FIBREOPTIC COMMUNICATION LINES

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Statistics of errors in fibre communication lines with a phase-modulation format and optical phase conjugation

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***Abstract*. Analytical formulas are derived to approximate the probability density functions of 'zero' and 'one' bits in a linear communication channel with a binary format of optical signal phase modulation. Direct numerical simulation of the propagation of optical pulses in a communication line with optical phase conjugation is performed. The results of the numerical simulation are in good agreement with the analytical approximation.**

Keywords: *optical fibre communication lines, modulation formats, optical phase conjugation, probability density, error rate.*

1. Introduction

A measure for estimating the quality of a communication line is the bit error rate (BER), which is the ratio of the number of incorrect bits to the total number of transmitted bits. Exact estimation of BER is important in designing optical communication systems. Gaussian approximation is often used for the probability density of 'one' and 'zero' bits. In this case, to calculate the BER, one must know the Q factor, which is determined from the formula $Q = (\mu_1 - \mu_0)/(\sigma_1 + \sigma_0)$, where μ_1 and μ_0 are the mean values of the currents of 'one' and 'zero' bits and σ_1 and σ_0 are their standard deviations; in this case,

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}.$$

The Gaussian approximation is fairly simple but predicts the error probability with a low accuracy.

Currently the main working regime of fibre communication lines is the amplitude modulation, at which a 'one' bit is transmitted in the form of a light pulse, while 'zero' bit corresponds to the absence of a pulse in a selected bit interval. An increase in the bit rate in a channel leads to an increase in the negative influence of nonlinear and dispersion effects and in the noise of amplified spontaneous emission. Therefore, an urgent problem is to study new formats for optical signal modulation (for example, phase-modulation format) and search for optimal configurations of optical communication lines based on these formats using mathematical simulation methods. In the simplest binary phase-modulation format with coding over the optical-phase difference (DBPSK format) data are coded using the phase difference between neighbouring bits. In contrast to the conventional amplitude modulation format, the format with coding data over the phase difference is based on coding logic 'zero' by the shift of the optical pulse phase in a bit interval by π with respect to the previous bit, while logic 'one' corresponds to identical phases of two neighbouring bits. Due to the uniform power distribution in all bit intervals and a random phase shift between neighbouring bits, this format is more stable to the negative influence of such nonlinear effects as the phase cross-modulation and four-wave mixing (see, for example, [1 – 7]).

In this study we derived simple analytical formulas for the error statistics in DBPSK communication lines with suppression of nonlinear effects. In addition, direct numerical simulation of the propagation of optical pulses was performed to check the applicability of these formulas. The analytical estimates are in good agreement with the results of numerical calculations.

2. Derivation of analytical formulas

When deriving analytical formulas, we assumed (as in [8]) that the main factor determining the signal distortion is the noise of spontaneous emission of amplifiers in the communication line. We used the model [8], where the noise is expressed in terms of a Fourier series with a period T, which coincides with the bit interval width. The real and imaginary parts of the Fourier coefficients are independent Gaussian random values with a 'zero' mean value and standard deviation σ.

The analytical formulas for the density of 'zeros' and 'ones' at the receiver were obtained for a linear transmission channel and the 'no return to zero' (NRZ) format.

A sequence of pulses arrives at the receiver in the end of the communication line. This sequence is set by the formula $a_n E_s(t) + e_n(t)$. Here, n is the bit number; $E_s(t) = E\exp(\mathrm{i}\omega_c t)$ is the complex representation of an optical pulse; t is the time; ω_c is the carrier frequency; and a_n is 1 or -1. The amplifier noise is set by the formula $e_n(t) = r_n(t) + \mathrm{i}s_n(t)$, where the imaginary and real parts of the noise $e_n(t)$ are Gaussian random values with a zero mean.

