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Doppler optical mixing spectroscopy in multiparticle scattering fluids

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Abstract. **We discuss the basic scheme of laser Doppler optical mixing spectroscopy for the analysis of media with multiparticle scattering. It is shown that the Rayleigh scheme, in contrast to the heterodyne and differential schemes, is insensitive to the effects of multiparticle scattering.**

Keywords: *Doppler optical mixing spectroscopy, laser Doppler anemometry.*

1. Introduction

Methods of Doppler optical mixing spectroscopy and laser Doppler anemometry have found wide applications in experimental physics [1, 2], biology [2, 3], fluid dynamics [4]. They are based on the analysis of the frequency structure of light fields scattered by a medium under study. The frequency structure of scattered light is transformed into a frequency structure of the electric current via the quadratic electric conversion (optical mixing) of light fields. In the case when the medium is a set of moving particles, there arises the problem of multiparticle scattering, which contributes to the frequency structure of the photoelectric current and distorts the results of measurements.

In papers [5, 6] the possibility of excluding the effect of multiparticle scattering was investigated for the case of laser Doppler imaging of the velocity fields in the optical frequency demodulation of scattered light. The optical frequency demodulation is a direct conversion of frequency into intensity for the light waves scattered by each particle. To this end, it differs fundamentally from optical mixing, in which the frequency structure of the photocurrent is determined by the power spectrum of scattered light fields formed on the photosensitive surface of the photodetector and, therefore, is nonlinearly related to the scattering functions of the particles in the probe field.

The possibility of minimising the effects of multiparticle scattering in the Doppler optical mixing spectroscopy is discussed in this paper.

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2. Effect of multiparticle scattering in basic schemes of Doppler optical mixing spectroscopy

The action of any Doppler optical mixing spectrometer is based on one of three basic schemes or their combinations discussed below.

2.1 Heterodyne scheme (scheme with a reference beam)

In this scheme, a medium under study is irradiated by a probe laser beam which is scattered by the particles present in this medium. Then, the light beam scattered in the given direction undergoes the heterodyne photoelectric conversion. As a reference (heterodyne) field use is made of the 'split-off' part of the unscattered probe beam. Consider Fig. 1. It shows the particles with numbers n and m in the field of the incident beam with wave vector $\boldsymbol{k}$. Dashed lines show the boundaries of the probe beam. Consider the structure of the light field scattered by the nth particle in the direction of the wave vector $\boldsymbol{k}_{\rm s}$, taking into account of the light incident on this particle which is scattered by the neighbouring mth particle. This field can be described by the expression

$$E_n = As_n \exp\{{\rm i}[\omega_0 + \boldsymbol{v}_n(\boldsymbol{k}_{\rm s} - \boldsymbol{k})]t + {\rm i}\varphi_n\}$$
$$+A\sum_m s_{nm} \exp\{{\rm i}[\omega_0 + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}) + \boldsymbol{v}_n(\boldsymbol{k}_{\rm s} - \boldsymbol{k}_{nm})]t + {\rm i}\varphi_{nm}\},$$

or by the expression

$$E_n = As_n \exp\{{\rm i}[\omega_0 + \boldsymbol{v}_n(\boldsymbol{k}_{\rm s} - \boldsymbol{k})]t + {\rm i}\varphi_n\}$$
$$+A\sum_m s_{nm} \exp[{\rm i}(\omega_0 + \boldsymbol{v}_{mn}\boldsymbol{k}_{nm} - \boldsymbol{v}_m\boldsymbol{k} + \boldsymbol{v}_n\boldsymbol{k}_{\rm s})t + {\rm i}\varphi_{nm}], \quad (1)$$

