INTEGRATED OPTICAL WAVEGUIDES

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Effects of surface roughness and absorption on light propagation in graded-profile waveguides

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Abstract. **This paper examines the effects of surface roughness and absorption on laser light propagation in graded-profile waveguiding structures. We derive analytical expressions for the scattering and absorption coefficients of guided waves and analyse these coefficients in relation to parameters of the waveguiding structure and the roughness of its boundary. A new approach is proposed to measuring roughness parameters of precision dielectric surfaces. Experimental evidence is presented which supports the main conclusions of the theory.**

Keywords: *surface roughness, graded-profile waveguide, wave attenuation, absorption, scattering, rms deviation.*

1. Introduction

The theory of light propagation in optical dielectric waveguides has been the subject of many studies [1 – 4]. There is particular interest in graded-profile waveguides because such systems have only one sharp interface and, accordingly, low scattering losses. Moreover, such structures can be produced in crystals possessing electro-optical, acousto-optic and nonlinear properties [1]. This enables high-performance planar optoelectronic and acousto-optic devices to be fabricated using such structures as key components.

The existing theory of light propagation in graded-profile waveguides is only approximate: it does not take into account a number of factors that cause guided wave attenuation. This refers first of all to the loss through scattering by a rough waveguide boundary. Another possible loss source is absorption. In particular, waveguides produced via Ag^+ ion exchange in glass [5] show wave attenuation due to the light absorption in silver nanoparticles.

The problem of light propagation in graded-profile absorbing waveguides with a rough boundary is of interest not only for accurately describing the properties and characteristics of such waveguides but also for devising novel techniques for measuring the surface roughness of dielectrics [6]. Such techniques take advantage of waveguide scattering and offer the highest sensitivity among optical methods [7]. At the same time, the ability to apply such techniques to absorbing structures depends on knowledge of actual waveguide parameters. A more perfect theory may allow one to determine not only surface roughness parameters but also the distribution of the complex dielectric permittivity in the near-surface region of materials.

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2. Formulation of the problem and approximations

We consider a graded-profile planar waveguide with parameters indicated in Fig. 1. The near-surface region of the dielectric, with permittivity ε_3 (substrate), has a region with increased permittivity: $\varepsilon(y) = \varepsilon_3 + \Delta\varepsilon'(y) + \mathrm{i}\varepsilon''(y) = \varepsilon'(y) + \mathrm{i}\varepsilon''(y)$. Here, $\Delta\varepsilon'(y)$ and $\varepsilon''(y)$ are the increments of the real and imaginary parts of $\varepsilon(y)$ due to the incorporation of atoms (molecules or ions) into the substrate. There is no absorption in the substrate or the medium with permittivity ε_1. The interface ($y = 0$) has rough morphology with an rms roughness height σ such that $\sigma \ll \lambda$, which corresponds to a rather smooth surface profile (λ is the wavelength of the light used).

Propagating through such a waveguide, a mode of a given type decays with an attenuation coefficient α through both scattering by the interfacial roughness and absorption in the near-surface region of the substrate. Our purpose is to

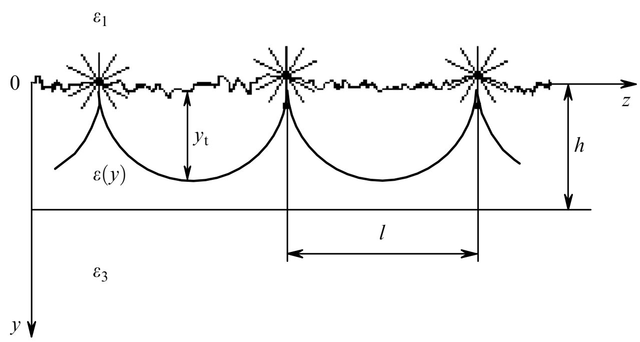

Figure 1. Schematic of wave propagation in a graded-profile planar waveguide.

find the relationship between the attenuation coefficient α, waveguide parameters and σ.

Since the increments of the real and imaginary parts of $\varepsilon(y)$ are proportional to the concentration of incorporated atoms (ions) [2], they can be taken to vary in the same way:

$$\varepsilon(y) = \varepsilon_3 + [\varepsilon'(0) - \varepsilon_3]f(y) + \mathrm{i}\varepsilon''(0)f(y). \quad (1)$$

Here $\varepsilon'(0)$ and $\varepsilon''(0)$ are the real and imaginary parts of ε at $y = 0$, and $f(0) = 1$. Structures of practical interest should meet the low absorption condition: $\varepsilon'(0) \gg \varepsilon''(0)$.