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At the input of the receiver the bit sequence is transformed into amplitude modulation using optical delay lines and interference. The interferometer transforms the signal into two sequences, corresponding to a current of 'zeros', $J_0(t)$, and a current of 'ones', $J_1(t)$. The electric current is proportional to the squared magnitude of the sum of the signal and noise. The transformation of optical pulses into electric current by the detector is set by the formulas

$$J_1(t) = K\left|\frac{a_n E_s(t) + e_n(t) + a_{n-1}E_s(t) + e_{n-1}(t)}{2}\right|^2,$$
$$J_0(t) = K\left|\frac{a_n E_s(t) + e_n(t) - a_{n-1}E_s(t) - e_{n-1}(t)}{2}\right|^2. \quad (1)$$

where K is the photodetector sensitivity. The receiver processes the difference between the currents of 'ones' and 'zeros': $J(t) = J_1(t) - J_0(t)$. The electric current is averaged over the bit interval T:

$$x = \frac{1}{T}\int_0^T J(t)\mathrm{d}t = \frac{1}{T}\int_0^T J_1(t)\mathrm{d}t - \frac{1}{T}\int_0^T J_0(t)\mathrm{d}t. \quad (2)$$

Generally, the cases $x < 0$ and $x > 0$ correspond to recording 'zero' and 'one' bits, respectively.

Let us find the probability densities of 'one' and 'zero' bits at the receiver. We will consider the coding of a 'one' bit; in this case $a_n = a_{n-1}$. Note that the functions

$$\eta^+(t) = \frac{e_n(t) + e_{n-1}(t)}{2} = \frac{r_n(t) + r_{n-1}(t) + \mathrm{i}[s_n(t) + s_{n-1}(t)]}{2}$$

and

$$\eta^-(t) = \frac{e_n(t) - e_{n-1}(t)}{2} = \frac{r_n(t) - r_{n-1}(t) + \mathrm{i}[s_n(t) - s_{n-1}(t)]}{2}$$

which correspond to the amplifier noises, are independent random values with a Gaussian distribution of the real and imaginary parts. Since a signal passes through an optical filter in the receiver, the expansion of the amplifier noise in a Fourier series contains a finite number of terms:

$$\eta^+(t) = \sum_{m=m_1}^{m_1+M} c_m \exp(\mathrm{i}\omega_m t), \quad \eta^-(t) = \sum_{m=m_1}^{m_1+M} c'_m \exp(\mathrm{i}\omega_m t),$$

where $\omega_m = (2\pi/T)m$ is frequency ($m = 0, +1, +2, \ldots$).

Let B_{opt} be the transmission band of the optical filter; then the $B_{\mathrm{opt}}T$ value sets the number M of the Fourier harmonics transmitted by the optical filter.

Then we find the characteristic function $F_1(\kappa)$ for the x_1, value, which is equal to the current of 'ones' $J_1(t)$, averaged over the bit interval T:

$$x_1 = \frac{1}{T}\int_0^T J_1(t)\mathrm{d}t. \quad (3)$$

The function

$$F_1(\kappa) = \int_0^\infty w_1(x_1)\exp(\mathrm{i}\kappa x_1)\mathrm{d}x_1,$$

where $w_1(x_1)$ is the density of the random value x_1.

Having substituted (1) into (3), we obtain

$$x_1 = \frac{1}{T}\int_0^T K[|E_s(t)|^2 + E_s(t)\overline{\eta^+}(t) + \overline{E_s}(t)\eta^+(t) + |\eta^+(t)|^2]\mathrm{d}t$$

(the bar above indicates complex conjugation). Since the functions $\exp(\mathrm{i}\omega_m t)$ are orthogonal in the interval T, and the characteristic function of a sum of independent random values is the product of the characteristic functions of the terms,

$$F_1(\kappa) = \exp\left(\mathrm{i}K|E_s|^2\kappa - \frac{2\sigma^2K^2|E_s|^2\kappa^2}{1 - 2\mathrm{i}\sigma^2K\kappa}\right)\frac{1}{(1 - 2\mathrm{i}\sigma^2K\kappa)^M}.$$