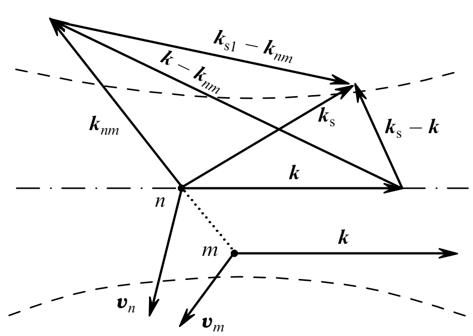

Figure 1. Scattering of radiation on two particles in the heterodyne measurement scheme.

where A is the amplitude of the incident light field with wave vector $\boldsymbol{k}$; s_n is a function of radiation scattering by the nth particle in the direction of the wave vector $\boldsymbol{k}_s$; s_{nm} is a function of the field scattering by the nth particle in the direction of the wave vector $\boldsymbol{k}_s$ for the light falling on it from the mth particle; $\boldsymbol{v}_n$ and $\boldsymbol{v}_m$ are the velocities of the nth and mth particles; ω_0 is the frequency of the incident light with the the wave vector $\boldsymbol{k}$; φ_n is the phase of the scattered light wave determined by the position of the nth particle with respect to the incident beam; $\boldsymbol{k}_{nm}$ is the wave vector of the light wave scattered by the mth particle in the direction of the nth particle; $\boldsymbol{v}_m(\boldsymbol{k}_{nm}-\boldsymbol{k})$ is the Doppler frequency shift of the light wave scattered by the mth particle in the direction of the nth particle; $\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k}_{nm})$ is the Doppler frequency shift of the light wave with the wave vector $\boldsymbol{k}_s$, scattered by the nth particle and incident on this particle from the side of the mth particle; $\boldsymbol{v}_{mn}=\boldsymbol{v}_m-\boldsymbol{v}_n$; φ_{nm} is the light field phase determined by the position the nth and mth particles in the incident light beam. Summation is performed over all particles, singly scattered light from which is incident on the nth particle. The account for single scattering only does not reduce the generality of the approach in studying the influence of multiparticle scattering on the whole.

Field (1) scattered by the nth particle is directed to a photosensitive surface of a quadratic photodetector and converted into a photoelectric current during optical mixing. As a reference light wave during optical mixing use can be made of a part of the unscattered light beam with frequency ω_0. Assume that on the photosensitive surface of the photodetector, the images of the scattering particles are formed in coherent light separately, and the cross-interference of the fields that form these images can be neglected. The photoelectric current is proportional to the intensity of the sum of the scattered light field (1) and field of the reference light wave $E_r=A_r\exp[\mathrm{i}(\omega_0+\Omega)t+\mathrm{i}\varphi_r]$:

$$
\begin{aligned}
i_n &= \rho|E_n+E_r|^2=\rho\Big|A_r\exp[\mathrm{i}(\omega_0+\Omega)t+\mathrm{i}\varphi_r] \\
&+As_n\exp\{\mathrm{i}[\omega_0+\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})]t+\mathrm{i}\varphi_n\} \\
&+A\sum_m s_{nm}\exp[\mathrm{i}(\omega_0+\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_m\boldsymbol{k}+\boldsymbol{v}_n\boldsymbol{k}_s)t+\mathrm{i}\varphi_{nm}]\Big|^2 \\
&=\rho\Big\{A_r^2+A^2s_n^2+A^2s_n\sum_m s_{nm}\exp[\mathrm{i}\boldsymbol{v}_{mn}(\boldsymbol{k}_{nm}-\boldsymbol{k})t \\
&+\mathrm{i}(\varphi_{nm}-\varphi_n)]+A^2\sum_{m,q}s_{nm}s_{nq}\exp[\mathrm{i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_{mq}\boldsymbol{k} \\
&-\boldsymbol{v}_{qn}\boldsymbol{k}_{nq})t+\mathrm{i}(\varphi_{nm}-\varphi_{nq})]+A_rAs_n\exp\{\mathrm{i}[\Omega-\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})]t \\
&+\mathrm{i}(\varphi_r-\varphi_n)\}+A_rA\sum_m s_{nm}\exp\{\mathrm{i}[\Omega-\boldsymbol{v}_{mn}(\boldsymbol{k}_{nm}-\boldsymbol{k}) \\
&-\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})]t+\mathrm{i}(\varphi_r-\varphi_{nm})\}+\mathrm{c.c}\Big\}=i_p+i_D(\Omega),
\end{aligned}
$$

