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Chiral particles in a circularly polarised light field: new effects and applications

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***Abstract*. Cross sections of scattering, absorption, and light pressure for a chiral spherical particle in a circularly polarised light field are studied for different values of the radius, dielectric constant, permeability, and parameter of the chiral particle. The conditions are found under which the cross sections of absorption and light pressure differ significantly when nanoparticles are exposed to light with different polarisations, which can be used to improve synthesis of chiral nanoparticles of complex structure.**

Keywords: *chiral particle, light scattering, light absorption, light pressure.*

1. Introduction

Chirality is a geometric property of three-dimensional objects not to be superimposed with their reflection in the mirror under any shifts and rotations. This property, for example, is inherent in a human hand, DNA molecule, spring. Chiral properties of continuous media are associated with a particular geometry of the molecules of chiral substances [1]. Chiral media exhibit a number of unique optical phenomena such as rotation of the plane of polarisation, circular dichroism, etc. [2 – 5]. The examples of chiral substances are a solution of sugar, nucleic acids, quartz, etc. Artificial chiral objects are the basis for creation of metamaterials with negative refractive properties of electromagnetic waves [6 – 8].

It is known that a material object, placed in a light field, is subjected to the light pressure force [9]. This phenomenon becomes particularly noticeable in a laser field, when there is an opportunity to observe the levitation of transparent dielectric particles [10], to capture and confine them [11]. Currently, much attention has been devoted to studying the forces of light pressure exerted by laser beams on dielectric particles [12 – 15]. Works which address the effect of the light pressure on the objects made of 'unusual' materials (chiral, with negative refraction) are quite few. For example, Riyopoulos [16] studies the effect of the light pressure force on the flat interface between a dielectric medium and a medium with negative refraction, Kemp et al [17] – on a thin plane-parallel plate made of a material with negative refraction, and Ross and Lakhtakia [18] – on a plate made of a chiral material. Finally, Chen et al. [19] study the light pressure force acting on a spherical particle made of a hypothetical material with the property of an invisibility cloak. Investigation of the effect of the light pressure force on chiral spherical particles, to our knowledge, has not been performed.

At the same time the optical properties of chiral spherical particles have been studied quite intensively. To date, scattering of a monochromatic plane electromagnetic wave by homogeneous [3, 20] and inhomogeneous [21] chiral spherical particles, scattering from chiral shells covering dielectric and metal spherical particles [22, 23], as well as scattering of a Hermite beam field by a chiral microsphere [24] and scattering of electromagnetic waves by a chiral sphere located in a chiral medium [3, 25] been investigated in detail.

The aim of this paper is to study the characteristics of the action of the light pressure force on a chiral spherical particle in the field of a plane monochromatic electromagnetic wave. Main attention is devoted to left- and right-hand circularly polarised waves, which is due to the difference in their action on chiral particles. To calculate the light pressure we have used the formalism associated with the Maxwell stress tensor [9], which allows one to find an analytical expression for this force in the case of a chiral spherical particle of arbitrary radius.

2. Scattering of a circularly polarised electromagnetic wave by a chiral spherical particle

When solving the problem of an electromagnetic field in an isotropic chiral medium, we follow the method proposed in [26], by applying it to the medium described by constitutive equations

$$\boldsymbol{D} = \varepsilon_{\mathrm{p}}\boldsymbol{E} - \mathrm{i}\chi\boldsymbol{H}, \quad \boldsymbol{B} = \mu_{\mathrm{p}}\boldsymbol{H} + \mathrm{i}\chi\boldsymbol{E}, \qquad (1)$$

where $\boldsymbol{D}$, $\boldsymbol{E}$ and $\boldsymbol{B}$, $\boldsymbol{H}$ are the induction and strength of the electric and magnetic fields, respectively; ε_{p} and μ_{p} are the dielectric constant and permeability of a chiral medium; χ is a dimensionless chirality parameter, which, in general, depends on the frequency of incident radiation [2, 4]. Substituting (1) into Maxwell's equations, we obtain

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$$\operatorname{rot}\boldsymbol{E}+k_0\chi\boldsymbol{E}=\mathrm{i}k_0\mu_\mathrm{p}\boldsymbol{H},\quad \operatorname{rot}\boldsymbol{H}+k_0\chi\boldsymbol{H}=-\mathrm{i}k_0\varepsilon_\mathrm{p}\boldsymbol{E},\tag{2}$$

where $k_0=\omega/c$ is the wave number in vacuum; ω is frequency; c is the speed of light in vacuum.

The system of equations (2) can be conveniently reduced to the matrix form:

$$\begin{bmatrix}\operatorname{rot}\boldsymbol{E}\\ \operatorname{rot}\boldsymbol{H}\end{bmatrix}=K\begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{bmatrix},\quad K=\begin{bmatrix}-k_0\chi & \mathrm{i}k_0\mu_\mathrm{p}\\ -\mathrm{i}k_0\varepsilon_\mathrm{p} & -k_0\chi\end{bmatrix}.\tag{3}$$

With the help of a linear transformation of electromagnetic fields

$$\begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{H}\end{bmatrix}=A\begin{bmatrix}\boldsymbol{Q}_\mathrm{L}\\ \boldsymbol{Q}_\mathrm{R}\end{bmatrix},\quad A=\begin{bmatrix}1 & 1\\ -\dfrac{\mathrm{i}k_\mathrm{p}}{k_0\mu_\mathrm{p}} & \dfrac{\mathrm{i}k_\mathrm{p}}{k_0\mu_\mathrm{p}}\end{bmatrix},\tag{4}$$

where $k_\mathrm{p}=k_0\sqrt{\varepsilon_\mathrm{p}\mu_\mathrm{p}}$, we diagonalise the matrix K:

$$A^{-1}KA=\begin{bmatrix}k_\mathrm{p}-k_0\chi & 0\\ 0 & -k_\mathrm{p}-k_0\chi\end{bmatrix}=\begin{bmatrix}k_\mathrm{L} & 0\\ 0 & -k_\mathrm{R}\end{bmatrix},\tag{5}$$

where k_L and k_R are the wave numbers, which can propagate in a chiral medium, and have left- (L) or right-hand (R) circular polarisation. The components of the transformed field (4) satisfy the equations

$$\operatorname{rot}\boldsymbol{Q}_\mathrm{L}=k_\mathrm{L}\boldsymbol{Q}_\mathrm{L},\ \ \operatorname{rot}\boldsymbol{Q}_\mathrm{R}=-k_\mathrm{R}\boldsymbol{Q}_\mathrm{R},$$
$$\operatorname{div}\boldsymbol{Q}_\mathrm{L}=\operatorname{div}\boldsymbol{Q}_\mathrm{R}=0.\tag{6}$$

Thus, the electromagnetic field inside a chiral spherical particle can be represented as follows [see formula (4)]:

$$\boldsymbol{E}^\mathrm{t}=\boldsymbol{Q}_\mathrm{L}+\boldsymbol{Q}_\mathrm{R},\quad \boldsymbol{H}^\mathrm{t}=-\frac{\mathrm{i}k_\mathrm{p}}{k_0\mu_\mathrm{p}}(\boldsymbol{Q}_\mathrm{L}-\boldsymbol{Q}_\mathrm{R}).\tag{7}$$