The characteristic function $F_0(\kappa)$ for the current of 'zeros' averaged over the interval T, $x_0 = \frac{1}{T}\int_0^T J_0(t)\mathrm{d}t$, is determined by the formula

$$F_0(\kappa) = \int_0^\infty w_0(x_0)\exp(\mathrm{i}\kappa x_0)\mathrm{d}x_0,$$

where $w_0(x_0)$ is the density of x_0. As in [8],

$$F_0(\kappa) = \frac{1}{(1 - 2\mathrm{i}\sigma^2K\kappa)^M}.$$

Therefore, the characteristic function of the random value x [difference in the currents of 'ones' and 'zeros' (2)], averaged over the bit interval T, is set by the formula

$$\langle\exp(\mathrm{i}\kappa x)\rangle = \exp\left(\mathrm{i}K|E_s|^2\kappa - \frac{2\sigma^2K^2|E_s|^2\kappa^2}{1 - 2\mathrm{i}\sigma^2K\kappa}\right) \times[(1 + 4\sigma^4K^2\kappa^2)^M]^{-1}. \quad (4)$$

The inverse Fourier transform of expression (4) yields the density of 'one' bits $\rho_1(x)$. Let us introduce the designations $Z = 2K\sigma^2\kappa$, $I_1 = K|E_s|^2$, and $I_0 = 2\sigma^2KM$. Then

$$\rho_1(x) = \frac{1}{2\pi}\frac{M}{I_0}\int_{-\infty}^{\infty}\exp\left\{\frac{M}{I_0}\left[(\mathrm{i}I_1 - \mathrm{i}x)Z - I_1\frac{Z^2}{1 - \mathrm{i}Z}\right]\right\}\frac{\mathrm{d}Z}{(1 + Z^2)^M}. \quad (5)$$

Similarly to the derivation of the formula for ρ_1, the density function of 'zero' bits can be written as

$$\rho_0(x) = \frac{1}{2\pi}\frac{M}{I'_0}\int_{-\infty}^{\infty}\exp\left\{\frac{M}{I'_0}\left[(\mathrm{i}I'_1 - \mathrm{i}x)Z + I'_1\frac{Z^2}{1 + \mathrm{i}Z}\right]\right\}\frac{\mathrm{d}Z}{(1 + Z^2)^M}. \quad (6)$$

In the case of the NRZ format of optical pulses $I'_1 = I_1$, and $I'_0 = I_0$.

Formulas (5) and (6) were obtained under assumption that the communication channel is linear. In this case, the main source of errors is the amplifier noises. If the channel is nonlinear, one of the main factors of signal degradation is the Kerr nonlinearity. The nonlinearity effect is small when the initial pulses have low power. In the case of higher power initial pulses one can use optical phase conjugation to suppress the Kerr nonlinearity [9].

3. Results of numerical simulation

The propagation of Gaussian optical pulses was numerically simulated for a communication line with optical phase conjugation. It was found that formulas (5) and (6) give a good approximation for the statistics of the currents corresponding to 'zeros' and 'ones' at the receiver, in a system based on the 'return to zero' (RZ) format and using optical phase conjugation. The results of the numerical simulation demonstrate that optical phase conjugation effectively suppresses the Kerr nonlinearity.

We considered an optical communication line composed of 16 periodic sections with the configuration

SMF (85 km) + EDFA + DCF (14.85 km) + EDFA

followed by 16 periodic sections with the configuration

DCF (14.85 km) + EDFA + SMF (85 km) + EDFA.

Here, SMF is the standard single-mode fibre, DCF is the dispersion-compensating fibre, and EDFA is an erbium-doped fibre amplifier. An optical phase conjugation device was mounted in the middle of the line, after the first 16 sections.

The parameters of the optical fibres are listed in Table 1. The erbium-doped amplifiers had a noise coefficient of 4.5 dB and completely compensated for the signal decay at the fibre segment between amplifiers. The mean dispersion of a periodic section of the optical communication line was zero.

Gaussian pulses with a duration of 7.5 ps and a peak power of 5 mW were used as bits in the line of the RZ-DBPSK format. We considered the data transmission in one frequency channel with a rate of 40 Gbit s^{-1}.