The problem in hand can be solved in a low-loss approximation: the scattering-induced attenuation coefficient, α_{sc}, and absorption coefficient, α_{ab}, are independent of one another, and $\alpha = \alpha_{sc} + \alpha_{ab}$. Therefore, the problem can be divided into two independent parts, and we can take for either part that low losses do not change the transverse structure of guided modes.

3. Solution to the problem

To find α_{sc} (the first part of the problem), we use the geometric optics approach [8] and take $\varepsilon''(0) = 0$, which means that there is no absorption in the near-surface region of the substrate. Consider the solution for TE waves. It follows from the well-known Rayleigh relation [9] that, if a light beam incident on a rough surface of a metal of infinite electrical conductivity has a power P_0, the reflected beam power is given by

$$P = P_0 \exp\left[-\varepsilon'(0)\left(\frac{4\pi\sigma\cos\theta}{\lambda}\right)^2\right], \quad (2)$$

where θ is the angle of incidence of the beam on the rough interface. This expression is also valid in the case of total internal reflection from a rough interface between two dielectrics with $\sigma \ll \lambda$. On the other hand, a wave propagating along the z axis of a waveguide decays exponentially because of scattering:

$$P(z) = P_0 \exp(-\alpha_{sc} z). \quad (3)$$

Over a length l (the distance between two consecutive reflections from the rough surface in Fig. 1), the wave is scattered once. Taking $z = l$ in (3) and equating the exponents in (2) and (3), we obtain the following expression for α_{sc}:

$$\alpha_{sc} = \frac{\varepsilon'(0)}{l}\left(\frac{4\pi\sigma}{\lambda}\right)^2 \frac{\varepsilon'(0) - \gamma^2}{\gamma^2}. \quad (4)$$

Here we use the relation $\cos^2\theta = [\varepsilon'(0) - \gamma^2]/\gamma^2$, where γ is the phase delay coefficient of the wave, which can be found from the dispersion relation for a particular mode [1]. The expression for l can be derived from the corresponding dispersion relation for the graded-profile waveguide by differentiating it with respect to $k_0\gamma$ [7]:

$$l = \int_0^{y_t} \frac{2\gamma}{\sqrt{\varepsilon'(y) - \gamma^2}}\mathrm{d}y + \frac{2\gamma}{k_0\sqrt{\varepsilon'(y) - \gamma^2}\sqrt{\gamma^2 - \varepsilon_1}}. \quad (5)$$

Here y_t is the coordinate of the turning point, where $\varepsilon'(y_t) = \gamma^2$, and k_0 is the propagation constant in vacuum.

Thus, the first part of the problem is solved: α_{sc} is expressed through waveguide parameters and surface roughness.

Note that there is a well-developed algorithm for finding the permittivity profile $\varepsilon'(y)$ in the near-surface region of a substrate for ion exchange and solid-state diffusion processes [2]. Therefore, knowing $\varepsilon'(y)$ and other waveguide parameters, one can calculate the γ spectrum for all modes using appropriate dispersion relations. Next, relation (5) can be used to find l, and α_{sc} can be evaluated using (4).

To find an expression for α_{ab} (the second part of the problem), we use a perturbation method [3] with $\sigma = 0$. Given that the permittivity of the near-surface region of the substrate is a complex quantity and that $\varepsilon''(0)/\varepsilon'(0) \ll 1$, the longitudinal component of the propagation constant, k_z, should also be treated as a complex quantity: $k_z = k_0\gamma = k_0(\gamma' + \mathrm{i}\gamma'')$. Here $k_0\gamma'' = \alpha_{ab}$ is the absorption coefficient, and $\gamma''/\gamma' \ll 1$.

We substitute expressions for complex-valued $\varepsilon(y)$ and k_z into a standard dispersion relation for TE waves. Given that the $\varepsilon''(0)/\varepsilon'(0)$ and γ''/γ' ratios are small, we Taylor expand the dispersion relation about $k_0\gamma'$ and $\varepsilon'(0)$. Retaining only the first-order terms, we obtain

$$k_0\int_0^{y_t}\sqrt{\varepsilon'(y) - \gamma'^2}\mathrm{d}y - \arctan\left[\sqrt{\frac{\gamma'^2 - \varepsilon_1}{\varepsilon'(0) - \gamma'^2}}\right] - \left(m - \frac{3}{4}\right)\pi + \mathrm{i}\left[\frac{g\varepsilon''(0)}{\varepsilon'(0)} + \frac{q\gamma''}{\gamma'}\right] = 0. \quad (6)$$

Here m is the mode number, and the coefficients g and q depend on waveguide parameters and wavelength. Equating the real and imaginary parts to zero, we obtain two equations. One of them determines the real part of $k_z = k_0\gamma'$ and is the dispersion relation for TE waves in an ideal graded-profile waveguide [1].

