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# Quasi-periodic synchronisation of self-modulation oscillations in a ring chip laser by an external periodic signal

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*Abstract*. **The synchronisation of periodic self-modulation oscillations in a ring Nd : YAG chip laser under an external periodic signal modulating the pump power has been experimentally investigated. A new quasi-periodic regime of synchronisation of self-modulation oscillations is found. The characteristic features of the behaviour of spectral and temporal structures of synchronised quasi-periodic oscillations with a change in the external signal frequency are studied.**

***Keywords***: *solid-state ring laser, self-modulation regime of the first kind, quasi-periodic synchronisation.*

## 1. Introduction

Synchronisation of nonlinear self-oscillations by an external periodic signal is one of the fundamental phenomena that are observed in dynamic systems of different nature [1]. To date, the synchronisation of periodic and random nonlinear oscillations has been studied well (see, for example, [1, 2]). When periodic oscillations are synchronised by an external periodic signal, the general case of synchronisation of the order $m/n$ is characterised by the following relation between the frequency of induced oscillations $\omega$ and the external signal frequency $\Omega$: $n\omega = m\Omega$. The synchronisation of quasi-periodic nonlinear oscillations has been studied much more poorly; only recently the main regularities determining this process were analysed [3 – 7]. Most studies on the synchronisation of quasi-periodic oscillations by an external signal were either performed for model systems or based on numerical simulation. The experimental study of these phenomena in real dynamic systems that are used in practice is of undoubted interest.

In the previous theoretical and experimental works the synchronisation of periodic self-modulation oscillations by an external periodic signal was studied in monolithic solid-state ring lasers (SSRLs) based on Nd : YAG [8, 9] and in erbium-fibre [10] and semiconductor [11] lasers. Recently the synchronisation of quasi-periodic self-modulation oscillations by an external periodic signal was also investigated in semiconductor lasers [12].

It is known that SSRLs, along with steady states (characterised by constant intensities of counterpropagating waves), allow for self-modulation oscillations, among which the self-modulation oscillations of the first kind is most important for practice. In an autonomous SSRL this regime is stable in a wide range of laser parameters, except for the cases where the frequency of self-modulation oscillations is close to the doubled main relaxation frequency. In this situation the parametric interaction between the self-modulation and relaxation oscillations leads to the instability of the self-modulation regime of the first kind and to the occurrence of a number of other self-modulation regimes of generation, for example, the regime with a doubled modulation period [13] and the quasi-periodic self-modulation regime [14].

In this paper we report the results of the experimental study of the quasi-periodic synchronisation of self-modulation oscillations, which arises in an SSRL with a periodic modulation of pump power.

## 2. Experimental setup

The investigations were performed on a monolithic ring Nd : YAG laser. The geometric perimeter of the nonplanar ring cavity was 28 mm, and the nonplanarity angle was 85°. The laser worked at a wavelength of 1.06 μm and was pumped by a semiconductor laser diode ($\lambda_{\rm p} = 0.810$ μm). A generator of periodic oscillations was inserted in the circuit of the laser diode power supply to modulate the pump power.

In the presence of pump modulation the pump excess over the threshold, $\eta(t)$, can be written as

$$\eta(t) = \eta + h\sin(2\pi f_{\rm p} t), \qquad (1)$$

where $\eta$ is the pump excess over the threshold in the absence of modulation, $h$ is the pump modulation depth, and $f_{\rm p}$ is the pump modulation frequency. In the experiments the modulation frequency was changed in the range of 200 – 250 kHz, while the modulation depth was constant: 0.2.

The thermal stabilisation system made it possible to change gradually the single-block temperature within 15 – 30 °C. As a result, the ratio of the moduli of the feedback coefficients of counterpropagating waves could be gradually changed. At working points the temperature of the laser studied was maintained with an error of 0.5 °C.

During the experiments the dependences of the temporal and spectral characteristics of the intensity of counterpropagating waves on the pump-modulation frequency were recorded simultaneously. Signals were detected using

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a 20-12-PCI ADC and a broadband Tektronix TDS 2014 digital oscilloscope.

## 3. Experimental results

### 3.1 Conditions for the quasi-periodic regime of self-modulation

The previous studies [8, 9] showed that periodic pump modulation in an SSRL may cause synchronisation of self-modulation oscillations in the self-modulation regime of the first kind. In these studies only periodic synchronisation regimes were observed. Here, we demonstrate that the quasi-periodic regime of synchronisation of self-modulation oscillations can be implemented under certain conditions (see below).

