Quantum Electronics **41** (9) 776–781 (2011)

PACS numbers: 42.55.Tv; 42.60.Da
DOI: 10.1070/QE2011v041n09ABEH014604

Two-dimensional distributed feedback lasers based on static and dynamic Bragg structures*

V.R. Baryshev, N.S. Ginzburg

Abstract. **In order to increase the output power of DFB lasers, we consider the possibility of using two-dimensional distributed feedback. Within the framework of this scheme, the feedback circuit includes four partial wave fluxes propagating in mutually orthogonal directions, which makes it possible to provide coherent radiation from a spatially extended planar active medium characterised by large values of the Fresnel parameter. By analogy with the one-dimensional distributed feedback, the wave coupling can be ensured by using both the structures with a periodically varying effective refractive index (static two-dimensional Bragg structures) and the gain modulation (photo-induced two-dimensional Bragg structures). Within the semiclassical approximation, the initial conditions and nonlinear dynamics of lasers with the above-described two-dimensional Bragg structures are analysed. Self-similarity conditions are found, allowing one to scale the laser parameters with increasing active region size, which is accompanied by an increase in the integrated output power.**

Keywords: *DFB laser, two-dimensional Bragg structures, static and light-induced gratings.*

1. Introduction

Already the first papers [1–4] devoted to the use of distributed feedback (DFB) lasers described two types of periodic Bragg structures, where two counterpropagating electromagnetic waves should experience distributed re-reflections. The structures of the first type are static gratings with a periodically varying refractive index or dielectric layer thickness. An alternative method is to periodically change the gain. For practical realisation of such a scheme, the inversion of the active medium should be ensured by two intersecting light beams of the pump source, forming a standing wave. Bragg structures of the second type are called dynamic or photo-induced structures [5, 6]. To date, there are many implementations of DFB lasers with both static and dynamic Bragg structures [5–8], some of which are already a traditional component of laser technology [9].

In this paper, in order to drastically increase the power of DFB lasers, we consider the possibility of using two-dimensional distributed feedback [10] (Fig. 1). In the framework such a scheme, the feedback circuit includes not two, but four partial wave fluxes propagating in orthogonal directions. This makes it possible to provide coherent radiation from a spatially extended planar active medium, characterised by large values of the Fresnel parameter with respect to propagation directions of the partial wave fluxes. In this case, by analogy with the one-dimensional distributed feedback, the wave coupling can be ensured by using both the structures with a periodically varying refractive index or thickness of the dielectric layer (static two-dimensional Bragg structures) and the gain modulation (dynamic two-dimensional Bragg structures). In the class of static two-dimensional Bragg structures, the most simple in terms of practical implementation is a dielectric plate with the chequered corrugation of one of the surfaces [11, 12] (Fig. 1). To implement the dynamic gain grating it is

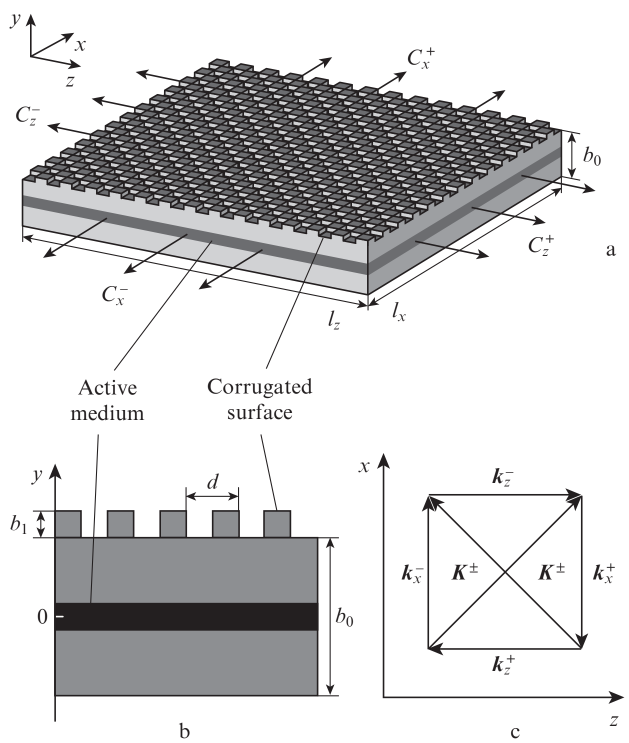

Figure 1. General scheme of a two-dimensional DFB laser based on a corrugated dielectric waveguide (a), checkerboard approximation of the waveguide surface in an enlarged scale (b) and diagram illustrating the coupling of the partial waves ($\boldsymbol{k}_{x,z}^{\pm}$ are the wave vectors of the partial waves, $\boldsymbol{K}^{\pm}$ are the translation vectors of the grating) (c).

