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Polarisation singularities in the electric field at a sum-frequency generated by two collinear elliptically polarised Gaussian beams in the bulk of a nonlinear gyrotropic medium

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Abstract. **Polarisation singularities in the electric field at a sum-frequency generated in the bulk of an isotropic gyrotropic medium with a quadratic nonlinearity are predicted to appear in the case of the collinear interaction of two uniformly elliptically polarised Gaussian beams. The parameters of the fundamental waves are found, corresponding to the formation of lines with circular and linear polarisations (C- and L-lines) in the cross section of the beam at the sum-frequency as well as to the appearance of the regions in the signal beam where the polarisation state varies smoothly from the left-hand circularly polarised state to the right-hand circularly polarised. In this case, the ellipticity degree of the polarisation ellipse takes all possible values, while its orientation remains unchanged.**

Keywords: *sum-frequency generation, elliptic polarisation, Gaussian beam, gyrotropy, polarisation singularities of C- and L-types.*

Polarisation singularities of the light field – points or lines in the cross section of a propagating beam in which the intensity of one of its orthogonally polarised components becomes zero – are the subject of numerous theoretical and experimental investigations in linear optics (see, for example, [1–12]). In one of the pioneering papers [1], the terminology extensively used at present was defined, according to which the locus of points in space where the propagating radiation is circularly (linearly) polarised, became known as C-lines (L-surfaces). The latter in the beam cross section become C-points and L-lines. Unlike conventional optical vortices, or spiral phase dislocations (where the intensity of the 'scalar' field is zero) studied usually in the approximation of constancy of polarisation of propagating radiation, the C-points, where the orientation of the polarisation ellipse of the electric field strength of the wave is not defined, can be called 'component' optical vortices. In their vicinity, there can be three types of morphological distributions of the polarisation ellipses of the light field, called in the literature as 'star', 'lemon', and 'monstar' (see [1 – 6]).

Later, the formation conditions and the dynamics of polarisation singularities were studied in different problems of linear optics, including the evolution of random vector fields in isotropic media [2], propagation of laser radiation in chiral crystals [3], scattering of light by the Earth's atmosphere [4], propagation of optical vortices in birefringent crystals [10, 11], coherent interaction of orthogonally polarised Bessel beams [12]. The fundamental association of the anisotropy of the Stokes parameters near the C-points with their morphology was established in [5]. The physical mechanisms were intensively investigated, leading to the appearance of singularities during the propagation of beams with a regular initial intensity and polarisation distribution in inhomogeneous media, optical fibres and laser resonators. Separately, we should note highly developed experimental methods for detecting the light beams with the points of singularity of the phase and polarisation [7 – 10]. A detailed review of these and other works is presented in [6].

Despite the popularity and wide range of the considered problems of singular linear polarisation optics, investigations of the origin and dynamics of the polarisation singularities in nonlinear optical processes are virtually absent. Meanwhile, the studies of the transverse spatial distribution of the electric field of the signal wave, generated in various nonlinear optical processes in media with a spatial dispersion [13 – 17], indicate the possibility of the appearance of the polarisation singularities of the electric field in it. In particular, the possibility of sum-frequency generation (SFG) in the bulk of an isotropic gyrotropic medium in the case of the collinear geometry of interaction of fundamental-frequency beams was justified in principle in [18], and formulas describing completely the nonuniform transverse spatial distribution of light polarisation formed in the signal beam were obtained in [13]. In the space of the incident radiation parameters, regions were constructed, corresponding to the formation of two different types of polarisation distributions in the beam at the sum-frequency. In the first case, the direction of the electric field vector rotation is the same at all points of the signal-beam cross section. In the second case, the direction of the electric field vector rotation is reversed when crossing the boundaries between the four sectors, formed by two straight lines passing through its centre. Within each sector, the direction of the electric field vector rotation does not change. The equations of these lines, which are in fact L-lines, along with the conditions of their appearance are given in [13].

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The present work focuses on the study of formation of C-type polarisation singularities in the beam cross section at the sum frequency generated in the bulk of an isotropic gyrotropic medium by uniformly collinear elliptically polarised Gaussian fundamental radiation beams. Polarisation of radiation in the signal-beam cross section in this case [13] is determined only by the polar angle and is the same along any arbitrarily chosen radial direction defined by the vector which originates from the beam centre, where the intensity is zero. The cylindrical symmetry of the problem prohibits the appearance of C-points in the plane of the beam cross section at the sum frequency, but allows for the appearance of C-lines. Investigation of the conditions of their appearance is the subject of this paper.