The pump excess over threshold ($\eta$) in the experiments reported here was changed from 0.15 to 0.2. Under these conditions the frequency of self-modulation oscillations $\omega_{\rm m}$ is close to the doubled frequency of relaxation oscillations ($\omega_{\rm m} \approx 2\omega_{\rm r}$), and strong parametric interaction between the self-modulation and relaxation oscillations occurs in the laser [13, 14]. At $\eta \geqslant 0.22$ a bifurcation arises, which is related to the doubling of the self-modulation oscillation period, and the self-modulation regime of the first kind is transformed into the periodic self-modulation regime with a doubled period. As was shown in [14], bistable self-modulation regimes of generation may exist under certain conditions in this range: the self-modulation regime of the first kind and the quasi-periodic self-modulation regime. The occurrence of bistability is significantly affected by the noise in the laser, in particular, the noise component of the pump power. Note that the bistability of self-modulation oscillations may occur only when the moduli of the feedback coefficients of counterpropagating waves, $m_{1,2}$, differ [14].

Our study showed that a variation in the monoblock temperature $T$ leads to a change in the moduli of the coupling coefficients of counterpropagating waves, because a variation in $T$ changes the ratio of the counterpropagating wave intensities. Figure 1 shows the experimental temperature dependence of the ratio of the mean intensities of counterpropagating waves, $I_1/I_2$. It can be seen that this value is almost constant (equal to 2) in the temperature range from 15 to 20°, whereas in the range of 20–25° it is close to unity. The $I_1/I_2$ value can be used to determine the ratio of the moduli of feedback coefficients [15, 16]. On the assumption that the phase difference of the feedback coefficients is zero and that the ring cavity has no phase and amplitude nonreciprocities, the ratio of the mean intensities of the counterpropagating waves is determined by the formula

$$I_1/I_2 = m_1/m_2. \tag{2}$$

The results of the experiments that are reported below were obtained at the single-block temperature $T = 19\,^{\circ}{\rm C}$. At this temperature, according to formula (2), the ratio of the moduli of feedback coefficients is close to 2.

We investigated the synchronisation of the order 1/1.

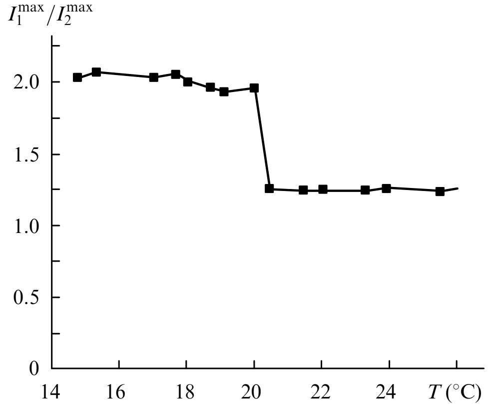

**Figure 1.** Experimental dependence of the ratio of mean intensities of counterpropagating waves, $I_1^{\rm max}/I_2^{\rm max}$, on temperature $T$.

### 3.2 Synchronisation of self-modulation oscillations at a periodic modulation of pump power

In the absence of pump modulation by an external signal the laser under consideration worked in the self-modulation regime of the first kind. At a pump excess over the threshold $\eta = 0.2$, the frequency of self-modulation oscillations $\omega_{\rm m}/2\pi = 220$ kHz and the main relaxation frequency $\omega_{\rm r}/2\pi = 99.5$ kHz. The lasing power spectrum for one of the counterpropagating waves in this regime (Fig. 2a) consists of one component at the frequency $\omega_{\rm m}/2\pi = 220$ kHz. Figure 2b shows the time dependence for this wave.

Figures 2c and 2d present the power spectra and the time dependences of the intensity of one of the counterpropagating waves in the quasi-periodic regime of synchronisation of self-modulation oscillations by an external signal with the frequency $f_{\rm p} = 213.6$ kHz. As can be seen in Fig. 2c, the power spectrum contains two independent components: one at the frequency of self-modulation oscillations, which is captured by the external signal ($\omega_{\rm m}/2\pi = f_{\rm p}$), and the other at the frequency $f_1 = 94.9$ kHz, which is close to the main relaxation frequency $\omega_{\rm r}/2\pi$. The other spectral components are combinations: $f_{\rm p} - f_1$ and $f_{\rm p} + f_1$. According to the data in Fig. 2d, in this synchronisation regime self-modulation oscillations have a low-frequency envelope, which corresponds to the second independent frequency $f_1$. In this case, the modulation depth at the envelope frequency is close to 100 %.