where

$$
\begin{aligned}
i_D(\Omega) &= 2\rho A_rAs_n\Big\{\cos\{[\Omega-\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})]t+\varphi_r-\varphi_n\} \\
&+2A_rA\sum_m s_{nm}\cos\{[\Omega-\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})- \\
&-\boldsymbol{v}_{mn}(\boldsymbol{k}_{nm}-\boldsymbol{k})]t+\varphi_r-\varphi_{nm}\}\Big\}; \qquad (2)
\end{aligned}
$$

i_p and $i_D(\Omega)$ is the low-frequency pedestal and high-frequency Doppler component of the photocurrent; ρ is the coefficient taking into account the sensitivity and amplification of the photodetector; Ω is the given modulation frequency introduced in the reference wave to determine the sign of the Doppler frequency shift; A_r and φ_r are the amplitude and initial phase of the reference wave. The high-frequency component with a carrier frequency is separated by a band-pass filter. It, as follows from (2), is described by the expression

$$
\begin{aligned}
i_D(\Omega) &= 2\rho A_rA\Big\{s_n\cos[(\Omega-\omega_D)t+\varphi_{rn}] \\
&+\sum_m s_{nm}\cos[(\Omega-\omega_D+\omega_{nm})t+\varphi_{rn}+\varphi_m]\Big\}, \qquad (3)
\end{aligned}
$$

where $\omega_D=\boldsymbol{v}_n(\boldsymbol{k}_s-\boldsymbol{k})$ is the Doppler frequency shift proportional to the projection of the velocity vector of the nth particle on the axis, given by the difference between the wave vectors of the scattered ($\boldsymbol{k}_s$) and the incident ($\boldsymbol{k}$) beams; $\omega_{nm}=\boldsymbol{v}_{nm}(\boldsymbol{k}_{nm}-\boldsymbol{k})$; $\varphi_{rn}=\varphi_r-\varphi_n$. One can see from (3) that in the signal at the output of the scheme with the reference beam, the sum of the components with frequencies $\Omega-\omega_D+\omega_{nm}$ is superimposed on the component with the frequency $\Omega-\omega_D$, where ω_{nm} is the frequency noise, whose source is the multiparticle scattering in the probe field. This noise can contribute significantly to the measurement error when the media with a high concentration of scattering particles is studied. If the velocity gradient is absent in the probe field ($\boldsymbol{v}_{nm}=0$), we have $\omega_{nm}=0$ and expression (3) takes the form

$$
\begin{aligned}
i_D(\Omega) &= 2\rho A_rA\Big\{s_{nm}\cos[(\Omega-\omega_D)t+\varphi_{rn}] \\
&+\sum_m s_{nm}\cos[(\Omega-\omega_D)t+\varphi_{rn}+\varphi_m]\Big\} \\
&= I_D\cos[(\Omega-\omega_D)t+\varphi_{rn}-\psi_m], \qquad (4)
\end{aligned}
$$

where

$$
I_D=2\rho A_rA\Big[\Big(s_n+\sum_m s_{nm}\cos\varphi_m\Big)^2+\Big(\sum_m s_{nm}\sin\varphi_m\Big)^2\Big]^{1/2};
$$

$$
\psi_m=\arctan\frac{\sum_m s_{nm}\sin\varphi_m}{s_n+\sum_m s_{nm}\cos\varphi_m}.
$$

According to (4) in the absence of the velocity gradient in the probe field ($\boldsymbol{v}_n=\boldsymbol{v}_m$) the influence of the multiparticle scattering is minimal and limited by the presence of the phase noise ψ_m. Sensitivity to multiparticle scattering is a characteristic feature and the drawback of optical schemes with the reference beam used in the Doppler optical mixing spectroscopy and laser anemometry.

