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# Estimation of the reliability of heterolasers subjected to ageing under irradiation by a fast particle flux

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*Abstract.* Relations for estimating the reliability of heterolasers operating under irradiation conditions are calculated based on the probabilistic analysis. The accumulation of defects in their active regions is considered to be the physical cause of their failure.

Keywords: heterolasers, defects, reliability, irradiation.

## 1. Introduction

Currently, heterolasers (HLs) are successfully used in various technical applications. In some cases, an important factor is the HL reliability under natural irradiation by fast particles, for example, in open space. To determine this parameter, it is necessary to develop methods for estimating the probability of failure-free HL operation under irradiation.

It has been shown previously that application of probabilistic estimates for 'ageing' (degradation) of cw HLs is efficient for determining their service life [1-6]. Below, we consider an approach to estimate the reliability of HLs in the case of their degradation under simultaneous effect of both the operation time and the irradiation dose by fast particles of different types on the degradation rate.

The up-to-date concept of the HL failure statistics as the most adequate approach to reliability prediction is based on the Weibull distribution (see, for example, [2, 5, 6] and references therein). A cumulative distribution within the Weibull statistics is presented as 1 - R(x), where the function R(x) has the form

$$R(x) = \begin{cases} \exp[-(x/\theta)^{\beta}] \text{ at } x \ge 0, \\ 0 \quad \text{at } x < 0. \end{cases}$$
(1)

Here,  $\theta$ , and  $\beta > 0$  are numerical parameters. Within this approach the probability of preserving the HL operation capacity, for example, for an operation time  $t \ge 0$  is

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$$R(t) = \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right],\tag{2}$$

whereas the failure probability for this time is 1 - R(t).

The distribution parameters  $\theta$  and  $\beta$  are only determined empirically based on failure tests of a series of HLs, beyond their relationship with any processes occurring in the HL. Accordingly, in a number of cases some very important conditions of the HL operation remain disregarded. An example is the HL operation in space, where reliability is a very urgent problem. These conditions are characterised by an additional (with respect to terrestrial conditions) effect of penetrating cosmic rays, which leads to enhanced HL ageing and increases the failure probability. The purpose of this study is to analyse the HL reliability from the point of view of the aforementioned additional effects, proceeding from the physical concepts of the HL ageing.

#### 2. Failure distribution

Let us first consider the properties of the Weibull distribution and the physical meaning of the parameters entering it. According to Eqn (2), the HL failure probability during a sufficiently narrow time interval  $\Delta t$  (between instants  $t_0$  and  $t_0 + \Delta t$ ) is

$$r(t_0)\Delta t = -\frac{\mathrm{d}R}{\mathrm{d}t}\Big|_{t=t_0}\Delta t = R(t_0)\left(\frac{t_0}{\theta}\right)^{\beta-1}\left(\frac{\beta}{\theta}\right)\Delta t.$$
 (3)

For the particular cases  $\beta = 1$  and  $\beta = 2$  the probability density distributions r(t) (3) are, respectively, the well known exponential and Rayleigh distributions.

The distribution  $r(t_0)$  characterises the conditional probability of a failure event, in which the condition is the probability  $R(t_0)$  of the sample operation capacity by the previous instant  $t \le t_0$ . The failure probability v for an operating sample during the period between  $t_0$  and  $t_0 + \Delta t$ can be written as

$$v = \frac{r(t_0)\Delta t}{R(t_0)} = -\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t}\Delta t = \left(\frac{\beta}{\theta}\right)$$
$$\times \left(\frac{t_0}{\theta}\right)^{\beta-1}\Delta t = FR(t_0)\Delta t, \tag{4}$$

where *F* is the failure rate. It follows from (4) that for the Weibull distribution *F* is generally a power-law function of time *t*, with an exponent  $\beta = 1$ . Obviously, for  $\beta = 1$  the failure rate  $F \equiv 1/\theta$ ; it is time-independent. Physically, the

elementary systems as an excited atom or excited nucleus. The probabilities of their spontaneous transitions to the ground state are determined by the corresponding matrix elements and world constants and are also time-independent. As a result, we deal with an exponential distribution law for these elementary excited systems. Obviously, if the internal parameters of a system are aged, *F* increases with time and the distribution law (2) must have an exponent  $\beta > 1$ .