*Reported at the Conference on 'Laser Optics', St. Petersburg, Russia, 2011.

V.R. Baryshev, N.S. Ginzburg Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603950 Nizhnii Novgorod, Russia; e-mail: baryshev@appl.sci-nnov.ru

Received 25 February 2011; revision received 21 July 2011
Kvantovaya Elektronika **41** (9) 776–781 (2011)
Translated by I.A. Ulitkin

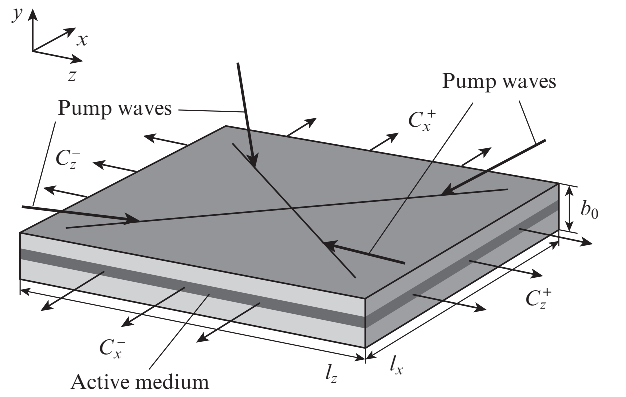

Figure 2. Scheme of a two-dimensional photo-induced DFB laser.

possible to affect the medium with resonant quantum transitions by four pump beams (Fig. 2).

In this paper, using the semiclassical approximation we describe the dynamics of two-dimensional distributed feedback lasers based on both static and dynamic Bragg structures. We find the threshold generation conditions as well as stable single-frequency regions where the use of the two-dimensional DFB allows one to synchronise radiation of the active medium that is planar extended in two orthogonal directions.

2. Static and dynamic two-dimensional Bragg structures

In the optical spectrum, a two-dimensional Bragg resonator can be implemented with the help of a dielectric plate with a doubly periodic sinusoidal modulation of one of the surfaces (see Fig. 1b),

$$b(x,y) = b_0 + b_1[\cos\bar{h}(x+z) + \cos\bar{h}(x-z)], \qquad (1)$$

with the translation vectors $\boldsymbol{K}^{\pm} = \bar{h}\boldsymbol{x}_0 \pm \bar{h}\boldsymbol{z}_0$ directed perpendicular to each other. Here, $\bar{h} = 2\pi/d$ is the absolute value of projections of the translation vectors to the directions x and z; d is the modulation period along the specified coordinates. The two-dimensional Bragg structure (2) ensures coupling and mutual scattering of four partial waves fluxes, which propagate in the directions $\pm z$ ($C_z^{\pm}$) and $\pm x$ ($C_x^{\pm}$) and are given by the vector-potentials

$$\boldsymbol{A} = \mathrm{Re}\{[\boldsymbol{a}_1(y)(C_z^+ \mathrm{e}^{-\mathrm{i}hz} + C_z^- \mathrm{e}^{\mathrm{i}hz}) + \boldsymbol{a}_2(y)(C_x^+ \mathrm{e}^{-\mathrm{i}hx} + C_x^- \mathrm{e}^{\mathrm{i}hx})]\mathrm{e}^{\mathrm{i}\omega t}\}, \qquad (2)$$

where $\boldsymbol{a}_{1,2}(y)$ are the transverse mode structures of a planar dielectric waveguide. We assume that the average waveguide thickness b_0 is limited by the condition of propagation of the only lowest TM_1 waveguide mode. The effective coupling of the partial waves on structure (1) takes place when the resonance condition

$$h \approx \bar{h} \qquad (3)$$

is fulfilled (see Fig. 1c). In this case, the waves $C_z^{\pm}$ scatter into the waves $C_x^{\pm}$ on a two-dimensional Bragg structure given by expression (1), and the direct coupling between the counter-propagating waves $_z^+$ and $C_x^- \leftrightarrow C_x^+$ is absent.