For the spherical geometry under consideration, the fields $\boldsymbol{Q}_\mathrm{L}$ and $\boldsymbol{Q}_\mathrm{R}$ have the form [26]

$$\boldsymbol{Q}_\mathrm{L}=\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left[A^\mathrm{L}_{mn\mathrm{e}}\left(\boldsymbol{n}^\mathrm{L}_{mn\mathrm{e}}+\boldsymbol{m}^\mathrm{L}_{mn\mathrm{e}}\right)+A^\mathrm{L}_{mn\mathrm{o}}\left(\boldsymbol{n}^\mathrm{L}_{mn\mathrm{o}}+\boldsymbol{m}^\mathrm{L}_{mn\mathrm{o}}\right)\right],$$
$$\boldsymbol{Q}_\mathrm{R}=\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}\left[A^\mathrm{R}_{mn\mathrm{e}}\left(\boldsymbol{n}^\mathrm{R}_{mn\mathrm{e}}-\boldsymbol{m}^\mathrm{R}_{mn\mathrm{e}}\right)+A^\mathrm{R}_{mn\mathrm{o}}\left(\boldsymbol{n}^\mathrm{R}_{mn\mathrm{o}}-\boldsymbol{m}^\mathrm{R}_{mn\mathrm{o}}\right)\right],\tag{8}$$

where $A^\mathrm{L}_{mn\mathrm{e}}$, $A^\mathrm{L}_{mn\mathrm{o}}$ and $A^\mathrm{R}_{mn\mathrm{e}}$, $A^\mathrm{R}_{mn\mathrm{o}}$ are the expansion coefficients, which can be found using the boundary condition of continuity of tangential components of the electric and magnetic fields on the surface of a chiral particle. Spherical vector functions in (8) have the form [27] ($j=\mathrm{L},\mathrm{R}$)

$$\left\{\begin{matrix}\boldsymbol{n}^{(j)}_{mn\mathrm{e}}\\ \boldsymbol{n}^{(j)}_{mn\mathrm{o}}\end{matrix}\right\}=\frac{n(n+1)}{(rk_j)^2}\psi_n(rk_j)P^m_n(\cos\theta)\left\{\begin{matrix}\cos(m\varphi)\\ \sin(m\varphi)\end{matrix}\right\}\boldsymbol{e}_r+\frac{1}{rk_j}\psi'_n(rk_j)\frac{\partial}{\partial\theta}P^m_n(\cos\theta)\left\{\begin{matrix}\cos(m\varphi)\\ \sin(m\varphi)\end{matrix}\right\}\boldsymbol{e}_\theta+\frac{m}{rk_j\sin\theta}\psi'_n(rk_j)P^m_n(\cos\theta)\left\{\begin{matrix}-\sin(m\varphi)\\ \cos(m\varphi)\end{matrix}\right\}\boldsymbol{e}_\varphi,$$
$$\left\{\begin{matrix}\boldsymbol{m}^{(j)}_{mn\mathrm{e}}\\ \boldsymbol{m}^{(j)}_{mn\mathrm{o}}\end{matrix}\right\}=\frac{m}{rk_j\sin\theta}\psi_n(rk_j)P^m_n(\cos\theta)\left\{\begin{matrix}-\sin(m\varphi)\\ \cos(m\varphi)\end{matrix}\right\}\boldsymbol{e}_\theta-\frac{1}{rk_j}\psi_n(rk_j)\frac{\partial}{\partial\theta}P^m_n(\cos\theta)\left\{\begin{matrix}\cos(m\varphi)\\ \sin(m\varphi)\end{matrix}\right\}\boldsymbol{e}_\varphi,\tag{9}$$

where $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_\varphi$ are the vectors of the spherical coordinate system; $0\leqslant r<\infty$, $0\leqslant\theta<2\pi$ and $0\leqslant\varphi<2\pi$ are the coordinates; $\psi_n(x)=\sqrt{\pi x/2}\,J_{n+1/2}(x)$; $J_{n+1/2}(x)$ is the Bessel function [28]; the prime in the function is the derivative of its argument; $P^m_n(x)$ are the associated Legendre functions [28]. Substituting (8) in (7), we find that TM- and TE-waves cannot exist separately within a chiral particle, because the spherical vector functions $\boldsymbol{n}^{(j)}_{mn\mathrm{e}}$ ($\boldsymbol{n}^{(j)}_{mn\mathrm{o}}$) and $\boldsymbol{m}^{(j)}_{mn\mathrm{e}}$ ($\boldsymbol{m}^{(j)}_{mn\mathrm{o}}$) with different spatial structure enter into (8) as a sum or difference (but not separately). In the case if the particle is made of a material that does not have chiral properties, such separation can be achieved.

To investigate scattering of a plane monochromatic electromagnetic left- or right-hand circularly polarised wave by a chiral spherical particle, we consider an elliptically incident polarised wave:

$$\boldsymbol{E}^\mathrm{i}=(E^\mathrm{i}_x\boldsymbol{e}_x+E^\mathrm{i}_y\boldsymbol{e}_y)\exp(\mathrm{i}k_\mathrm{m}z-\mathrm{i}\omega t),$$
$$\boldsymbol{H}^\mathrm{i}=-\frac{\mathrm{i}}{k_0\mu_\mathrm{m}}\operatorname{rot}\boldsymbol{E}^\mathrm{i}=-\sqrt{\frac{\varepsilon_\mathrm{m}}{\mu_\mathrm{m}}}(E^\mathrm{i}_y\boldsymbol{e}_x-E^\mathrm{i}_x\boldsymbol{e}_y)\exp(\mathrm{i}k_\mathrm{m}z-\mathrm{i}\omega t),\tag{10}$$

where ε_m, $\mu_\mathrm{m}>0$ are the dielectric constant and permeability of the medium in which the particle is located. If $E^\mathrm{i}_x=-\mathrm{i}E^\mathrm{i}_y=E_0$, the incident wave is left-hand circularly polarised, and if $E^\mathrm{i}_x=\mathrm{i}E^\mathrm{i}_y=E_0$, it is right-hand circularly polarised. The field (10) can be represented as an expansion in spherical vector functions [27]:

$$\boldsymbol{E}^\mathrm{i}=-\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\mathrm{i}^n\frac{2n+1}{n(n+1)}\left(\mathrm{i}E^\mathrm{i}_x\boldsymbol{n}^\mathrm{i}_{1n\mathrm{e}}+E^\mathrm{i}_y\boldsymbol{m}^\mathrm{i}_{1n\mathrm{e}}\right)-\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\mathrm{i}^n\frac{2n+1}{n(n+1)}\left(\mathrm{i}E^\mathrm{i}_y\boldsymbol{n}^\mathrm{i}_{1n\mathrm{o}}-E^\mathrm{i}_x\boldsymbol{m}^\mathrm{i}_{1n\mathrm{o}}\right),$$
$$\boldsymbol{H}^\mathrm{i}=-\sqrt{\frac{\varepsilon_\mathrm{m}}{\mu_\mathrm{m}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\mathrm{i}^n\frac{2n+1}{n(n+1)}\left(E^\mathrm{i}_x\boldsymbol{m}^\mathrm{i}_{1n\mathrm{e}}-\mathrm{i}E^\mathrm{i}_y\boldsymbol{n}^\mathrm{i}_{1n\mathrm{e}}\right)-\sqrt{\frac{\varepsilon_\mathrm{m}}{\mu_\mathrm{m}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\mathrm{i}^n\frac{2n+1}{n(n+1)}\left(E^\mathrm{i}_y\boldsymbol{m}^\mathrm{i}_{1n\mathrm{o}}+\mathrm{i}E^\mathrm{i}_x\boldsymbol{n}^\mathrm{i}_{1n\mathrm{o}}\right),\tag{11}$$

where the vector function $\boldsymbol{n}^\mathrm{i}_{1n\mathrm{e}}$, $\boldsymbol{n}^\mathrm{i}_{1n\mathrm{o}}$ and $\boldsymbol{m}^\mathrm{i}_{1n\mathrm{e}}$, $\boldsymbol{m}^\mathrm{i}_{1n\mathrm{o}}$ can be obtained from (9) in a special case $m=1$, with successive substitutions: the indices $j\to\mathrm{i}$ and wave number $k_\mathrm{j}\to k_\mathrm{m}=k_0\sqrt{\varepsilon_\mathrm{m}\mu_\mathrm{m}}$.