Papers [13, 18] describe in detail the approximations made and the method for finding the solution of the parabolic equation for the vortex component of the signal wave at the sum-frequency in an isotropic gyrotropic medium. The right-hand side of this solution contains a vortex component of the vector of matter polarisation, which appears as a result of the collinear propagation of two uniformly elliptically polarised incident Gaussian fundamental radiation beams with frequencies $\omega_{1,2}$.

If the symmetry axes of these beams coincide with the Z axis and the waist plane $z = l_0$ is common for them and located at a distance l_0 from the plane boundary of the medium, in the cylindrical coordinates (r, φ, z) the circularly polarised electric-field components at the sum-frequency $E_{\pm}^{\rm SF}(r, \varphi, z, t)$ are expressed by the formulas [13, 18]:

$$E_{\pm}^{\rm SF}(r,\varphi,z,t) = F_0(r,z)E_{\pm}(\varphi)$$
$$\times \exp[-{\rm i}(\omega_1+\omega_2)t + {\rm i}k_{\rm SF}(z-l_0)], \qquad (1)$$

where $k_{\rm SF} = n_{\rm SF}(\omega_1+\omega_2)/c$ is the wavenumber of the wave at the sum-frequency in a gyrotropic medium; $n_{\rm SF} = \sqrt{\varepsilon_{\rm SF}}$ is the refractive index; $\varepsilon_{\rm SF}$ is the dielectric constant of the material at the frequency $\omega_1+\omega_2$. The function $F_0(r,z)$, whose explicit form is given in [13, 18], vanishes at $r = 0$ and does not affect the degree of ellipticity

$$M(\varphi) = \frac{|E_+^{\rm SF}|^2 - |E_-^{\rm SF}|^2}{|E_+^{\rm SF}|^2 + |E_-^{\rm SF}|^2} = \frac{|E_+(\varphi)|^2 - |E_-(\varphi)|^2}{|E_+(\varphi)|^2 + |E_-(\varphi)|^2} \qquad (2)$$

of the wave at the sum-frequency. In formula (2)

$$E_+(\varphi) = 0.5\big\{\big[\sqrt{(1+M_{01})(1-M_{02})} + (1-k)$$
$$\times\sqrt{(1+M_{02})(1-M_{01})}\exp(2{\rm i}\Psi)\big]\exp({\rm i}\varphi)$$
$$+\sqrt{(1+M_{01})(1+M_{02})}\exp[2{\rm i}\Psi - {\rm i}\varphi]\big\}$$
$$\equiv A_+\exp[{\rm i}(\alpha_+ + \varphi)] + B_+\exp[{\rm i}(\beta_+ - \varphi)],$$
$$E_-(\varphi) = -0.5\big\{\sqrt{(1-M_{01})(1-M_{02})}\exp({\rm i}\varphi) \qquad (3)$$
$$+\big[(1-k)\sqrt{(1+M_{01})(1-M_{02})} + k\exp(2{\rm i}\Psi)$$
$$\times\sqrt{(1+M_{02})(1-M_{01})}\,\big]\exp(-{\rm i}\varphi)\big\}$$
$$\equiv A_-\exp[{\rm i}(\alpha_- + \varphi)] + B_-\exp[{\rm i}(\beta_- - \varphi)].$$

Here, Ψ is the angle between the principal axes of the polarisation ellipses of the fundamental waves at the medium input; M_{01} and M_{02} are the degrees of ellipticity of the ellipses; $k = 1/(1+k_2/k_1)$; $k_{1,2}$ are the wave vectors of the fundamental waves; $A_{\pm}$, $B_{\pm}$, $\alpha_{\pm}$,and $\beta_{\pm}$ are the real functions of k, $M_{01,02}$ and Ψ. The principal axis of the polarisation ellipse of the wave with the frequency ω_1 is oriented along the straight line $\varphi = 0$.

One can see from (2), (3) that the degree of the wave ellipticity at the sum-frequency is determined only by the polar angle φ and also has a period π. Therefore, the roots of the equation $M(\varphi) = 0$ set the angles, which define, in the plane of the beam cross section, the directions of the lines that are L-type lines. Under the same parameters of the fundamental waves, when this equation has two roots [13], in the beam cross section there are two L-lines intersecting at its centre, these lines being the boundaries of the four sectors. The directions of rotation of the electric field vectors in two adjacent sectors are opposite.