The other equation determines the imaginary part of k_z, i.e. the absorption coefficient for TE waves:

$$\alpha_{ab} = \frac{\varepsilon''(0)}{l}\left[k_0\int_0^{y_t}\frac{f(y)}{\sqrt{\varepsilon'(y) - \gamma'^2}}\mathrm{d}y + \frac{1}{\varepsilon'(0) - \varepsilon_1}\sqrt{\frac{\gamma'^2 - \varepsilon_1}{\varepsilon'(0) - \gamma'^2}}\right]. \quad (7)$$

The absorption coefficient for TM waves is given by

$$\alpha_{ab} = \frac{\varepsilon''(0)}{l_{TM}}\left[k_0\int_0^{y_t}\frac{f(y)}{\sqrt{\varepsilon'(y) - \gamma'^2}}\mathrm{d}y + \frac{\varepsilon'(0)}{\varepsilon_1[\varepsilon'(0) - \varepsilon_1]}\sqrt{\frac{\gamma'^2 - \varepsilon_1}{\varepsilon'(0) - \gamma'^2}}\right]. \quad (8)$$

The expression for l_{TM} can be found in the same way as (5), by differentiating the dispersion relation for TM modes.

Thus, the two parts of the above problem are solved, and we can turn to analysis of the main relations.

4. Analysis of the scattering and absorption coefficients

We begin analysis of the main relations (4) and (7) from the scattering coefficient α_{sc}. Clearly, α_{sc} is a quadratic function of σ/λ. The influence of waveguide parameters is not so obvious. The most important and interesting dependences are those of α_{sc} on guiding layer thickness for different modes (Fig. 2). Calculations were performed for a waveguide with a Gaussian permittivity profile, $\varepsilon'(y) = \varepsilon_3 + [\varepsilon'(0) - \varepsilon_3]\exp[-(y/h)^2]$, and the following parameters: $\varepsilon'(0) = 2.506$, $\varepsilon_3 = 2.292$, $\varepsilon_1 = 1$, $\lambda = 0.6328$ μm and $\sigma = 5$ nm. Such profiles are typical of waveguides produced by solid-state diffusion [2].

It is known [3] that a wave can propagate in a waveguide when $h > h_{cr}$, where h_{cr} is a critical guiding layer thickness for a particular mode. It can be seen from Fig. 2 that, when h approaches the critical thickness h_{cr} for a given mode, the scattering coefficient sharply rises with layer thickness, reaches a maximum and then gradually falls off. Such behaviour is exhibited by all the modes, but the peak value of α_{sc} decreases with increasing mode number. The shape of the curves is governed by the variation of l with h. It should be emphasised that, at large guiding region thicknesses, the scattering coefficient increases with mode number, and the fundamental mode has the lowest loss. At the same time, near the critical thickness (of the order of 1 – 2 μm in Fig. 2) various relationships between the α_{sc} of different modes are possible.

It follows from (4) that α_{sc} increases with $\varepsilon'(0)$. In addition, h_{cr} decreases.

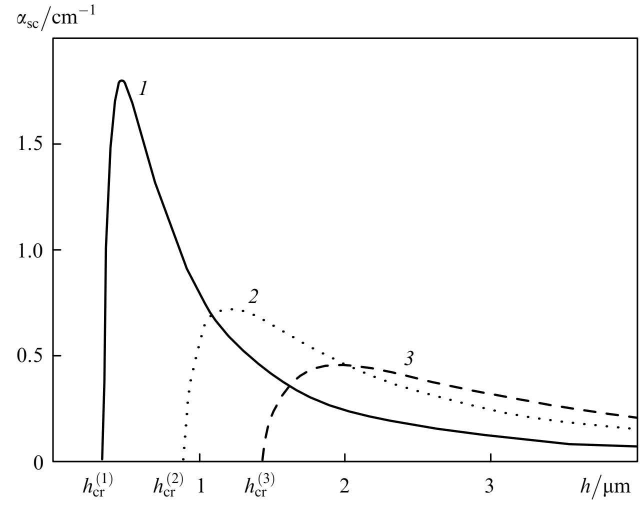

Figure 2. Scattering coefficient α_{sc} as a function of h for different TE modes. Numbers at the curves specify the mode number.