The induced electromagnetic field outside a spherical particle has the form

$$\boldsymbol{E}^\mathrm{r}=\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(A^\mathrm{r}_{mn\mathrm{e}}\boldsymbol{n}^\mathrm{r}_{mn\mathrm{e}}+B^\mathrm{r}_{mn\mathrm{e}}\boldsymbol{m}^\mathrm{r}_{mn\mathrm{e}}\right)+$$

$$+\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(A^{\mathrm{r}}_{mn\mathrm{o}}\boldsymbol{n}^{\mathrm{r}}_{mn\mathrm{o}}+B^{\mathrm{r}}_{mn\mathrm{o}}\boldsymbol{m}^{\mathrm{r}}_{mn\mathrm{o}}\right),$$

$$\boldsymbol{H}^{\mathrm{r}}=-\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(A^{\mathrm{r}}_{mn\mathrm{e}}\boldsymbol{m}^{\mathrm{r}}_{mn\mathrm{e}}+B^{\mathrm{r}}_{mn\mathrm{e}}\boldsymbol{n}^{\mathrm{r}}_{mn\mathrm{e}}\right)-\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(A^{\mathrm{r}}_{mn\mathrm{o}}\boldsymbol{m}^{\mathrm{r}}_{mn\mathrm{o}}+B^{\mathrm{r}}_{mn\mathrm{o}}\boldsymbol{n}^{\mathrm{r}}_{mn\mathrm{o}}\right), \tag{12}$$

where $A^{\mathrm{r}}_{mn\mathrm{e}}$, $B^{\mathrm{r}}_{mn\mathrm{e}}$ и $A^{\mathrm{r}}_{mn\mathrm{o}}$, $B^{\mathrm{r}}_{mn\mathrm{o}}$ are the expansion coefficients, which can be found from boundary conditions. Explicit expressions for the spherical vector functions $\boldsymbol{n}^{\mathrm{r}}_{mn\mathrm{e}}$, $\boldsymbol{n}^{\mathrm{r}}_{mn\mathrm{o}}$ and $\boldsymbol{m}^{\mathrm{r}}_{mn\mathrm{e}}$, $\boldsymbol{m}^{\mathrm{r}}_{mn\mathrm{o}}$ are obtained from (9) by successive substitutions: the indices $j\to\mathrm{r}$, wave numbers $k_{\mathrm{j}}\to k_{\mathrm{m}}$, and functions $\psi_n(x)\to\zeta_n(x)=\sqrt{\pi x/2}\,H^{(1)}_{n+1/2}(x)$, where $H^{(1)}_{n+1/2}(x)$ is the Hankel function of first kind [28].

Thus, the induced electromagnetic field in the problem on scattering of an electromagnetic wave (10) by a chiral spherical particle has the form (7) and (12). To find the unknown coefficients of series (8) and (12) we should use the condition of continuity of tangential components of electric and magnetic fields on the surface of a chiral particle of radius a:

$$\left[\boldsymbol{E}^{\mathrm{t}},\boldsymbol{e}_r\right]\big|_{r=a}=\left[\left(\boldsymbol{E}^{\mathrm{i}}+\boldsymbol{E}^{\mathrm{r}}\right),\boldsymbol{e}_r\right]\big|_{r=a}, \qquad \left[\boldsymbol{H}^{\mathrm{t}},\boldsymbol{e}_r\right]\big|_{r=a}=\left[\left(\boldsymbol{H}^{\mathrm{i}}+\boldsymbol{H}^{\mathrm{r}}\right),\boldsymbol{e}_r\right]\big|_{r=a}. \tag{13}$$

Using equations (13) we find the following explicit expressions for the coefficients of series (12):

$$\begin{aligned}
A^{\mathrm{r}}_{mn\mathrm{e}}&=\delta_{1m}\mathrm{i}^{n+1}\frac{2n+1}{n(n+1)}\left(a_nE^{\mathrm{i}}_x-c_nE^{\mathrm{i}}_y\right),\\
A^{\mathrm{r}}_{mn\mathrm{o}}&=\delta_{1m}\mathrm{i}^{n+1}\frac{2n+1}{n(n+1)}\left(c_nE^{\mathrm{i}}_x+a_nE^{\mathrm{i}}_y\right),\\
B^{\mathrm{r}}_{mn\mathrm{e}}&=\delta_{1m}\mathrm{i}^{n}\frac{2n+1}{n(n+1)}\left(b_nE^{\mathrm{i}}_y+c_nE^{\mathrm{i}}_x\right),\\
B^{\mathrm{r}}_{mn\mathrm{o}}&=\delta_{1m}\mathrm{i}^{n}\frac{2n+1}{n(n+1)}\left(c_nE^{\mathrm{i}}_y-b_nE^{\mathrm{i}}_x\right),
\end{aligned} \tag{14}$$

where δ_{1m} is the Kronecker delta. In (14) we introduced the notations

$$\begin{aligned}
a_n&=\frac{V^{\mathrm{L}}_nA^{\mathrm{R}}_n+V^{\mathrm{R}}_nA^{\mathrm{L}}_n}{V^{\mathrm{L}}_nW^{\mathrm{R}}_n+V^{\mathrm{R}}_nW^{\mathrm{L}}_n},\\
b_n&=\frac{B^{\mathrm{L}}_nW^{\mathrm{R}}_n+B^{\mathrm{R}}_nW^{\mathrm{L}}_n}{V^{\mathrm{L}}_nW^{\mathrm{R}}_n+V^{\mathrm{R}}_nW^{\mathrm{L}}_n},\\
c_n&=\mathrm{i}\,\frac{A^{\mathrm{L}}_nW^{\mathrm{R}}_n-A^{\mathrm{R}}_nW^{\mathrm{L}}_n}{V^{\mathrm{L}}_nW^{\mathrm{R}}_n+V^{\mathrm{R}}_nW^{\mathrm{L}}_n},
\end{aligned} \tag{15}$$

which were written using the functions ($j=\mathrm{L},\mathrm{R}$)

$$\begin{aligned}
W^{(j)}_n&=P\psi_n(ak_j)\zeta'_n(ak_{\mathrm{m}})-\psi'_n(ak_j)\zeta_n(ak_{\mathrm{m}}),\\
V^{(j)}_n&=\psi_n(ak_j)\zeta'_n(ak_{\mathrm{m}})-P\psi'_n(ak_j)\zeta_n(ak_{\mathrm{m}}),\\
A^{(j)}_n&=P\psi_n(ak_j)\psi'_n(ak_{\mathrm{m}})-\psi'_n(ak_j)\psi_n(ak_{\mathrm{m}}),\\
B^{(j)}_n&=\psi_n(ak_j)\psi'_n(ak_{\mathrm{m}})-P\psi'_n(ak_j)\psi_n(ak_{\mathrm{m}}),
\end{aligned} \tag{16}$$