If the HL ageing is characterised as accumulation of internal defects with some effective conditional rate q, the HL degradation state by an instant t will depend on the total number of accumulated defects with a conditional measure Q. These defects can be dislocations in the active region; microscopic damaged portions in this region, induced by thermoelastic and other stresses; clusters of lattice defects, for example, vacancies or interstitials migrated from the surface to the bulk of the active region; deviations from stoichiometry; etc. Their conditional measure Q can be, for example, the total number of defects or the total volume occupied by the defects accumulated during the HL operation time t. Anyhow, it is obvious that their measure increases during the process, for example, as follows:

$$Q = qt. \tag{5}$$

It is also obvious that F should increase with increasing the number of accumulated defects. However, it is difficult to predict the specific form (for example, power-law or some other) of this dependence by a given instant. Certainly, the simplest version is a linear dependence. In any case, at sufficiently small Q (which is quite allowable for high-quality reliable HLs), the function F(Q) can be expressed in terms of a Taylor series expansion with m first powers retained:

$$F \approx F_0 + \sum_{n=1}^m a_n Q^n = F_0 + \sum_{n=1}^m a_n (qt)^n$$
  
=  $F_0 + \sum_{n=1}^m \alpha_n t^n$ , (6)  
 $a_n = \frac{1}{n!} \frac{\mathrm{d}^{(n)} F}{\mathrm{d} Q^{(n)}} \Big|_{Q=0}$ ,  $\alpha_n = a_n (q)^n$ ,

where  $F_0$  is a constant, which is independent of Q, and, correspondingly, of time t. Obviously, a Weibull distribution is implemented when one of the terms that are proportional to  $t^{(\beta-1)}$  dominates in the right-hand side of (6).

Note that in the initial stage of HL tests one can always find a small time interval t in which the inequality

 $F_0 >> \alpha_n t^n$ 

is satisfied.

In this interval the failure statistics will have a distribution similar to exponential, because F is almost constant. This fact is in agreement with the experimental data of [6]. Practically, this test time interval is used for laser 'training' in order to reject obviously unfit samples. Indeed, since  $F_0$  is time-independent, its value is only determined by the number  $Q_0$  of 'frozen' initial defects, which were formed during laser fabrication.

Let us analyse the case of HL ageing under penetrating radiation. Each *i*th type of penetrating radiation with an intensity  $I^{(i)}$  forms specific defects  $Q^{(i)}$ , which additively contribute to *F*. This additive contribution is characterised by the individual coefficient  $a_n^{(i)}$ , which enters Eqn (6). Moreover, the formation rate  $q^{(i)}$  of these defects is also individual for each *i*th radiation type. It is natural to assume that  $q^{(i)}$  is proportional to the intensity  $I^{(i)}$  with a proportionality factor  $\gamma^{(i)}$ :

$$q^{(i)} = \gamma^{(i)} I^{(i)}.$$
 (7)

As a result, for a combination of several ageing mechanisms, the  $\alpha_n$  values in (6) take the form

$$\alpha_n = \sum_i \alpha_n^{(i)}, \quad \alpha_n^{(i)} = a_n^{(i)} (\gamma^{(i)} I^{(i)})^n.$$
(8)

Finally, the probability of failure-free operation during a time interval t can be written as

$$R(t) = \exp(-\xi),\tag{9}$$

where

$$\xi = t \sum_{n=1}^{m} \frac{\alpha_n}{n} t^n.$$
<sup>(10)</sup>

Formula (10) refers to the selected (subjected to training) HL samples; therefore,  $\xi$  does not contain the first-power term in *t*, related to  $F_0$ . Each *i*th term in the sum (8) for  $\alpha_n$  is determined by the specific ageing mechanism. Obviously, the first (*i* = 1) term that is necessarily present in these sums is due to the laser operation. Thus,  $\gamma^{(1)}$  in (7) is a proportionality factor between the rate  $q^{(1)}$  of the defect production caused by laser operation and, for example, the laser pump current  $I^{(1)}$ .