Note that in terms of practical realisation a doubly periodic sinusoidal corrugation can be replaced by a chequered-type corrugated surface.

By analogy with conventional one-dimensional Bragg structures, resonators formed due to periodic modulation of the characteristics of a dielectric plate (effective refractive index) should be called static.

In the case of dynamic (photo-induced) Bragg gratings, the waves are coupled due to the gain modulation of the medium. In the two-dimensional variant, the doubly periodic grating of the active medium gain that is similar to (1) can be induced due to the interference of four plane linearly polarised (S-polarisation) pump waves (Fig. 2):

$$\boldsymbol{A}_{\mathrm{p}} = A_{\mathrm{p}}\cos(k_{\perp}y + \omega_{\mathrm{p}}t)\,\mathrm{Re}\Big\{(\boldsymbol{x}^0 + \boldsymbol{z}^0)\Big(\exp\Big[\frac{k_{\|}}{\sqrt{2}}(z-x)\Big] + \exp\Big[-\frac{k_{\|}}{\sqrt{2}}(z-x)\Big]\Big) + (\boldsymbol{x}^0 - \boldsymbol{z}^0)\cos\Big(\exp\Big[\frac{k_{\|}}{\sqrt{2}}(z+x)\Big] + \exp\Big[-\frac{k_{\|}}{\sqrt{2}}(z+x)\Big]\Big)\Big\}, \qquad (4)$$

where $k_{\|} = (\omega_{\mathrm{p}}/c)\cos\theta$ and $k_{\perp} = (\omega_{\mathrm{p}}/c)\sin\theta$ are the moduli of the wave-vector projections of the pump fields on the active material plane and perpendicular to it. As a result, the time-averaged pump intensity in the plane of the active material $y = \mathrm{const}$ is expressed as

$$\langle|\boldsymbol{A}|^2\rangle_t = \frac{A_{\mathrm{p}}^2}{\sqrt{2}}\{2 + \cos[\sqrt{2}\,k_{\|}(z-x)] + \cos[\sqrt{2}\,k_{\|}(z+x)]\}. \qquad (5)$$

Therefore, in the active medium we will have two induced singly periodic diagonal (with respect to the axes x, z) gratings of inversion and gain, which prove additive due to orthogonality of polarisations of the mentioned pairs (see Fig. 2).

The condition of the Bragg resonance ensuring the coupling of four partial wave fluxes is written in this case as

$$h = \sqrt{2}(\omega_{\mathrm{p}}/c)\cos\theta. \qquad (6)$$

The change in the angle θ of incidence of the pump waves onto the plane of the active medium obviously allows changing the wave number, and, of course, the frequency of generated radiation. Thus, similar to the one-dimensional prototypes, the two-dimensional DFB lasers with photo-induced gratings are frequency tunable.

3. Nonstationary model of two-dimensional DFB lasers

We will describe the interaction of the active medium with the electromagnetic field within the semiclassical approach. Representing the electromagnetic field as a set of four partial wave fluxes [see (2)], we will write the resonance part of the polarisation $\boldsymbol{P}$ and inversion ρ of the active medium in the form [13]:

$$\boldsymbol{P} = \boldsymbol{y}_0\,\mathrm{Re}[\mathrm{i}(P_z^+ \mathrm{e}^{\mathrm{i}\bar{h}z} + P_z^- \mathrm{e}^{-\mathrm{i}\bar{h}z} + P_x^+ \mathrm{e}^{\mathrm{i}\bar{h}z} + P_x^- \mathrm{e}^{-\mathrm{i}\bar{h}z})\mathrm{e}^{\mathrm{i}\omega_0 t}],$$

$$\rho = \rho_0 + \mathrm{Re}(\rho_{2z} e^{2i\bar{h}z} + \rho_{2x} e^{2i\bar{h}z} + \rho_{z-x} e^{2i\bar{h}(z-x)} + \rho_{z+x} e^{2i\bar{h}(z+x)}), \quad (7)$$

where $P_z^{\pm}, \rho_0, \rho_{2z}, \rho_{2x}, \rho_{z\pm x}$ are the slowly varying amplitudes of the corresponding harmonics.