2.2 Differential scheme

Consider the differential optical scheme shown in Fig. 2. The probe field is formed by the intersection of two Gaussian beams with the wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. The centres of the beam waists are combined. The intersecting beams are shown in Fig. 2 by dashed lines. The probe field contains particles, which scatter light. Consider the structure of the light field E_n, scattered by the nth particle from the light beams forming the probe field. We will take into account the contribution of the field incident on the nth particle to this structure, the field being scattered by the neighbouring mth particle. We restrict ourselves to a model of single multiparticle scattering:

$$E_n = E_{n1} + E_{n2},$$

where

$$\begin{aligned} E_{n1} &= As_{n1}\exp\{\mathrm{i}[\omega_0 + \Omega + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_1)]t + \mathrm{i}\varphi_{n1}\} \\ &\quad + A\sum_m s_{nm1}\exp\{\mathrm{i}[\omega_0 + \Omega + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_1) \\ &\quad + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_{nm})]t + \mathrm{i}\varphi_{nm1}\}; \\ E_{n2} &= As_{n2}\exp\{\mathrm{i}[\omega_0 + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_2)]t + \mathrm{i}\varphi_{n2}\} \\ &\quad + A\sum_m s_{nm2}\exp\{\mathrm{i}[\omega_0 + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_2) \\ &\quad + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_{nm})]t + \mathrm{i}\varphi_{nm2}\}. \end{aligned} \tag{5}$$

Radiation scattered by the nth particle falls on the photodetector, which optically mixes the light fields E_{n1} and E_{n2}. We assume that on the photosensitive photodetector surface images of the scattering particles in coherent light are formed separately. Therefore, the cross-interference component of the fields forming these images can be neglected. The photoelectric current is proportional to the intensity of the light field scattered by the nth particle:

$$\begin{aligned} i_n &= \rho|E_n|^2 = \rho\Big|As_{n1}\exp\{\mathrm{i}[\omega_0 + \Omega + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_1)]t + \mathrm{i}\varphi_{n1}\} \\ &\quad + A\sum_m s_{nm1}\exp\{\mathrm{i}[\omega_0 + \Omega + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_1) + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_{nm})]t + \\ &\quad + \mathrm{i}\varphi_{nm1}\} + As_{n2}\exp\{\mathrm{i}[\omega_0 + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_2)]t + \mathrm{i}\varphi_{n2}\} \\ &\quad + A\sum_m s_{nm2}\exp\{\mathrm{i}[\omega_0 + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_2) + \boldsymbol{v}_n(\boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_{nm})]t \\ &\quad + \mathrm{i}\varphi_{nm2}\}\Big|^2 = \rho\Big\{I_\mathrm{p} + A^2 s_{n1}s_{n2}\exp\{\mathrm{i}[\Omega + v_n(\boldsymbol{k}_2 - \boldsymbol{k}_1)]t + \mathrm{i}\varphi_{n12}]\} \\ &\quad + A^2\sum_m\sum_q s_{nm1}s_{nq2}\exp\{\mathrm{i}[\Omega + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_1) - \boldsymbol{v}_n\boldsymbol{k}_{nm} \\ &\quad - \boldsymbol{v}_q(\boldsymbol{k}_{nq} - \boldsymbol{k}_2) + \boldsymbol{v}_n\boldsymbol{k}_{nq}]t + \mathrm{i}\varphi_{nmq12}\} + A^2 s_{n1}\sum_m s_{nm2} \\ &\quad \times\exp\{\mathrm{i}[\Omega + \boldsymbol{v}_n(\boldsymbol{k}_{nm} - \boldsymbol{k}_1) - \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_2)]t + \mathrm{i}\varphi_{nm12}\} \\ &\quad + A^2 s_{n2}\sum_m s_{nm1}\exp\{\mathrm{i}[\Omega + \boldsymbol{v}_m(\boldsymbol{k}_{nm} - \boldsymbol{k}_1) - \boldsymbol{v}_n(\boldsymbol{k}_{nm} - \boldsymbol{k}_2)]t \\ &\quad + \mathrm{i}\varphi_{nm21}\} + \mathrm{c.c.}\Big\} = i_\mathrm{p} + i_\mathrm{D}(\Omega). \end{aligned} \tag{6}$$