Singularity of the C-type polarisation appears at those points of the beam cross section at the sum-frequency in which one of its circularly polarised components vanishes. One can see from equalities (3) that $E_+^{\rm SF} = 0$ at $A_+ = B_+$ and $\varphi = (\beta_+ - \alpha_+)/2 + \pi/2 + \pi n$, and $E_-^{\rm SF} = 0$ at $A_- = B_-$ and $\varphi = (\beta_- - \alpha_-)/2 + \pi/2 + \pi n$, where $n = 0$, 1. In other words, the C-line

$$\varphi = \varphi_+ = -0.5\arg\big[k\sqrt{(1+M_{01})(1-M_{02})} + (1-k)$$
$$\times\exp(2{\rm i}\Psi)\sqrt{(1-M_{01})(1+M_{02})}\,\big] + \Psi + \pi/2, \qquad (4)$$

at the points of which the electric field vector rotates clockwise, appears when the condition

$$k^2(1+M_{01})(1-M_{02}) + (1-k)^2(1-M_{01})(1+M_{02})$$
$$+2k(1-k)\sqrt{\big(1-M_{01}^2\big)\big(1-M_{02}^2\big)}\cos 2\Psi$$
$$-(1+M_{01})(1+M_{02}) = 0 \qquad (5)$$

is met, and the C-line

$$\varphi = \varphi_- = 0.5\arg\big[k\exp(2{\rm i}\Psi)\sqrt{(1-M_{01})(1+M_{02})}$$
$$+(1-k)\sqrt{(1+M_{01})(1-M_{02})}\,\big] + \pi/2 \qquad (6)$$

at the points of which the electric field vector rotates counter-clockwise, appears if

$$k^2(1-M_{01})(1+M_{02}) + (1-k)^2(1+M_{01})(1-M_{02})$$
$$+2k(1-k)\sqrt{\big(1-M_{01}^2\big)\big(1-M_{02}^2\big)}\cos 2\Psi$$
$$-(1-M_{01})(1-M_{02}) = 0. \qquad (7)$$

Figure 1 shows in grey the domains of the ellipticity degree of the fundamental waves, for which L-type polarisation singularities do not appear in the beam cross section at the sum-frequency. For other values of M_{01} and M_{02}, there are two L-lines intersecting on the symmetry axis of the beam. At the points of solid curves bounding the gray areas (their equations are given in [13]), two L-lines merge

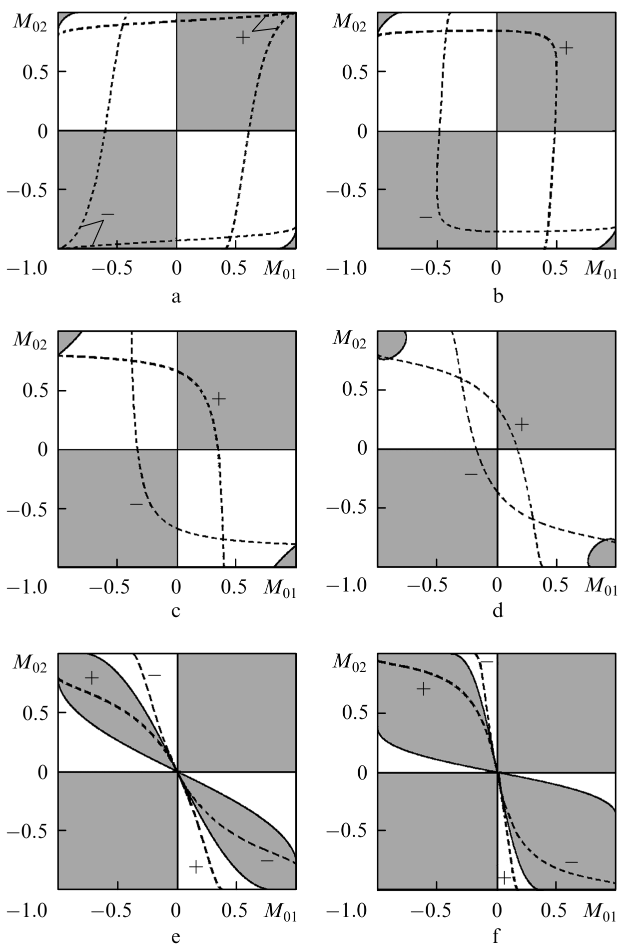

Figure 1. Domains of the ellipticity degree of the fundamental waves corresponding to the generation of polarisation singularities of C- and L-types in the beam cross section at the sum-frequency at $k_2/k_1 = 2$ and $\Psi_2 = \pi/2$ (a), $\pi/3$ (b), $\pi/4$ (c), and $\pi/6$ (d), as well as at $\Psi_2 = 0$ and $k_2/k_1 = 2$ (e) and 5 (f).

into one. The dashed curves in Fig. 1 relate the values of M_{01} and M_{02} for which C-type lines appear in the beam cross section at the sum-frequency. Next to them, we put the plus sign if at fixed k and Ψ the dependence of M_{02} on M_{01} follows from equality (5), and the minus sign if the curve is constructed on the basis of equality (7).