The dynamics of optical pulses was described using the generalised nonlinear Schrödinger equation for the complex envelope A of electromagnetic field [10]:

$$\mathrm{i}\frac{\partial A}{\partial z}-\frac{\beta_2(z)}{2}\frac{\partial^2 A}{\partial t^2}+\sigma(z)|A|^2A = \mathrm{i}\left[-\gamma(z)+\sum_{k=1}^{N}r_k\delta(z-z_k)\right]A.$$

Here, z is the distance along the line; $N=32$; $|A|^2$ is the power; β_2 is the dispersion parameter of the group velocity; $\sigma=2\pi n_2/(\lambda_0 A_{\rm eff})$ is the Kerr nonlinearity coefficient; z_k are the amplifier location points; $\gamma(z)$ is the signal decay coefficient; and r_k is the gain. The σ and β_2 values are presented as functions of z to take into account the change in these parameters at a transition from one type of optical fibre to another.

Communication systems with dispersion control use optical fibres with chromatic dispersion of opposite sign, which makes it possible to control the dispersion broadening of pulses. If the average dispersion of a communication line is zero, in the linear case and in the absence of decay and noise, the signal shape is completely recovered in the end of the line [10]. The model of generalised nonlinear Schrödinger equation, which describes the propagation of optical pulses, takes into account the following effects that contribute to signal distortion: Kerr nonlinearity, dispersion broadening, and the amplifier spontaneous emission noise.

The data were statistically processed after the propagation of optical signals at a distance of 3200 km. The receiver included a rectangular optical filter with a transmission band $B_{\rm opt}=100$ GHz and a third-order Butterworth filter.

We compared the statistics of 'zero' and 'one' bits transmitted through a line with optical phase conjugation, with the statistics of 'zeros' and 'ones' for the transmission without phase conjugation. In addition, a comparison with the statistics of 'zeros' and 'ones' in the linear channel was performed. The statistics of 'zeros' and 'ones' in a linear channel were obtained by adding the noise of all amplifiers of the communication line to the initial signal, without transmission of pulses through the fibre line. All obtained samples of 'zero' and 'one' bits were normalised to the mean current of 'one' bits in the communication line with optical phase conjugation.

Figure 1 shows the probability density functions PDF for 'zero' and 'one' bits, depending on the normalised electric current. The samples of 'ones' and 'zeros' included 12892 and 12708 values, respectively. The dotted lines show similar density functions for the linear channel, and the dashed lines are the probability densities of 'one' bits (on the right) and 'zero' bits (on the left) after the transmission through a communication line without optical phase conjugation.

It can be seen that optical phase conjugation effectively suppresses the Kerr nonlinearity. Without the phase conjugation the PDFs of 'zero' and 'one' bits are overlapped, and about 5 % bits are incorrect. As can be seen in Fig. 1, the signal quality in the system with optical phase conjugation is much better. In this case, the probability densities of 'zero' and 'one' bits differ only slightly from those for the linear channel.

Figure 2 shows the probability densities of 'one' and 'zero' bits on the logarithmic scale after the transmission through the communication line with optical phase conjugation and the analytical approximation set by formulas (5) and (6). Due to the normalisation of the samples to the average value of 'one' bits, $I_1=1$. The approximation parameters I_1' and M/I_0' for 'zero' bits and the parameter M/I_0 for 'one' bits were found by the least squares method from a sample of bits at the receiver. The following values of the approximation parameters were obtained: $I_1'=0.93$, $M/I_0=M/I_0'=112$. The number M of the harmonics transmitted through the optical filter is three. One can see good coincidence of the analytical approximation of the probability densities of 'zeros' and 'ones' with the results of direct numerical simulation.

Table 1. Parameters of optical fibres.