where $P=k_{\mathrm{p}}\mu_{\mathrm{m}}/(k_{\mathrm{m}}\mu_{\mathrm{p}})$. In the special case of a particle without chiral properties ($\chi=0$), the coefficient $c_n=0$ is equal to zero, and the coefficients a_n and b_n take a well-known form of Mie coefficients [27]:

$$\begin{aligned}
a_n&=\frac{(k_{\mathrm{p}}/k_{\mathrm{m}})\psi_n(ak_{\mathrm{p}})\psi'_n(ak_{\mathrm{m}})-(\mu_{\mathrm{p}}/\mu_{\mathrm{m}})\psi'_n(ak_{\mathrm{p}})\psi_n(ak_{\mathrm{m}})}{(k_{\mathrm{p}}/k_{\mathrm{m}})\psi_n(ak_{\mathrm{p}})\zeta'_n(ak_{\mathrm{m}})-(\mu_{\mathrm{p}}/\mu_{\mathrm{m}})\psi'_n(ak_{\mathrm{p}})\zeta_n(ak_{\mathrm{m}})},\\
b_n&=\frac{(\mu_{\mathrm{p}}/\mu_{\mathrm{m}})\psi_n(ak_{\mathrm{p}})\psi'_n(ak_{\mathrm{m}})-(k_{\mathrm{p}}/k_{\mathrm{m}})\psi'_n(ak_{\mathrm{p}})\psi_n(ak_{\mathrm{m}})}{(\mu_{\mathrm{p}}/\mu_{\mathrm{m}})\psi_n(ak_{\mathrm{p}})\zeta'_n(ak_{\mathrm{m}})-(k_{\mathrm{p}}/k_{\mathrm{m}})\psi'_n(ak_{\mathrm{p}})\zeta_n(ak_{\mathrm{m}})}.
\end{aligned} \tag{17}$$

In the case when the size of a chiral spherical particle is much smaller than the wavelength of incident radiation, i.e., in the case of nanoparticles, the sphere can be represented as a point particle with nonzero electric and magnetic dipole moments. To obtain explicit expressions for these moments, it is necessary to find the asymptotics of the induced field (12) at large distances from the particle ($r\to\infty$). Note that the main contribution will be made by the angular radiation components damping proportionally to $1/r$; in this case, it is also necessary to expand the coefficients (12) in a series over $k_0a\to 0$, using only the principal terms, and to take into account that the main contribution is made by the coefficients with index $n=1$. Comparing the thus obtained expressions for the induced fields with the known expression for the total fields of electric and magnetic dipole sources located at one point (see, for example, [27]), we find the expressions for the electric ($\boldsymbol{d}_0$) and magnetic ($\boldsymbol{m}_0$) dipole moments of a chiral spherical nanoparticle in the field of the incident electromagnetic circularly polarised wave:

$$\begin{aligned}
d^{(j)}_{0x}&=\frac{(\varepsilon_{\mathrm{p}}-\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3E_0-(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\frac{3\chi\sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}}}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3E_0,\\
d^{(j)}_{0y}&=-\frac{3\mathrm{i}\chi\sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}}}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3E_0+\mathrm{i}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\,\frac{(\varepsilon_{\mathrm{p}}-\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3E_0,\\
d^{(j)}_{0z}&=0,
\end{aligned} \tag{18}$$

$$\begin{aligned}
m^{(j)}_{0x}&=\frac{3\mathrm{i}\chi\sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}}}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3H_0-\mathrm{i}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\,\frac{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}-\mu_{\mathrm{m}})-\chi^2}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3H_0,\\
m^{(j)}_{0y}&=\frac{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}-\mu_{\mathrm{m}})-\chi^2}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3H_0-
\end{aligned} \tag{19}$$

$$-(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\frac{3\chi\sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}}}{(\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}})(\mu_{\mathrm{p}}+2\mu_{\mathrm{m}})-\chi^2}\,a^3H_0,$$

$$m_{0z}^{(j)}=0,$$

where $H_0=\sqrt{\varepsilon_{\mathrm{m}}/\mu_{\mathrm{m}}}\,E_0$. If in (18) and (19) we put $\delta_{\mathrm{L}j}=\delta_{\mathrm{R}j}=0$, we obtain the known expressions for the electric and magnetic dipole moments of a spherical chiral particle located in the field of a plane monochromatic electromagnetic linearly polarised wave [3, 29].

To find the time-averaged power extinctions (attenuation) and scattering of a plane monochromatic electromagnetic circularly polarised wave by a chiral spherical particle, it is necessary to calculate [with respect to the surface of a sphere of infinite radius ($r\to\infty$)] the integrals [27]:

$$W_{\mathrm{ext}}=-\frac{c}{8\pi}\int_0^{\pi}\mathrm{d}\theta\sin\theta\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{Re}\big(\boldsymbol{e}_r[\boldsymbol{E}^{\mathrm{i}},\boldsymbol{H}^{\mathrm{r}*}]\big)r^2\big|_{r\to\infty}$$
$$-\frac{c}{8\pi}\int_0^{\pi}\mathrm{d}\theta\sin\theta\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{Re}\big(\boldsymbol{e}_r[\boldsymbol{E}^{\mathrm{r}},\boldsymbol{H}^{\mathrm{i}*}]\big)r^2\big|_{r\to\infty}, \quad (20)$$
$$W_{\mathrm{scat}}=\frac{c}{8\pi}\int_0^{\pi}\mathrm{d}\theta\sin\theta\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{Re}\big(\boldsymbol{e}_r[\boldsymbol{E}^{\mathrm{r}},\boldsymbol{H}^{\mathrm{r}*}]\big)r^2\big|_{r\to\infty},$$

where the asterisk denotes complex conjugation. After integration, normalising the obtained expressions by $W_0=c\varepsilon_{\mathrm{m}}|E_0|^2/(4\pi\sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}})$ – the energy flux density in the incident circularly polarised wave, we find the expressions for cross sections of extinction and scattering:

$$\sigma_{\mathrm{ext}}^{(j)}=\frac{2\pi}{k_{\mathrm{m}}^2}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}(a_n+b_n)$$
$$+\frac{4\pi}{k_{\mathrm{m}}^2}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\sum_{n=1}^{\infty}(2n+1)\mathrm{Im}\,c_n, \quad (21)$$

$$\sigma_{\mathrm{scat}}^{(j)}=\frac{2\pi}{k_{\mathrm{m}}^2}\sum_{n=1}^{\infty}(2n+1)\big(|a_n|^2+|b_n|^2+2|c_n|^2\big)$$
$$-\frac{4\pi}{k_{\mathrm{m}}^2}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})\sum_{n=1}^{\infty}(2n+1)\mathrm{Im}\big[(a_n+b_n)c_n^*\big]. \quad (22)$$

Expression (21) and (22) coincide with those given in [26]. In the case of a nonchiral spherical particle, the extinction and scattering cross sections will not depend on the polarisation of the incident plane monochromatic electromagnetic wave [in (21) and (22) we should put $c_n=0$].