Figure 2 shows the typical distribution patterns of polarisation ellipses in the plane of the beam cross section at the sum-frequency. The sum of the squares of the lengths of the semiaxes of each ellipse in this figure is proportional to the intensity at the beam point, which coincides with its centre; the ratio of the lengths of the semiaxes is uniquely expressed in terms of the degree of ellipticity of the wave at the same point in space; and the angle of inclination of its principal axis coincides with $\Phi(\varphi) = 0.5\arg(E_+^{\rm SF} E_-^{\rm SF*})$ in the centre of the ellipse. If the values of M_{01} and M_{02} correspond to the points of the dashed curve in Fig. 1, which are within the grey area, then in the beam cross section at the sum-frequency there appears only one C-line (Fig. 2a), if to the points inside the white area there appear two L-lines and one C-line (Fig. 2b).

Our studies showed that $M(\varphi_\pm) = \mp 1$, and $|\Phi(\varphi_\pm + 0) - \Phi(\varphi_\pm - 0)| = \pi/2$, i.e., the rotation angle of the principal axis of the polarisation ellipse at the points $\varphi_\pm$ experiences a jump by $\pi/2$. This is clearly seen from Fig. 3a, which shows

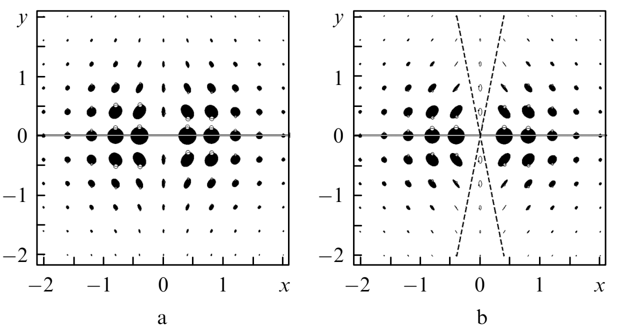

Figure 2. Transverse spatial distribution of polarisation in the beam at the sum-frequency at $M_{01} = -0.25$, $k_2/k_1 = 2$, $\Psi_2 = \pi/2$ and $M_{02} = -0.938$ (a) and 0.903 (b). Solid lines show the C-lines, and dashed lines – L-lines.

the degree of ellipticity and rotation angle of the principal axis of the polarisation ellipse versus the polar angle φ, where $0 \leqslant \varphi \leqslant \pi$. For selected values of k, $M_{01,02}$ and Ψ, the equation of the C-line has the form $\varphi = 0$, because $\varphi_- = 0$. For other values of parameters of the fundamental radiation beams the jump $\Phi(\varphi)$ by π at the point $\varphi = \pi/2$ may be absent.

If $\Psi \neq 0$, then for the values of $M_{01,02}$, corresponding to the intersection point of the dashed curves (Fig. 1) constructed from equalities (5) and (7), respectively, there can appear two L-lines and two C-lines in beam cross section at the sum-frequency. The beam cross section is divided in this case by two C-lines into the four sectors, within each of

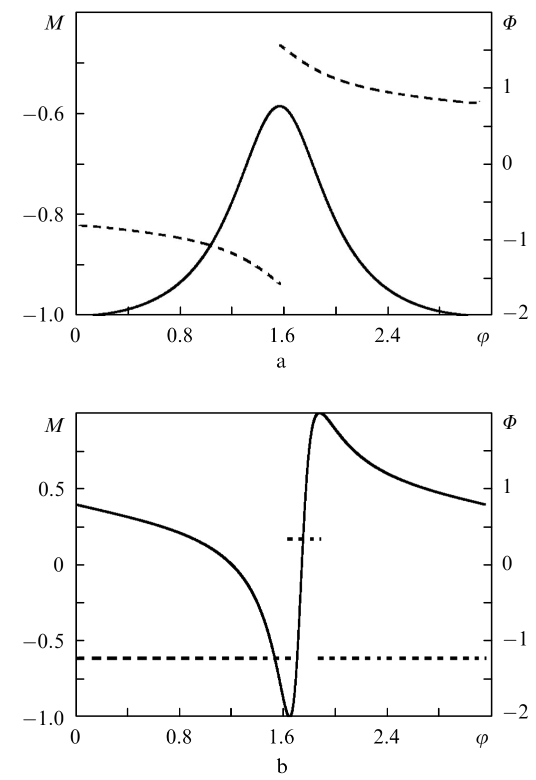

Figure 3. Dependences of the degree of ellipticity (solid curves) and the rotation angle of the principal axis of the polarisation ellipse (dashed curves) on the polar angle in the beam cross section at the sum-frequency at $k_2/k_1 = 2$ and $M_{01} = -0.25$, $M_{02} = -0.938$, $\Psi_2 = \pi/2$ (a), $M_{01} = -0.3$, $M_{02} = 0.6$, $\Psi_2 = \pi/6$ (b).