Optical fibres	Optical loss γ at 1550 nm/dB km^{-1}	Effective mode area $A_{\rm eff}/\mu\mathrm{m}^2$	Group velocity dispersion D/ps nm^{-1} km^{-1}	Dispersion slope $\frac{\mathrm{d}D}{\mathrm{d}\lambda}$/ps nm^{-2} km^{-1}	Nonlinear refractive index n_2/m^2 W^{-1}
SFM	0.2	80	17	0.07	2.7×10^{-20}
DCF	0.65	19	−100	−0.41	2.7×10^{-20}

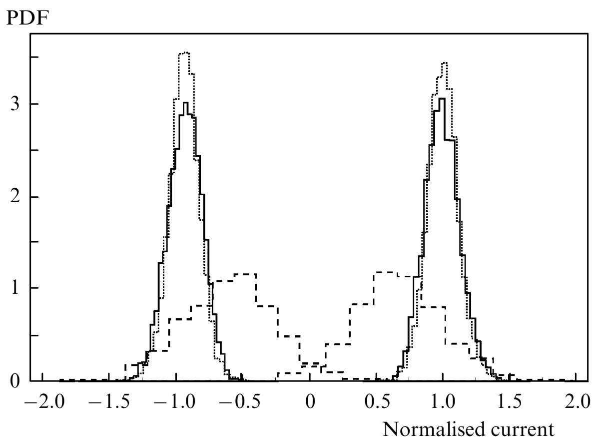

Figure 1. Probability densities of 'zero' and 'one' bits for different propagation modes as functions of normalised current.

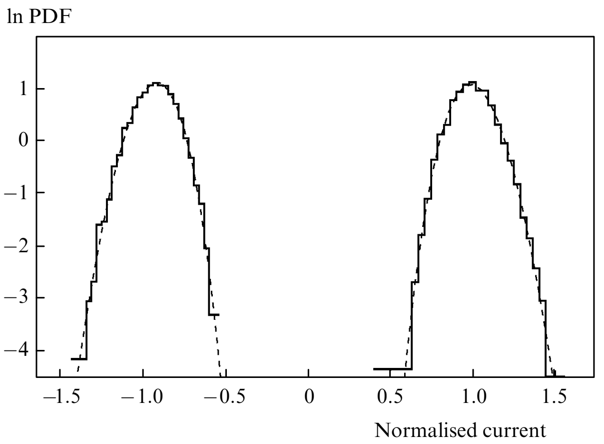

Figure 2. Probability densities of 'zero' and 'one' bits as functions of normalised current: (solid lines) the calculation results and (dashed lines) the analytical approximation.

Note that formulas (5) and (6) set the behaviour of the probability density of 'zero' and 'one' bits. The error probability depends on the parameters M/I_0 and M/I_0', which are determined by numerical calculation from a sample of 'zero' and 'one' bits at the receiver. The larger these parameters, the higher the signal quality. To determine them, it is sufficient to have a fairly small sample of 'zeros' and 'ones' at the receiver. Thus, the amount of calculations necessary for precise determination of the probability error in a communication line can be significantly reduced.

In the case of data transfer in the RZ format, generally speaking, the set of the parameters I_1, I_0 should not coincide with the set I_1', I_0'. The reason is as follows: when coding 'zero' bit, the function of the electromagnetic field envelope A is zero at the boundary of the bit intervals, whereas, when coding a 'one' bit, this condition does not hold true because the characteristic width of Gaussian optical pulses is $0.3-0.5$ of the bit period. Since the electric current at the receiver is averaged over the bit period, the mean of the currents corresponding to 'one' bits exceeds in magnitude the mean of the currents corresponding to 'zeros'.

4. Conclusions

We derived analytical formulas, which give a good approximation of the densities of 'zero' and 'one' bits at the receiver in optical communication systems with suppression of nonlinear effects. The probability density functions for 'zero' and 'one' bits, calculated from these formulas, were compared with similar functions obtained by direct numerical simulation of an optical communication line with transmission capacity of 40 Gbit s^{-1}, based on a standard single-mode optical fibre and a dispersion-compensating fibre. It is shown that optical phase conjugation effectively suppresses the Kerr nonlinearity and reduces the error rate of the system.

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