If the substance of which the chiral particle is made absorbs incident radiation, we can calculate the absorption cross sections as the difference between extinction and scattering cross sections [27]: $\sigma_{\mathrm{abs}}^{(j)}=\sigma_{\mathrm{ext}}^{(j)}-\sigma_{\mathrm{scat}}^{(j)}$. For nonabsorbing particles the scattering and extinction cross sections are the same.

If the size of chiral particles is much smaller than the wavelength of incident radiation which takes place in the case of nanoparticles ($k_0a\to 0$), we can find asymptotic expressions for cross sections (21) and (22), using electric and magnetic dipole moments (18) and (19). Using general expressions for cross sections [9], we obtain

$$\sigma_{\mathrm{abs}}^{(j)}=\frac{2\pi k_{\mathrm{m}}}{|E_0|^2}\,\mathrm{Im}\big\{\big[d_{0x}^{(j)}-\mathrm{i}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})d_{0y}^{(j)}\big]E_0^*\big\}$$
$$+\frac{2\pi k_{\mathrm{m}}}{|H_0|^2}\,\mathrm{Im}\big\{\big[\mathrm{i}(\delta_{\mathrm{L}j}-\delta_{\mathrm{R}j})m_{0x}^{(j)}+m_{0y}^{(j)}\big]H_0^*\big\}, \quad (23)$$

$$\sigma_{\mathrm{scat}}^{(j)}=\frac{4\pi k_{\mathrm{m}}^4}{3|E_0|^2}\big(|d_{0x}^{(j)}|^2+|d_{0y}^{(j)}|^2\big)$$
$$+\frac{4\pi k_{\mathrm{m}}^4}{3|H_0|^2}\big(|m_{0x}^{(j)}|^2+|m_{0y}^{(j)}|^2\big). \quad (24)$$

In the case of nanoparticles, the absorption cross section exceeds the scattering cross section. We easily find from (23) that for right-hand circularly polarised radiation and at

$$\varepsilon_{\mathrm{p}}\approx-2\varepsilon_{\mathrm{m}}+\chi\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}},\quad \mu_{\mathrm{p}}\approx-2\mu_{\mathrm{m}}+\chi\sqrt{\frac{\mu_{\mathrm{m}}}{\varepsilon_{\mathrm{m}}}} \quad (25)$$

the absorption cross section of light tends to zero, and consequently, the nanoparticles absorb only left-hand circularly polarised light. And vice versa, in the case of left-hand polarised radiation and at

$$\varepsilon_{\mathrm{p}}\approx-2\varepsilon_{\mathrm{m}}-\chi\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}},\quad \mu_{\mathrm{p}}\approx-2\mu_{\mathrm{m}}-\chi\sqrt{\frac{\mu_{\mathrm{m}}}{\varepsilon_{\mathrm{m}}}} \quad (26)$$

the cross section of light absorption also tends to zero, and consequently, the nanoparticles absorb only right-hand polarised light.

3. Light pressure force acting on a chiral spherical particle in the circularly polarised electromagnetic wave

To calculate the light pressure force acting on a chiral spherical particle in the field of a plane monochromatic electromagnetic circularly polarised wave, we will use the formalism related to the Maxwell stress tensor [9, 27]. In this case, we do not take into account the mechanical deformations of the medium, which arise under the action of the electromagnetic field, and their attendant effects. The general expression for finding the time-averaged light pressure force acting on the particle under study has the form

$$\boldsymbol{F}=\frac{1}{2}\,\mathrm{Re}\int_S\mathrm{d}S\big(\boldsymbol{n}\hat{T}\big),$$
$$\hat{T}=\frac{\varepsilon_{\mathrm{m}}}{4\pi}\Big(\boldsymbol{E}^{\mathrm{s}}\otimes\boldsymbol{E}^{\mathrm{s}*}-\frac{1}{2}|\boldsymbol{E}^{\mathrm{s}}|^2\hat{I}\Big)+\frac{\mu_{\mathrm{m}}}{4\pi}\Big(\boldsymbol{H}^{\mathrm{s}}\otimes\boldsymbol{H}^{\mathrm{s}*}-\frac{1}{2}|\boldsymbol{H}^{\mathrm{s}}|^2\hat{I}\Big), \quad (27)$$

where S is an arbitrary surface, covering the considered particle; $\boldsymbol{n}$ is the vector of the outward normal to S; $\hat{T}$ is the Maxwell stress tensor; $\boldsymbol{E}^{\mathrm{s}}=\boldsymbol{E}^{\mathrm{i}}+\boldsymbol{E}^{\mathrm{r}}$ and $\boldsymbol{H}^{\mathrm{s}}=\boldsymbol{H}^{\mathrm{i}}+\boldsymbol{H}^{\mathrm{r}}$ are the total electric and magnetic fields in a medium outside the particle; the symbol $\otimes$ denotes the direct product of the vectors; $\hat{I}$ is the unit tensor. To calculate the force (27), the surface S, covering the spherical particle, is convenient to choose as a sphere of infinite radius ($r\to\infty$). As a result, (27) transforms into the expression

$$\boldsymbol{F}=\frac{\varepsilon_{\mathrm{m}}}{8\pi}\int_0^{\pi}\mathrm{d}\theta\sin\theta\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{Re}\Big[(\boldsymbol{e}_r\boldsymbol{E}^{\mathrm{s}*})\boldsymbol{E}^{\mathrm{s}}-\frac{1}{2}|\boldsymbol{E}^{\mathrm{s}}|^2\boldsymbol{e}_r\Big]r^2\Big|_{r\to\infty}+$$

$$+\frac{\mu_{\rm m}}{8\pi}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi{\rm Re}\left[\left(\boldsymbol{e}_r\boldsymbol{H}^{\rm s*}\right)\boldsymbol{H}^{\rm s}-\frac{1}{2}|\boldsymbol{H}^{\rm s}|^2\boldsymbol{e}_r\right]r^2\bigg|_{r\to\infty}. \tag{28}$$

After further simplifications [30] and integration in (28), we find that the light pressure force has only one nonzero component oriented along the z axis [the direction of propagation of the incident wave (10)]. Normalising this component by $P_0 = \sqrt{\varepsilon_{\rm m}\mu_{\rm m}}W_0/c$ – the momentum flux density in the incident wave, we obtain the explicit expression for the cross section of the light pressure:

$$\sigma_{\rm pr}^{(j)} = \sigma_{\rm ext}^{(j)} - \eta^{(j)}\sigma_{\rm scat}^{(j)}, \tag{29}$$

where

$$\eta^{(j)}\sigma_{\rm scat}^{(j)} = \frac{4\pi}{k_{\rm m}^2}\sum_{n=1}^{\infty}{\rm Re}\left[\frac{2n+1}{n(n+1)}\left(a_nb_n^* + |c_n|^2\right)\right.$$
$$\left. + \frac{n(n+2)}{n+1}\left(a_na_{n+1}^* + b_nb_{n+1}^* + 2c_nc_{n+1}^*\right)\right]$$
$$- \frac{4\pi}{k_{\rm m}^2}(\delta_{{\rm L}j} - \delta_{{\rm R}j})\sum_{n=1}^{\infty}{\rm Im}\left\{\frac{2n+1}{n(n+1)}(a_n+b_n)c_n^*\right.$$
$$\left. + \frac{n(n+2)}{n+1}\left[(a_n+b_n)c_{n+1}^* + (a_{n+1}+b_{n+1})c_n^*\right]\right\}. \tag{30}$$

The value of $\eta = \langle\cos\theta\rangle$ is called the asymmetry factor and can be calculated by averaging the cosine of the spherical angle θ when using the intensity distribution over θ as a weighting function [31]. In the particular case of a spherical particle made of a material without chiral properties, the well-known Debye expression for the cross section of the light pressure [31, 32], which does not depend on the polarisation of the incident electromagnetic wave, follows from (29).