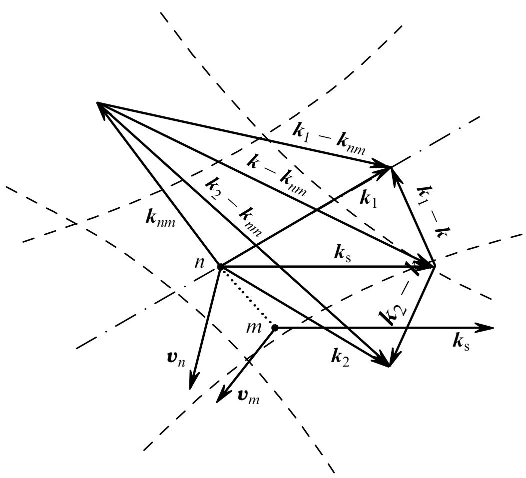

Figure 2. Scattering of radiation on two particles in the differential measurement scheme.

Here, $i_\mathrm{p} = \rho I_\mathrm{p}$ is the low-frequency pedestal; $i_\mathrm{D}(\Omega)$ is the high-frequency Doppler component.

We restrict ourselves to the photocurrent component

$$\begin{aligned} i(\Omega) &= 2\rho A^2\Big\{s_{n1}s_{n2}\cos[(\Omega + \omega_\mathrm{D})t + \varphi_{n12}] \\ &\quad + \sum_{m,q} s_{nm1}s_{nq2}\cos\{[\Omega + \omega_\mathrm{D} + \boldsymbol{v}_{mn}\boldsymbol{k}_1 - \boldsymbol{v}_{nq}(\boldsymbol{k}_2 - \boldsymbol{k}_{nq})]t \\ &\quad + \varphi_{n12} + \varphi_{mq}\} + s_{n1}\sum_m s_{nm2}\cos[(\Omega + \omega_\mathrm{D} + \boldsymbol{v}_{mn}\boldsymbol{k}_2 + \boldsymbol{v}_{nm}\boldsymbol{k}_{nm})t \\ &\quad + \varphi_{n12} + \varphi_m] + s_{n2}\sum_m s_{nm1}\cos[(\Omega + \omega_\mathrm{D} + \boldsymbol{v}_{nm}\boldsymbol{k}_1 + \boldsymbol{v}_{mn}\boldsymbol{k}_{nm})t \\ &\quad + \varphi_{n12} + \varphi_m]\Big\}, \end{aligned} \tag{7}$$

filtered in the vicinity of the frequency Ω. Here, $\omega_\mathrm{D} = \boldsymbol{v}_n(\boldsymbol{k}_2 - \boldsymbol{k}_1)$ is the Doppler frequency shift proportional to the projection of the velocity vector of the nth particle on the axis, given by the difference between the wave vectors of the incident beams. The contribution of multiparticle scattering is determined by the Doppler frequency shift proportional to the difference between the velocities $\boldsymbol{v}_{nm}$, $\boldsymbol{v}_{nq}$ of particles in the probe field. In the absence of the velocity gradient in the probe field ($\boldsymbol{v}_{nm} = 0$), the expression for the high-frequency component is reduced to

$$\begin{aligned} i(\Omega) &= 2\rho A^2\Big\{s_{n1}s_{n2}\cos[(\Omega + \omega_\mathrm{D})t + \varphi_{n12}] \\ &\quad + \sum_{m,q} s_{nm1}s_{nq2}\cos[(\Omega + \omega_\mathrm{D})t + \varphi_{n12} + \varphi_{mq}] \\ &\quad + s_{n1}\sum_m s_{nm2}\cos[(\Omega + \omega_\mathrm{D})t + \varphi_{n12} + \varphi_m] \\ &\quad + s_{n2}\sum_m s_{nm1}\cos[(\Omega + \omega_\mathrm{D})t + \varphi_{n12} + \varphi_m]\Big\}. \end{aligned} \tag{8}$$