Consider now Eqn (7), which relates the absorption coefficient, α_{ab}, to parameters of the graded-profile waveguide. Clearly, the absorption coefficient increases with $\varepsilon''(0)$. Figure 3 plots the absorption coefficient, α_{ab}, against guiding layer thickness, h, for the three lowest order TE modes in a waveguide with the same parameters as in Fig. 2, $\sigma = 0$ and $\varepsilon''(0) = 10^{-5}$.

As seen in Fig. 3, $\alpha_{ab} = 0$ at the critical guiding layer thickness, but the absorption coefficient rises sharply with increasing h. The relatively low α_{ab} values near h_{cr} can be accounted for by the low guided wave power in the guiding region, where there is absorption. For $h \to \infty$, the absorption coefficient tends towards the α_{ab} value for an infinite medium with $\varepsilon' = \varepsilon'(0)$ and $\varepsilon'' = \varepsilon''(0)$. It is worth noting that, at a given thickness, lower order modes have greater attenuation coefficients, whereas α_{sc} varies with the mode number in the opposite way.

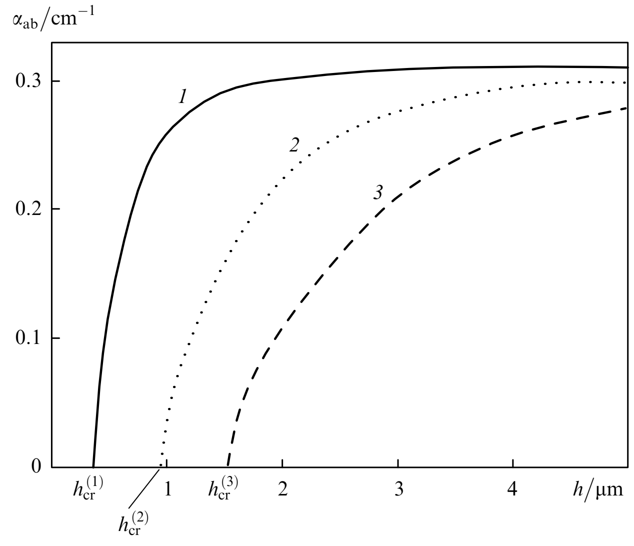

Figure 3. Absorption coefficient α_{ab} as a function of h for different TE modes. Numbers at the curves specify the mode number.

It is of interest to analyse the total attenuation coefficient α as a function of h when there are both surface roughness and absorption. Such data are presented in Fig. 4. The calculations were performed with the same graded-profile waveguide parameters as above. The rms deviation is constant ($\sigma = 5$ nm) and $\varepsilon''(0)$ takes various values.

As seen in Fig. 4, when the waveguide thickness approaches the critical one, low losses can be ensured even at high $\varepsilon''(0)$ and σ values because the mode energy in the guiding region is low. It should, however, be kept in mind that the mode stability is then lower: a very slight

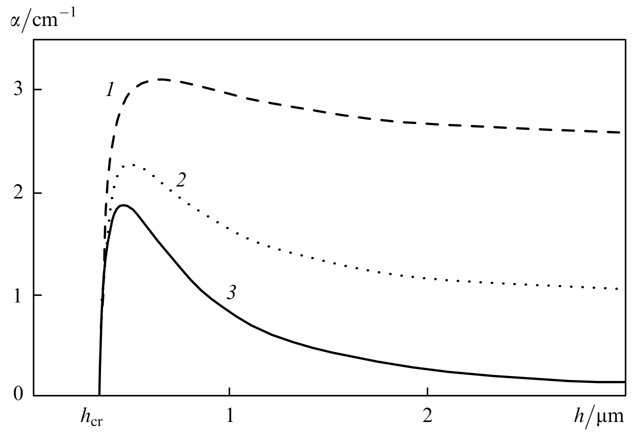

Figure 4. Total attenuation coefficient α as a function of h for the fundamental TE mode at $\varepsilon''(0) = 8 \times 10^{-5}$ (*1*), 3×10^{-5} (*2*) and 0 (*3*).

decrease in h may disturb waveguiding and lead to energy emission into the substrate. At low $\varepsilon''(0)$ values ($\sim 10^{-5}$), the dominant attenuation mechanism (at a given value of σ) is scattering. With increasing $\varepsilon''(0)$, the absorption contribution to attenuation increases throughout the h range examined.

The above analysis of wave attenuation in a graded-profile waveguide was carried out for TE waves. Very similar results were obtained for TM waves. The main distinction is that, at the same parameters of the waveguiding system, α_{ab} and α_{sc} for TM waves are, respectively, smaller and greater than those for TE modes.