which the degree of ellipticity changes from -1 to 1 and the rotation angle of the principal axis of the polarisation ellipse remains unchanged. To this end, it discontinues by $\pi/2$ at the intersection of each of the C-lines. Figure 3b shows one of these sectors and parts of two adjacent sectors to the right and left. As the Ψ decreases, the points of intersection of the dashed curves, denoted by signs 'plus' and 'minus', move to the point $M_{01,02} = 0$ in Fig. 1. In this case, the sizes of two of the four aforementioned sectors are reduced and the radiation intensity in smaller sectors drops substantially. In the limiting case $M_{01,02} = 0$, 'small' sectors collapse into a line on which the radiation intensity is zero. If $\Psi = 0$, then the appearance of two C-lines is impossible.

Concluding the article, we would like to note that we predicted for the first time the appearance of the polarisations singularities of C- and L-types in beam cross section at the sum-frequency generated in the bulk of an isotropic gyrotropic medium by two coaxial collinearly propagating elliptically polarised Gaussian beams. We found the parameters of the fundamental waves, at which one or two C-lines and two L-lines appear in the beam cross section at the sum-frequency, and also analysed the possible characteristics of emerging transverse inhomogeneous distributions of polarisation.

Such polarisation singularities can apparently emerge during the second harmonic and sum-frequency generation in the case of reflection from the surface of a nonlinear optically active medium, during the self-focusing in a medium with a nonlocal optical response, as well as in some other problems of nonlinear optics.

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References

1. Nye J.F. *Proc. Royal. Soc. Lond. A*, **389**, 279 (1983).
2. Berry M.V., Dennis M.R. *Proc. Royal. Soc. Lond. A*, **457**, 141 (2001).
3. Berry M.V., Dennis M.R. *Proc. Royal. Soc. Lond. A*, **459**, 1261 (2003).
4. Berry M.V., Dennis M.R., Dennis M.R., Lee R.L. *New J. Phys.*, **6**, 162 (2004).
5. Dennis M.R. *Opt. Lett.*, **33**, 2572 (2008).
6. Dennis M.R. *Opt. Commun.*, **213**, 201 (2002).
7. Angelsky O.V., Mokhun I.I., Mokhun A.I., Soskin M.S. *Phys. Rev. E*, **65**, 036602 (2002).
8. Chen Y.F., Lu T.H., Huang K.F. *Phys. Rev. Lett.*, **96**, 033901 (2006).
9. O'Holleran K., Flossmann F., Dennis M.R., Padgett M.J. *J. Opt. A: Pure Appl. Opt.*, **11**, 094020 (2009).
10. Egorov Yu.A., Fadeyeva T.A., Volyar A.V. *J. Opt. A: Pure Appl. Opt.*, **6**, 217 (2004).
11. Flossmann F., Ulrich T., Schwarz U.T., Maier M., Dennis M.R. *Phys. Rev. Lett.*, **95**, 253901 (2005).
12. Bogatyryova G.V., Felde K.V., Polyanskii P.V., Soskin M.S. *Opt. Spektrosk.*, **97**, 833 (2004).
13. Volkov S.N., Makarov V.A., Perezhogin I.A. *Kvantovaya Elektron.*, **36**, 860 (2006) [*Quantum Electron.*, **36**, 860 (2006)].
14. Makarov V.A., Perezhogin I.A. *Opt. Commun.*, **281**, 3906 (2008).
15. Makarov V.A., Perezhogin I.A. *J. Opt. A: Pure Appl. Opt.*, **11**, 074008 (2009).
16. Golubkov A.A., Makarov V.A., Perezhogin I.A. *Vestnik Mosk. Univer. Ser. Fiz. Astronom.*, (1), 52 (2009).
17. Makarov V.A., Perezhogin I.A. *Kvantovaya Elektron.*, **39**, 627 (2009) [*Quantum Electron.*, **39**, 627 (2009)].
18. Koroteev N.I., Makarov V.A., Volkov S.N. *Laser Phys.*, **9**, 655 (1999).