes place, particularly when filling the conical aperture of the amplifiers [4] and in some schemes of multipass amplifiers [5].

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