Our study showed that the intensity of the second independent component (with the frequency $f_1$) depends on the external signal frequency. Figures 2e and 2f present the power spectrum and the time dependence of the intensity for one of the counterpropagating waves at the modulation frequency $f_{\rm p} = 217$ kHz. In this case, the intensity of the second independent spectral component is lower than the intensity of the main component by a factor of 200; therefore, the intensity of synchronised self-modulation oscillations changes almost sinusoidally.

When processing the obtained power spectra, we found the dependence of the ratio of the self-modulation and modulation frequencies, $f_{\rm m}/f_{\rm p}$, on the modulation frequency (Fig. 3). It can be seen that there is a range of synchronisation of the self-modulation oscillation frequency by an external signal, where $f_{\rm m}/f_{\rm p} = 1$. In this range the quasi-periodic regime of synchronisation with independent frequencies $f_{\rm p}$ and $f_1$ is implemented. The dependence of the frequency of the second independent spectral component $f_1$ on the external signal frequency $f_{\rm p}$ is shown in Fig. 4; one can see that $f_1$ depends weakly on $f_{\rm p}$.

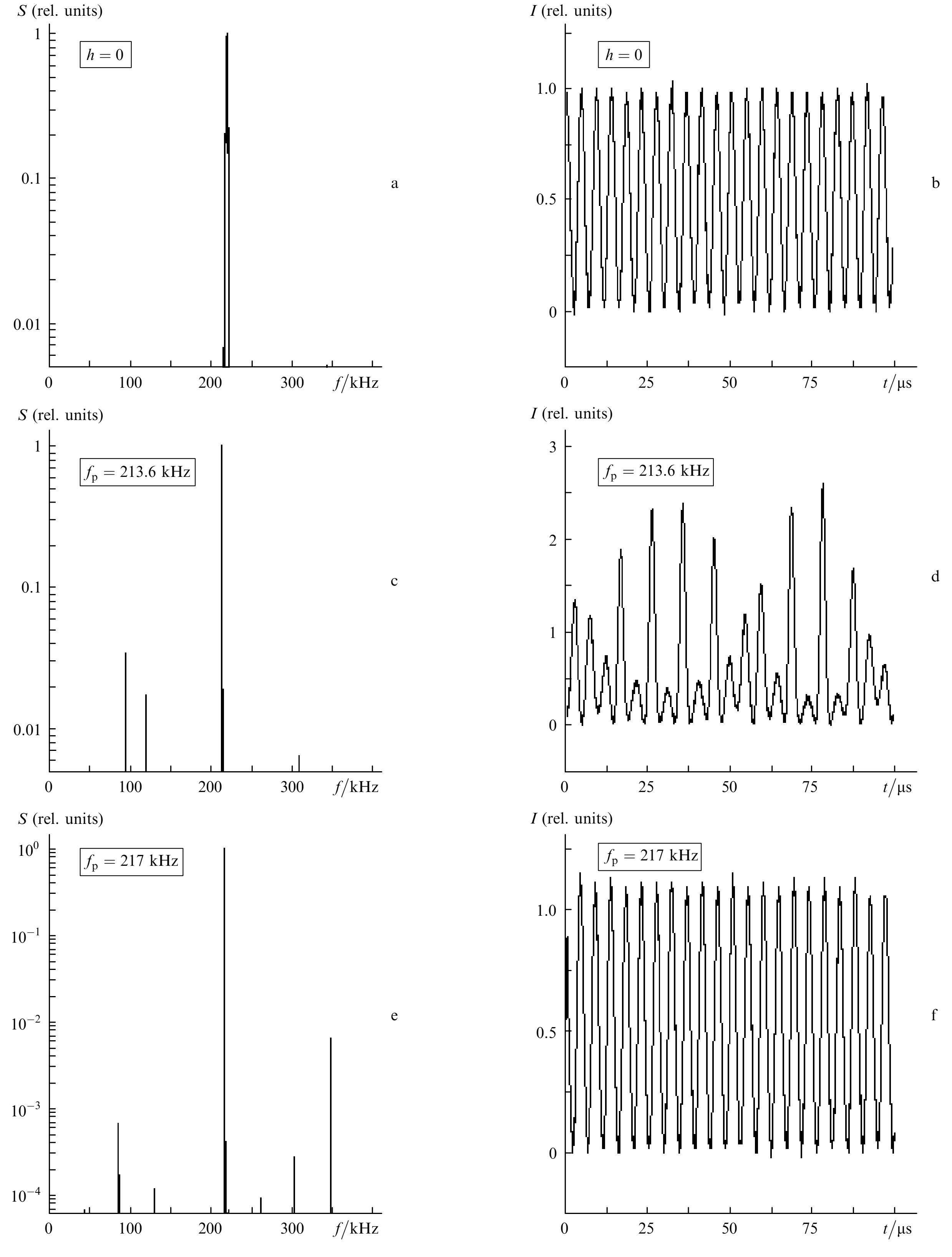

**Figure 2.** Power spectra and time dependences of the intensity of one of the counterpropagating waves in the absence of modulation by an external signal ($h = 0$) and under pump modulation by an external signal with frequencies of 213.6 and 217 kHz.

### 3.3 Results and discussion

In the previous studies of the synchronisation of self-modulation oscillations by an external periodic signal only a periodic synchronisation regime was found [8, 9], whereas our experiments revealed a quasi-periodic regime. It is due to two factors: (i) a significant difference in the moduli of feedback coefficients for the counterpropagating waves, $m_{1,2}$ (the mean intensity of one of the waves exceeded that of the other wave by a factor of about two), and (ii) the

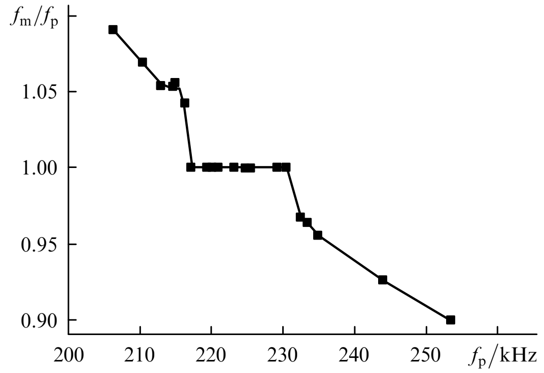

**Figure 3.** Dependence of the ratio of self-modulation and modulation frequencies, $f_m/f_p$, on the modulation frequency.

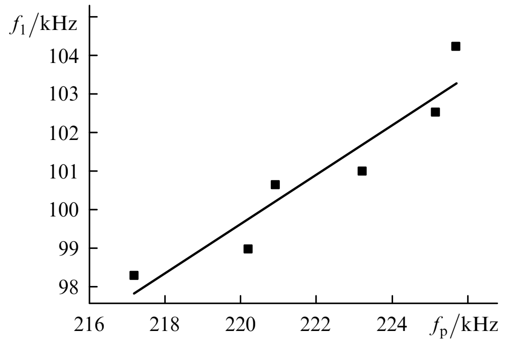