The asymptotic expression for $\eta^{(j)}\sigma_{\rm scat}^{(j)}$ in the case of particles of very small radii (nanoparticles) can be found if we expand the coefficients (15) in a series over $k_0a \to 0$, while keeping only the leading terms, and take into account the fact that the main contribution will be made by the coefficients with index $n = 1$. However, to derive a more compact asymptotic expression use should be made of the general relation from paper [33]. As a result, for the desired asymptotic of (30), we obtain

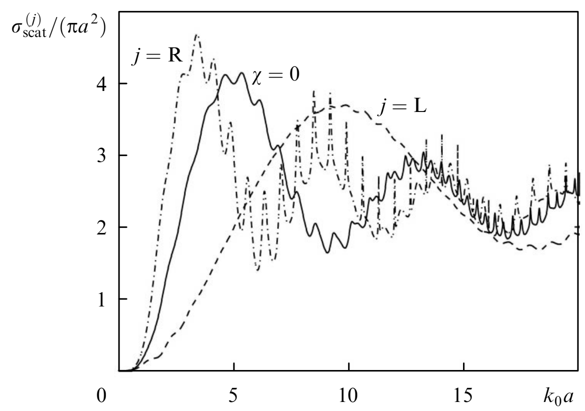
$$\eta^{(j)}\sigma_{\rm scat}^{(j)} = \frac{4\pi k_{\rm m}^4}{3E_0H_0^*}{\rm Re}\left(d_{0x}^{(j)}m_{0y}^{(j)*} - d_{0y}^{(j)}m_{0x}^{(j)*}\right). \tag{31}$$

Using (23), (24) and (31), we find from (29) an explicit asymptotic expression for the light pressure cross section, which is suitable in the case of chiral spherical nanoparticles. Note that for the nanoparticles ($k_0a \to 0$) the contribution of $\eta^{(j)}\sigma_{\rm scat}^{(j)}$ in expression (29) is small compared with the contribution of $\sigma_{\rm ext}^{(j)}$; therefore, the characteristic features of the light pressure cross section will be the same as those of the extinction (absorption) cross section, and hence, the maximum radiation pressure will be determined by conditions (25) or (26).

4. Discussion of the results

The analytic results obtained in the previous sections are valid for chiral spherical particles with arbitrary properties, i.e. the properties of a dielectric ($\varepsilon_{\rm p}$, $\mu_{\rm p} > 0$), metal ($\varepsilon_{\rm p} < 0$, $\mu_{\rm p} > 0$), material with negative refraction ($\varepsilon_{\rm p}$, $\mu_{\rm p} < 0$) and magnetic plasma ($\varepsilon_{\rm p} > 0$, $\mu_{\rm p} < 0$). In this section, using the numerical examples we will study in detail the most interesting, in our opinion, cases of particles made of a dielectric and of a material with negative refraction. Without loss of generality, we consider a chiral particle in vacuum ($\varepsilon_{\rm m} = \mu_{\rm m} = 1$).

Figure 1 shows the scattering cross section of a plane monochromatic electromagnetic circularly polarised wave by the chiral spherical particle, normalised to the geometric cross section πa^2, as a function of the particle size k_0a. One can see from Fig. 1a that if the dielectric particle has chiral properties, the scattering cross sections of left- and right-hand polarised waves are different. Nevertheless, regardless of the incident wave polarisation and the chirality parameter, we can note the general character of the dependences of the normalised cross section on the size of the chiral particle having the form of a slowly oscillating function with more rapid oscillations against its background [31, 34]. The presence of chirality results in either an increase in the amplitude of fast oscillations in the case of the incident right-hand circularly polarised wave, or in their smoothing in the case of a left-hand circularly polarised wave. It should



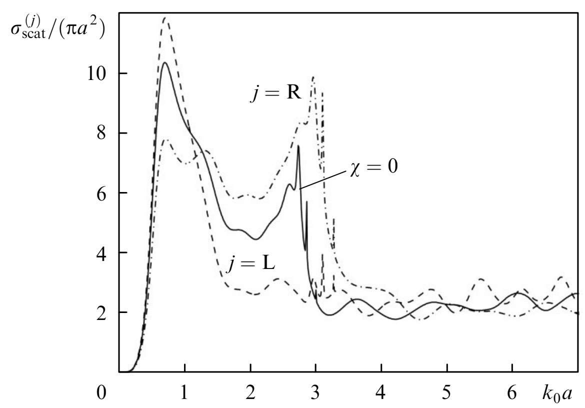


Figure 1. Normalised scattering cross section of the electromagnetic left- ($j = {\rm L}$) and right-hand ($j = {\rm R}$) circularly polarised wave incident on a chiral spherical nanoparticle made of a dielectric ($\varepsilon_{\rm p} = 2$, $\mu_{\rm p} = 1$) (a) and of a material with negative refraction ($\varepsilon_{\rm p} = -3$, $\mu_{\rm p} = -1$) (b) as a function of k_0a. The chirality parameter of the particle is $\chi = 0.2$.

also be noted that depending on the incident wave polarisation, the period of slow oscillations of the normalised cross section varies: it can be smaller (the curve for $j = \mathrm{R}$ in Fig. 1a) or larger (the curve for $j = \mathrm{L}$) than the same period for a dielectric particle without chiral properties (the curve for $\chi = 0$). Both these effects are due to the difference in the wave vectors of right- and left-hand polarised waves in the particle [see formula (5)].

Optical properties of a chiral spherical particle made of a material with negative refraction are more complicated (Fig. 1b). Characteristic of the normalised scattering cross section in this case is the presence of one or more major peaks at relatively small values of k_0a; with increasing the particle radius, we observe an oscillatory dependence of the cross section on k_0a with a small amplitude of oscillations. Unlike the dielectric particle it is obviously due to the fact that for the given negative values of ε_{p} and μ_{p} there exist such values of the radius at which the denominators of coefficients (15) will be close to zero. This corresponds to the excitation condition in a spherical particle made of a material with negative refractions of different types: plasmon, high-Q surface modes and whispering gallery modes [35]. At the same time, only whispering gallery modes can be excited in a dielectric particle [36].

Figure 2 shows the normalised cross section of the light pressure of a plane monochromatic electromagnetic circularly polarised wave incident on the chiral spherical particle as a function of k_0a. One can see from Fig. 2a that for right-hand polarised light, the number of fast oscillations increases against the background of a slowly varying dependences of the light pressure cross section with increasing size of the dielectric particle [30, 31]. In this case, on average the normalised cross section first increases, reaching a maximum, and then slowly decreases. The amplitude of fast oscillations is larger in the case of an incident electromagnetic right-hand circularly polarised wave. For a left-hand polarised wave the amplitude of fast oscillations is noticeably smaller. For a dielectric particle without chiral properties (Fig. 2a, the curve for $\chi = 0$) the corresponding cross section of the light pressure is in the range between the values of $\sigma_{\mathrm{pr}}^{\mathrm{L}}$ and $\sigma_{\mathrm{pr}}^{\mathrm{R}}$ for the chiral dielectric particle.