Note an interesting application of the problem in question. Recall that the total attenuation coefficient is the sum of the scattering and absorption coefficients. The former depends only on σ (at a given set of waveguide parameters, wavelength and mode number), and the latter depends only on $\varepsilon''(0)$. Therefore, the total attenuation coefficient of mode m can be represented in the form $\alpha_m = A_m\sigma^2 + B_m\varepsilon''(0)$, where A_m and B_m depend on the waveguide parameters, wavelength and mode number. Similarly, for mode $m+1$ we have $\alpha_{m+1} = A_{m+1}\sigma^2 + B_{m+1}\varepsilon''(0)$. In practice, A and B can be determined with high accuracy.

Therefore, measuring the attenution coefficients of at least two modes and knowing A and B, one can determine both σ and $\varepsilon''(0)$ by solving the resultant system of equations. The sensitivity of this approach to changes in σ is markedly higher in comparison with other optical methods. In addition to the high sensitivity and ease for application, this method, as distinct from profilometry, eliminates limitations stemming from a finite tip radius and vibrations and averages the measurand over a considerably larger area.

5. Experimental investigation of guided wave attenuation in graded-profile waveguides

To verify the above theory of light propagation in graded-profile absorbing waveguides with a rough boundary, we carried out experimental studies. Waveguides were produced via ion exchange (Ag^+ ions) in glass substrates at 225 °C by a known procedure [2].

First, the phase delays of all TE modes supported by one of the waveguides were measured by the prism coupling method with an accuracy of 2×10^{-4} (Table 1, column 1). From these data, we derived the $\varepsilon'(y)$ distribution and its parameters: $\varepsilon' = 2.519$ and $h = 3.68$ μm. Next, we measured the total attenuation coefficient of the same modes, α_{meas}, with an accuracy of 3×10^{-2} cm^{-1} (column 4).

As mentioned above, when the waveguide parameters are known the expression for the total attenuation coefficient contains two quantities that are independent of mode number. These are the rms roughness height σ and $\varepsilon''(0)$, which can be found by jointly solving two equations for different pairs of modes. Solving six such systems of equations for different sets of modes, we obtained the following averages: $\sigma = 6.96 \pm 0.36$ nm and $\varepsilon''(0) = (4.42 \pm 0.35) \times 10^{-5}$.

Using these values and Eqs (4) and (7), we calculated α_{sc} and α_{ab} for different modes. The calculation results are presented in Table 1. The average error of determination of α_{sc} and α_{ab} was 6×10^{-2} cm^{-1}. In accordance with theoretical predictions, α_{sc} increases and α_{ab} decreases with increasing mode number. The total attenuation coefficients determined as the sum of α_{sc} and α_{ab} are listed in Table 1 (column 5). These values are in reasonable agreement with the experimental data, suggesting that adequate approximations were made in describing light propagation in graded-profile absorbing waveguides with a rough boundary.

Table 1. Main parameters of TE wave attenuation in a graded-profile waveguide.

Mode	Column				
	1	2	3	4	5
	γ	α_{sc}/cm^{-1}	α_{ab}/cm^{-1}	α_{meas}/cm^{-1}	α_{calc}/cm^{-1}
1	1.5764	0.17	1.20	1.31	1.37
2	1.5588	0.46	0.96	1.43	1.42
3	1.5424	0.75	0.77	1.54	1.52
4	1.5259	1.06	0.63	1.65	1.69

6. Conclusions

It is well known [2] that the $\varepsilon(y)$ distribution in graded-profile waveguides depends on the waveguide fabrication process. In the above theory of light propagation in graded-profile waveguides, this distribution was arbitrary. Consequently, the present results can be used to analyse the absorption coefficient of waveguiding structures produced by different methods on different substrates using different materials as diffusants. An important issue for waveguides that are used as integrated optical devices or components is to find conditions and parameters that minimise losses. At the same time, when implementing advanced approaches to surface roughness measurements, it is desirable to maximise α at a given value of σ. This will allow one to maximise the measurement sensitivity and assess the roughness of very smooth dielectric surfaces.

When further developing the above theory, particular attention should be paid to describing scattering processes in the systems considered here. The Rayleigh relation (2) is only approximate and needs refinements, especially in the case of small $\varepsilon(0) - \varepsilon_1$ differences. Moreover, to more accurately describe scattering by a rough dielectric surface, one should take into account the actual distribution of the spectral density of surface roughness and, possibly, the imperfect surface layer [10].

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