In the absence of penetrating radiation the defect production rate is indeed controlled by the pump current. The latter determines both the temperature and the volume density of the laser energy in the cavity, which in turn determine the production rate of the defects leading to catastrophic degradation and HL failure [7]. For this reason, it is expedient to consider the quantity  $\gamma^{(1)}$  in formula (7) specifically as a proportionality factor between  $q^{(1)}$  and  $I^{(1)}$ . The other quantities  $q^{(i)}$  with  $i \ge 2$  are coefficients that characterise the increase in the number of defects due to the penetrating external radiation.

Due to the presence of different powers of time *t* in formula (10), the joint distribution (9) is not of Weibull type. Nevertheless, in practice, having again approximated the polynomial  $\xi$  by only the dominant term of power *n*, we finally have the Weibull distribution (2) with an integer value  $\beta = n + 1$  and  $\theta = (n/\alpha_n)^{1/n+1}$ . In this context we should note that the reason for using the fractional  $\beta$  values to fit experimental data with the Weibull distribution can be

arise from the fact that  $\xi$  is unsatisfactorily approximated by only one term with a specific power of *n* (i.e., the effective number *Q* of defects increases as a fractional power of time *t*).

Obviously, the polynomial coefficients  $\alpha_n$  can be found only experimentally, because, as was mentioned above, there is no complete understanding of the physical nature of the dependence FR(Q). Nevertheless, it is obvious that this dependence contains (as parameters) quantities characterising the HL operation conditions (for example, the heatsink temperature) and (as arguments) the  $I^{(i)}$  values.

Consider now the possibility of determining the factors  $\alpha_n$  from experimental results of HL reliability tests in the presence of penetrating radiation. Being initially based on the Weibull statistics, one can use in principle the experimental data on the laser mean time between failures (expected lifetime) *T* and on its variance  $\delta T^2$ , which are defined as

$$T \approx \frac{1}{N} \sum_{k=1}^{N} t_k r(t_k) \approx \frac{\theta}{\beta} \Gamma\left(\frac{1}{\beta}\right),\tag{11}$$

$$\overline{\delta T^2} \approx \frac{1}{N} \sum_{k=1}^{N} (t_k - T)^2 r(t_k)$$
$$\approx \theta^2 \left[ \frac{2}{\beta} \Gamma\left(\frac{2}{\beta}\right) - \frac{1}{\beta^2} \Gamma\left(\frac{1}{\beta}\right) \right], \tag{12}$$

where *N* is the number of HL samples subjected to tests,  $t_k$  is the failure time of the *k*th sample, and  $\Gamma(...)$  is the gamma function.

It follows from (11) and (12) that T and  $\overline{\delta T^2}$  are independent; therefore, when their experimental values are known, one can first find  $\beta$ , using the relation

$$\overline{\frac{\delta T^2}{T^2}} \approx \left[ 2\beta \frac{\Gamma(2/\beta)}{\Gamma(1/\beta)} - 1 \right] \left[ \Gamma\left(\frac{1}{\beta}\right) \right]^{-1}, \tag{13}$$

and then derive  $\theta$  from (11), using the known value of  $\beta$ .

For the experimental T and  $\delta T^2$  values, measured both in the presence and in the absence of penetrating radiation, one can select separately each  $\theta^{(i)}$  value.