The dependence of the normalised light pressure cross section on the radius of a particle made of a material with negative refraction (Fig. 2b), in general resembles a similar dependence for the scattering cross section (Fig. 1b). At relatively small values of k_0a, we again observe the main peak (corresponding to the main plasmon resonance), which is replaced by some resonances that are smaller in amplitude (corresponding to plasmon oscillations with a high multipolarity, high-Q surface modes and whispering gallery modes) with increasing the particle radius.

Figure 3 shows the dependence of the normalised light pressure cross section on the chirality parameter for several radii of the particle. One can see that as in the case of a dielectric particle so in the case of the particle made of a material with negative refraction, there are some optimal

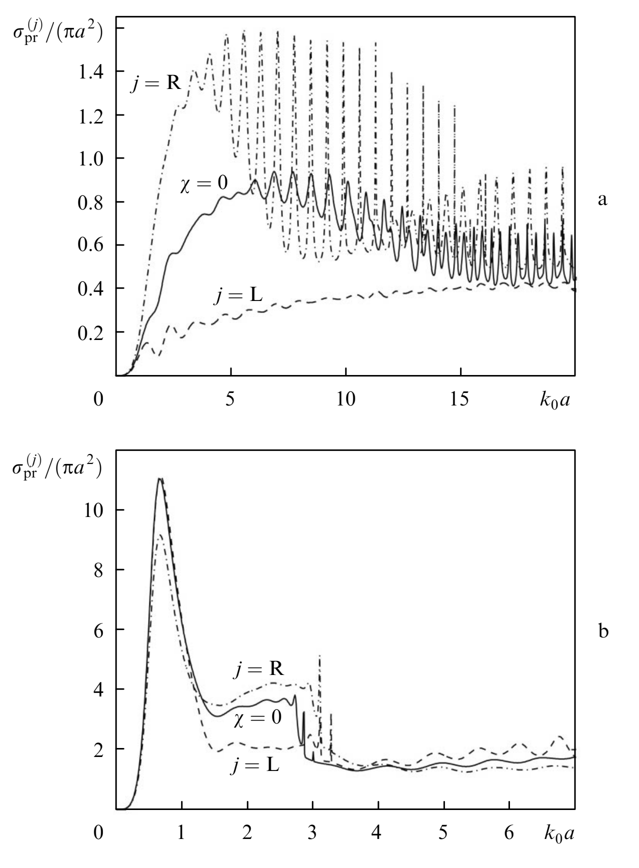


Figure 2. Normalised light pressure cross section of the electromagnetic left- ($j = \mathrm{L}$) and right-hand ($j = \mathrm{R}$) circularly polarised wave incident on a chiral spherical nanoparticle made of a dielectric ($\varepsilon_{\mathrm{p}} = 2$, $\mu_{\mathrm{p}} = 1$) (a) and of a material with negative refraction ($\varepsilon_{\mathrm{p}} = -3$, $\mu_{\mathrm{p}} = -1$) (b) as a function of k_0a. The chirality parameter of the particle is $\chi = 0.2$.

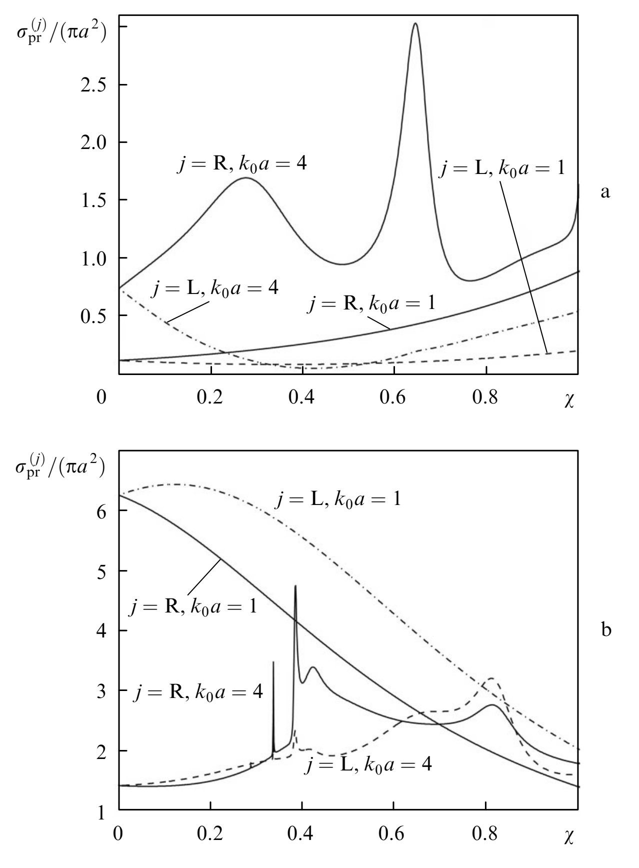


Figure 3. Normalised light pressure cross section of the electromagnetic left- ($j = \mathrm{L}$) and right-hand ($j = \mathrm{R}$) circularly polarised wave incident on a chiral spherical nanoparticle made of a dielectric ($\varepsilon_{\mathrm{p}} = 2$, $\mu_{\mathrm{p}} = 1$) (a) and of a material with negative refraction ($\varepsilon_{\mathrm{p}} = -3$, $\mu_{\mathrm{p}} = -1$) (b) as a function of the chirality parameter χ at different k_0a.

values of the chirality parameter χ, which make it possible to markedly increase (or decrease) the light pressure cross section for the given radius, dielectric constant and permeability if the polarisation of the incident electromagnetic wave is properly selected. Thus, finer tuning (within small variations in χ) is possible only in the case of sufficiently large k_0a, which is especially noticeable for particles made of a material with negative refraction, when high-Q surface modes are excited (curves for $k_0a = 4$ in Fig. 3b). A change in χ for particles with $k_0a \sim 1$ does not lead to a rapid change in the normalised cross section of the light pressure (cf. curves for $k_0a = 1$ and 4 in Fig. 3a and b).

The most important is the dependence of the light pressure cross section on the dielectric constant of the chiral spherical nanoparticle made of a material with negative refraction (Fig. 4). It is seen that in the region [specified by conditions (25)] of the values of the dielectric constant and permeability $\mathrm{Re}\varepsilon_p = -2 + \chi$ and $\mathrm{Re}\mu_p = -2 + \chi$, the choice of the incident electromagnetic left- or right-hand circularly polarised waves can significantly increase or decrease the light pressure cross section for the chiral particle compared with the cross section for the nanoparticle without chiral properties. It is important to note that these values of the dielectric constant and permeability at the same time correspond to the conditions of excitation of plasmon oscillations in the nanoparticle [the conditions of vanishing denominators in expressions (18) and (19)]. In this case, we will observe not only an increase in the light pressure cross section of the incident electromagnetic wave with one of circular polarisations compared with the wave with a different polarisation but also an increase in the cross section with decreasing imaginary parts of the dielectric constant and permeability.

The above demonstrated strong dependence of the characteristics of chiral spherical nanoparticles on the polarisation of incident radiation (Fig. 4) can be used to improve the synthesis of nanoparticles made of different metamaterials. Indeed, suppose that there is a set of synthesised nanoparticles with a negative refractive index. Then, by exposing them to right- or left-hand polarised light only those particles can be extracted from an ensemble that satisfy condition (25) or (26). Similar effects also take place for dielectric particles (see Fig. 3a).