**Figure 4.** Dependence of the frequency of the second independent spectral component $f_l$ on the modulating signal frequency $f_p$.

choice of the bifurcation point corresponding to the doubling of the period of self-modulation oscillations of the first kind. Under these conditions, as was shown in [14], SSRL, along with the periodic regime of the first kind, can also implement a stable quasi-periodic regime of self-modulation oscillations. However, it can be observed only at fairly small pump fluctuations, related to the technical noise. An application of an external periodic signal to modulate the pump power, apparently, induces a quasi-periodic regime, and partial synchronisation of quasi-periodic oscillations occurs in a certain range of modulation frequencies: the self-modulation frequency becomes equal to that of the external signal ($f_m = f_p$), and the second independent frequency ($f_l$) depends weakly on the external signal frequency.

The main features of the quasi-periodic synchronisation regime (the presence of two independent frequencies, one of which is the external-signal frequency, and the other is not synchronised with the external signal) are characteristic of the synchronisation of nonlinear quasi-periodic oscillations by an external signal, which was investigated in [3–7]. Here, we observed a periodic regime of self-modulation oscillations in the absence of an external signal. As far as we know, the case where synchronised quasi-periodic oscillations arise as a result of the effect of an external signal on nonlinear periodic oscillations has not been studied previously. In this context, the theoretical study of this phenomenon is of undoubted interest.

Thus, we experimentally investigated the specific features of synchronisation of self-modulation oscillations in a ring Nd:YAG chip laser by an external periodic signal. A new quasi-periodic regime of synchronisation of self-modulation oscillations was found. Within the synchronisation range we studied the effect of the external signal frequency on the spectral and temporal structures of quasi-periodic oscillations.

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