As was mentioned above within the Weibull statistics, due to the different mechanisms F is characterised by a power-law dependence with the same exponent and, therefore, the same  $\beta$  value. Obviously, this is a strict requirement, which means that the ratio  $\delta T^2/T^2$  is the same for all ageing types; therefore, the summation index nis absent in formulas (6). In this case, we omit n by replacing the power n with  $\beta - 1$  in the formulas.

Having successively tested the HL under the conditions of each effect, characterised by the reference intensity  $I_0^{(i)}$ , one can find  $\theta_0^{(i)}$  in the aforementioned way and then determine  $\gamma^{(i)}$ , which enters (7), according to the relation

$$\gamma^{(i)} = (\beta - 1)[\theta_0^{(i)}]^{-\beta} [I_0^{(i)}]^{1-\beta}.$$
(14)

The final  $\theta$  value for the joint distribution and arbitrary intensities of the effect  $I^{(i)}$  has the form

$$\theta = \left[\sum_{i} \theta_{0}^{(i)^{-\beta}} \left(\frac{I^{(i)}}{I_{0}^{(i)}}\right)^{\beta-1}\right]^{-1/\beta}.$$
(15)

Thus, we can completely determine the two-parameter ( $\theta$  and  $\beta$ ) Weibull distribution (2), and, correspondingly, the HL failure probability in the presence of external irradiation sources with arbitrary combinations of intensities  $I^{(i)}$ .

Along with the stringent requirements to the possibility of describing the reliability in terms of the Weibull statistics and under the conditions of combined irradiation (see above), we should also note some experimental difficulties, related to the measurement of T and  $\delta T^2$ . These difficulties are caused by the following. The time necessary to precisely measure them is rather long, because the possible range of variation in  $\delta T^2/T^2$  within the Weibull statistics is narrow: according to (13),  $0.71 \leq \overline{\delta T^2}/T^2 < 1$  for  $\beta$  changing from 1 to  $\infty$ . This time should obviously exceed T, which is especially problematic in the case of highly reliable HLs, for which T may be few years even under enhanced degradation conditions at elevated temperatures. In this context, it may be reasonable to determine the  $\alpha_n$  values, which directly enter Eqn (6) for F. One can experimentally find  $F(t_k)$ :

$$F(t_k) \approx \frac{\Delta N_k}{N(t_k)\Delta t},$$
 (16)

where  $\Delta N_k$  is the number of samples that failed during the time interval from  $t_k$  to  $t_k + \Delta t$ , and  $N(t_k)$  is the number of samples that functioned properly by the instant  $t_k$ . Note that, in view of the practical importance of the parameter F, it is often measured using a specially introduced unit 1 FIT (equal to  $10^{-9} h^{-1}$ ). When the number of measurements  $F(t_k)$  is sufficiently large, one can find the dependence F(t) for the time sequence  $t_k$  using interpolation of the  $F(t_k)$  values by a polynomial of some power m and thus determine the coefficients  $\alpha_n$ . Then, having found  $\tilde{\alpha}_n^{(i)}$  for each specific case of irradiation with a reference intensity  $I_0^{(i)}$  (see above), one can easily find the polynomial  $\xi$ , which enters (9), for the specific HL operation conditions, characterised by the values  $I^{(i)}$ :

$$\xi = t \sum_{n=1}^{m} \frac{1}{n} t^n \left[ \sum_i \tilde{\alpha}_n^{(i)} \left( \frac{I^{(i)}}{I_0^{(i)}} \right)^n \right].$$
(17)

Thus, the probability R of failure-free operation during a time interval t is given by relation (9), where  $\xi$  is determined from Eqn (17).

### 3. Conclusions

We determined the dependences of the HL failure-free operation probability as a function of time and intensity of penetrating radiation. The approach used to find these dependences is based on the following concept: the HL failure is physically caused by the accumulation of defects. Based on the sample tests, we proposed some ways for determining the numerical factors characterising the HL ageing rate. It was shown that the Weibull distribution, which is often used to describe the HL failure statistics, is not always adequate when several ageing mechanisms are simultaneously involved.

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