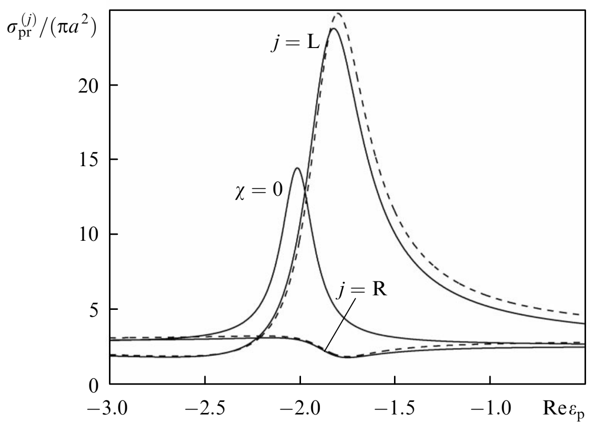


Figure 4. Normalised light pressure cross section of the electromagnetic left- ($j = \mathrm{L}$) and right-hand ($j = \mathrm{R}$) circularly polarised wave incident on a chiral spherical nanoparticle made of a material with negative refraction ($\varepsilon_p = \mathrm{Re}\varepsilon_p + \mathrm{i}0.1$, $\mu_p = -1.8 + \mathrm{i}0.1$) as a function of $\mathrm{Re}\varepsilon_p < 0$. The size of the nanoparticles $k_0a = 0.1$, the chirality parameter $\chi = 0.2$. Dashed curves are the asymptotics of normalised cross section (29) obtained with the help of expressions (23), (24), and (31).

5. Conclusions

Thus, we have studied the properties of a chiral spherical particle made of an arbitrary metamaterial, the particle being located in the field of a plane monochromatic electromagnetic circularly polarised wave. We have shown that, depending on the polarisation of the incident electromagnetic wave, the cross sections of scattering and light pressure for a chiral particle can be larger or smaller than the corresponding values for a particle made of a material with the same dielectric constant and permeability but without chiral properties. We have derived asymptotic expressions for cross sections of scattering, extinction and light pressure in the case of chiral spherical particles of very small radii. We have obtained explicit expressions for the induced electric and magnetic dipole moments of a chiral spherical nanoparticle in the field of a plane monochromatic electromagnetic circularly polarised wave. We have determined the conditions under which the absorption (extinction) and light pressure cross sections are significantly different for electromagnetic left- and right-hand circularly polarised waves.

Analytic results obtained in this paper are quite general in nature and can be used to calculate characteristics of chiral spherical micro- and nanoparticles in optical fields as well as to test the algorithms of numerical calculations of characteristics of chiral nonspherical particles.

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References

1. Barron L.D. *Molecular Light Scattering and Optical Activity* (Cambridge: Cambridge University Press, 1982).
2. Fedorov F.I. *Teoriya girotropii* (Theory of Gyrotropy) (Minsk: Nauka i tekhnika, 1976).
3. Lindell I.V., Sihvola A.H., Tretyakov S.A., Viitanen A.J. *Electromagnetic Waves in Chiral and Bi-isotropic Media* (Boston: Artech House Publishers, 1994).
4. Lekner J. *Pure Appl. Opt.*, **5**, 417 (1996).
5. Vinogradov A.P. *Elektrodinamika kompozitnykh materialov* (Electrodynamics of Composite Materials) (Moscow: URSS, 2001).
6. Wang B., Zhou J., Koschny T., Soukoulis C.M. *Appl. Phys. Lett.*, **94**, 151112 (2009).
7. Plum E., Zhou J., Dong J., Fedotov V.A., Koschny T., Soukoulis C.M., Zheludev N.I. *Phys. Rev. B*, **79**, 035407 (2009).
8. Zhou J., Dong J., Wang B., Koschny T., Kafesaki M., Soukoulis C.M. *Phys. Rev. B*, **79**, 121104(R) (2009).
9. Landau L.D., Livshits E.M. *Electrodynamics of Continuous Media* (Oxford: Pergamon Press, 1984; Moscow: Nauka, 1986).
10. Ashkin A. *Usp. Fiz. Nauk*, **110**, 101 (1973).
11. Ashkin A., Dziedzic J.M., Bjorkholm J.E., Chu S. *Opt. Lett.*, **11**, 288 (1986).
12. Barton J.P., Alexander D.R. *J. Appl. Phys.*, **66**, 2800 (1989).
13. Barton J.P., Alexander D.R., Schaub S.A. *J. Appl. Phys.*, **66**, 4594 (1989).
14. Ren K.F., Greha G., Gouesbet G. *Opt. Commun.*, **108**, 343 (1994).
15. Neves A.A.R., Fontes A., Pozzo L.d.Y., de Thomaz A.A., Chillce E., Rodriguez E., Barbosa L.C., Cesar C.L. *Opt. Express*, **14**, 13101 (2006).

16. Riyopoulos S. *Opt. Lett.*, **31**, 2480 (2006).
17. Kemp B.A., Kong J.A., Grzegorczyk T.M. *Phys. Rev. A*, **75**, 053810 (2007).
18. Ross B.M., Lakhtakia A. *Optik*, **119**, 7 (2008).
19. Chen H., Zhang B., Luo Y., Kemp B.A., Zhang J., Ran L., Wu B.-I. *Phys. Rev. A*, **80**, 011808(R) (2009).
20. Belichenko V.P., Fisanov V.V. *Rus. Phys. J.*, **37**, 1008 (1994).
21. Li L.W., Dan Y., Leong M.S., Kong J.A. *Progr. Electromagn. Res.*, **23**, 239 (1999).
22. Li L.-W., You D., Leong M.-S., Yeo T.-S., Kong J.A. *Progr. Electromagn. Res.*, **26**, 249 (2000).
23. Uslenghi P.L.E. *Electromagn.*, **10**, 201 (1990).
24. Yokota M., He S., Takenaka T. *J. Opt. Soc. Am. A*, **18**, 1681 (2001).
25. Hinders M.K., Rhodes B.A. *Il Nuovo Cimento*, **14**, 575 (1992).
26. Bohren C.F., Huffman D.R. *Absorption and Scattering of Light by Small Particles* (New York: Wiley-Interscience, 1983; Moscow: Mir, 1986).
27. Stratton J.A. *Electromagnetic Theory* (New York: McGraw-Hill, 1941; Moscow – Leningrad: GITTL, 1948).
28. Abramowitz M., Stegun I.A. (Eds) *Handbook of Mathematical Functions* (New York: Dover, 1964; Moscow: Nauka, 1979).
29. Lindell I.V., Sihvola A.H. *J. Electromagn. Waves Appl.*, **4**, 1223 (1990).
30. Guzatov D.V., Gaida L.S., Afanas'ev A.A. *Kvantovaya Elektron.*, **38**, 1155 (2008) [*Quantum Electron.,* **38**, 1155 (2008)].
31. Kerker M. *The Scattering of Light and Other Electromagnetic Radiation* (New York: Academic Press, 1969).
32. Debye P. *Ann. Physik*, **30**, 57 (1909).
33. Kats A.V. *Izv. Vyssh. Uchebn. Zaved. Ser. Radiofiz.*, **28**, 566 (1975).
34. Born M., Wolf E. *Principles of Optics* (London: Pergamon, 1970; Moscow: Nauka, 1973).
35. Klimov V.V. *Opt. Commun.*, **211**, 183 (2002).
36. Oraevsky A.N. *Kvantovaya Elektron.*, **32**, 377 (2002) [*Quantum Electron.*, **32**, 377 (2002)].