It follows from (8) that in the absence of the velocity gradient the influence of multiparticle scattering on the measurement result in the differential scheme is small, as in the scheme with the reference beam, and is reduced to the presence of the phase noise ψ_m:

$$i(\Omega) = I_{\rm D}\cos[(\Omega+\omega_{\rm D})t+\varphi_{n12}+\psi_m].$$

The amplitude $I_{\rm D}$ and the phase are determined in the same way as in the case of the heterodyne scheme. Therefore, the differential schemes of optical mixing spectroscopy and laser Doppler anemometry are subjected to the influence of multiparticle scattering on the results of measurements, especially in studying media with the velocity gradient.

2.3 Rayleigh scheme

In laser Doppler anemometry it is often called the inverse-differential scheme (Fig. 3). The probe field is formed by a laser beam with the wave vector $\boldsymbol{k}$. The boundaries of this beam are shown by dashed lines. Restricting ourselves to a model of single multiparticle scattering, we will write the expression for the field scattered by the nth particle in the direction of the wave vectors $\boldsymbol{k}_{\rm s1}$ и $\boldsymbol{k}_{\rm s2}$:

$$E_n = E_{n1}+E_{n2},$$

where

$$E_{n1} = As_{n1}\exp\{{\rm i}[\omega_0+\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k})]t+{\rm i}\varphi_n\}+A\sum_m s_{nm1}$$
$$\times\exp\{{\rm i}[\omega_0+\Omega+\boldsymbol{v}_m(\boldsymbol{k}_{nm}-\boldsymbol{k})+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{nm})]t+{\rm i}\varphi_{nm}\};$$

$$E_{n2} = As_{n2}\exp\{{\rm i}[\omega_0+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s2}-\boldsymbol{k})]t+{\rm i}\varphi_n\}+A\sum_m s_{nm2}$$
$$\times\exp\{{\rm i}[\omega_0+\boldsymbol{v}_m(\boldsymbol{k}_{nm}-\boldsymbol{k})+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s2}-\boldsymbol{k}_{nm})]t+{\rm i}\varphi_{nm}\}.$$

After elementary transformations the expression for E_{n1} and E_{n2} take the form

$$\begin{aligned}E_{n1} &= As_{n1}\exp\{{\rm i}[\omega_0+\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k})]t+{\rm i}\varphi_n\}\\ &+A\exp[{\rm i}(\omega_0+\Omega+\boldsymbol{v}_n\boldsymbol{k}_{\rm s1})t+{\rm i}\varphi_n]\\ &\times\sum_m s_{nm1}\exp[{\rm i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_m\boldsymbol{k})t-{\rm i}\varphi_m],\\ E_{n2} &= As_{n2}\exp\{{\rm i}[\omega_0+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s2}-\boldsymbol{k})]t+{\rm i}\varphi_n\}\\ &+A\exp[{\rm i}(\omega_0+\boldsymbol{v}_n\boldsymbol{k}_{\rm s2})t+{\rm i}\varphi_n]\\ &\times\sum_m s_{nm2}\exp[{\rm i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_m\boldsymbol{k})t-{\rm i}\varphi_m].\end{aligned} \tag{9}$$

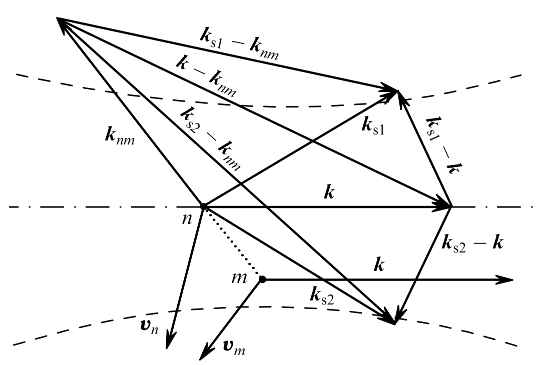

Figure 3. Scattering of radiation on two particles in the Rayleigh scheme.

The light beams with the wave vectors $\boldsymbol{k}_{\rm s1}$ and $\boldsymbol{k}_{\rm s2}$ are spatially combined and directed to the photodetector operating in the optical mixing regime. The photocurrent at the photodetector output is proportional to the intensity of the sum of light fields (9):

$$\begin{aligned}i &= \rho|E_{n1}+E_{n2}|^2 = \rho\Big|As_{n1}\exp\{{\rm i}[\omega_0+\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k})]t\\ &+{\rm i}\varphi_n\}+A\exp[{\rm i}(\omega_0+\Omega+\boldsymbol{v}_n\boldsymbol{k}_{\rm s1})t+{\rm i}\varphi_n]\\ &\times\sum_m s_{nm1}\exp[{\rm i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_m\boldsymbol{k})t-{\rm i}\varphi_m]+As_{n2}\\ &\times\exp\{{\rm i}[\omega_0+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s2}-\boldsymbol{k})]t+{\rm i}\varphi_n\}+A\exp[{\rm i}(\omega_0+\boldsymbol{v}_n\boldsymbol{k}_{\rm s2})t\\ &+{\rm i}\varphi_n]\sum_m s_{nm2}\exp[{\rm i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_m\boldsymbol{k})t-{\rm i}\varphi_m]\Big|^2 = i_{\rm p}+i_{\rm D}(\Omega),\end{aligned}$$

where $i_{\rm p}$ is the low-frequency pedestal;

$$\begin{aligned}i_{\rm D}(\Omega) &= \rho A^2\Big\{s_{n1}s_{n2}\exp\{{\rm i}[\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})]t\}\\ &+\exp\{{\rm i}[\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})]t\}\sum_m\sum_q s_{nm1}s_{nq2}\\ &\times\exp[{\rm i}(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_{qn}\boldsymbol{k}_{nq}-\boldsymbol{v}_{mq}\boldsymbol{k})t+{\rm i}\varphi_{mq}]\\ &+s_{n1}\exp\{{\rm i}[\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})]t\}\sum_m s_{nm2}\exp[-{\rm i}\boldsymbol{v}_{mn}(\boldsymbol{k}_{nm}-\boldsymbol{k})t\\ &-{\rm i}\varphi_m]+s_{n2}\exp\{{\rm i}[\Omega+\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})]t\}\\ &\times\sum_m s_{nm1}\exp[{\rm i}\boldsymbol{v}_{mn}(\boldsymbol{k}_{nm}-\boldsymbol{k})t-{\rm i}\varphi_m]+{\rm c.c.}\Big\}\end{aligned} \tag{10}$$

is the high-frequency Doppler component of the photocurrent.

Assume that $s_{n1}=s_{n2}=s_n$ and $s_{nm1}=s_{nm2}=s_{nm}$. This is fulfilled, for example, for a medium with spherical particles and configuration of the scattering beams, symmetrical with respect to the incident beam [when $(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})\boldsymbol{k}=0$)], or in the case of Rayleigh scattering. Under these conditions, expression (10) takes the form

$$\begin{aligned}i_{\rm D}(\Omega) &= 2\rho A^2\Big\{s_n^2\cos[(\Omega+\omega_{\rm D})t]+2s_n\cos[(\Omega+\omega_{\rm D})t]\\ &\times\sum_m s_{nm}\cos[\boldsymbol{v}_{mn}(\boldsymbol{k}-\boldsymbol{k}_{nm})t-\varphi_m]+\sum_{m,q}s_{nm}s_{nq}\\ &\times\cos[(\Omega+\omega_{\rm D}+\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_{qn}\boldsymbol{k}_{nq}-\boldsymbol{v}_{mq}\boldsymbol{k})t-\varphi_{mq}]\Big\},\end{aligned} \tag{11}$$

where $\omega_{\rm D}=\boldsymbol{v}_n(\boldsymbol{k}_{\rm s1}-\boldsymbol{k}_{\rm s2})$ is the Doppler frequency shift proportional to the projection of the velocity vector of the nth particle on the axis determined by the difference between the wave vectors $\boldsymbol{k}_{\rm s1}$ and $\boldsymbol{k}_{\rm s2}$. The double sum in (11) after elementary trigonometric transformations can be written as

$$\cos[(\Omega+\omega_{\rm D})t]\sum_{m,q}s_{nm}s_{nq}\cos[(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_{qn}\boldsymbol{k}_{nq}-\boldsymbol{v}_{mq}\boldsymbol{k})t$$
$$-\varphi_{mq}]+\sin[(\Omega+\omega_{\rm D})t]\sum_{m,q}s_{nm}s_{nq}\sin[(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm}-\boldsymbol{v}_{qn}\boldsymbol{k}_{nq}-$$

$$-\boldsymbol{v}_{mq}\boldsymbol{k})t - \varphi_{mq}] = \cos[(\Omega + \omega_{\mathrm{D}})t]\sum_{m,q} s_{nm}s_{nq}$$

$$\times \cos[(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm} - \boldsymbol{v}_{qn}\boldsymbol{k}_{nq} - \boldsymbol{v}_{mq}\boldsymbol{k})t - \varphi_{mq}], \quad (12)$$

since

$$\sum_{m,q} s_{nm}s_{nq}\sin[(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm} - \boldsymbol{v}_{qn}\boldsymbol{k}_{nq})t - \varphi_{mq}] = 0.$$

Because of the oddness of the function $\sin(\ldots)$. Substituting (12) into (11), for the high-frequency Doppler component of the photocurrent we obtain the expression

$$i_{\mathrm{D}}(\Omega) = A_{\mathrm{D}}\cos[(\Omega + \omega_{\mathrm{D}})t], \quad (13)$$

where

$$A_{\mathrm{D}} = 2\rho A^2\Big\{s_n^2 + 2s_n\sum_m s_{nm}\cos[\boldsymbol{v}_{mn}(\boldsymbol{k} - \boldsymbol{k}_{nm})t - \varphi_m]$$

$$+\sum_{m,q} s_{nm}s_{nq}\cos[(\boldsymbol{v}_{mn}\boldsymbol{k}_{nm} - \boldsymbol{v}_{qn}\boldsymbol{k}_{nq} - \boldsymbol{v}_{mq}\boldsymbol{k})t - \varphi_{mq}]\Big\}.$$

One can see from (13) that the Doppler component of the photocurrent $i_{\mathrm{D}}(\Omega)$ represents the amplitude-modulated narrowband signal whose frequency is independent of multiparticle scattering. It follows that the influence of multiparticle scattering on the result of measurements of the Doppler frequency shift in the Rayleigh scheme can be ignored, since it is reduced to the amplitude modulation of a narrowband signal.

3. Conclusions

We have studied the effect of multiparticle scattering and the possibility to minimise it in the basic schemes of Doppler optical mixing spectroscopy and laser Doppler anemometry. It is shown that this effect takes place in heterodyne and differential schemes. It is minimised when the medium has no velocity gradients of scattering particles. In the Rayleigh scheme the multiparticle scattering effect on the result of measurements of the Doppler frequency shift can be neglected, since it is reduced to the amplitude noise in a narrowband signal.

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