Assuming that the Fresnel parameter for each of the partial wave fluxes is sufficiently large,

$$l_x^2/l_z\lambda \gg 1, \ l_z^2/l_x\lambda \gg 1,$$

propagation of the wave fluxes will be described in geometric optics approximation, neglecting diffraction effects. In these approximations the amplification process of partial waves (2) in the active medium, and their mutual scattering at the static Bragg grating (1) or the nonlinear grating formed by modulation of the inversion of the medium (5), can be expressed by the system of averaged equations:

$$\left(\pm\frac{\partial}{\partial Z} + \frac{\partial}{\partial \tau}\right)\hat{C}_z^{\pm} + i\alpha(\hat{C}_x^{+} + \hat{C}_x^{-}) = \hat{P}_z^{\pm},$$

$$\left(\pm\frac{\partial}{\partial X} + \frac{\partial}{\partial \tau}\right)\hat{C}_x^{\pm} + i\alpha(\hat{C}_z^{+} + \hat{C}_z^{-}) = \hat{P}_x^{\pm},$$

$$\frac{\partial\hat{\rho}_0}{\partial\tau} + \frac{\hat{\rho}_0 - 1}{\hat{T}_1} = -2\,\mathrm{Re}(\hat{C}_z^{+}\hat{P}_z^{+*} + \hat{C}_z^{-}\hat{P}_z^{-*} + \hat{C}_x^{+}\hat{P}_x^{+*} + \hat{C}_x^{-}\hat{P}_x^{-*}),$$

$$\frac{\partial\hat{\rho}_{2z}}{\partial\tau} + \frac{\hat{\rho}_{2z}}{\hat{T}_1} = -(\hat{C}_z^{+*}\hat{P}_z^{-} + \hat{C}_z^{-}\hat{P}_z^{+*}), \quad (8)$$

$$\frac{\partial\hat{\rho}_{2x}}{\partial\tau} + \frac{\hat{\rho}_{2x}}{\hat{T}_1} = -(\hat{C}_x^{+*}\hat{P}_x^{-} + \hat{C}_x^{-}\hat{P}_x^{+*}),$$

$$\frac{\partial\hat{\rho}_{z+x}}{\partial\tau} + \frac{\hat{\rho}_{z+x} - \rho_g}{\hat{T}_1} = -(\hat{C}_z^{+*}\hat{P}_x^{-} + \hat{C}_z^{-}\hat{P}_x^{+*} + \hat{C}_x^{+*}\hat{P}_z^{-} + \hat{C}_x^{-}\hat{P}_z^{+*}),$$

$$\frac{\partial\hat{\rho}_{z-x}}{\partial\tau} + \frac{\hat{\rho}_{z-x} - \rho_g}{\hat{T}_1} = -(\hat{C}_z^{+*}\hat{P}_x^{+} + \hat{C}_z^{-}\hat{P}_x^{-*} + \hat{C}_x^{+}\hat{P}_z^{+*} + \hat{C}_x^{-}\hat{P}_z^{-*}).$$

Here, $X = x/l_z$, $Z = z/l_z$, $\tau = v_{gr}t/l_z$ are the normalised spatial coordinates and time;

$$\hat{\rho} = \frac{\rho}{\rho_e}; \quad \hat{P}_z^{\pm} = P_z^{\pm}\left(\frac{\pi b_a l_z}{\rho_e h\omega_0 c v_{gr} b^{eff}}\right)^{1/2};$$

$$\hat{C}_{x,z}^{\pm} = C_{x,z}^{\pm}\left(\frac{b^{eff}\omega_0}{\pi\rho_e h c v_{gr} b_a}\right)^{1/2}; \quad \hat{T}_1 = \frac{v_{gr}T_1}{l_z};$$

$$b^{eff} = b_0 + \frac{2}{\sqrt{h^2 - k^2}\,(h^2/\varepsilon k^2 + h^2/k^2 - 1)};$$

$k = \omega_0/c$; ω_0 is the Bragg frequency; v_{gr} is the group velocity of the partial waves in a regular dielectric waveguide; ρ_g is the ratio of the intensities of the periodic and the uniform components of the pump [for the pump intensity distribution defined by (5), $\rho_g = 0.5$]; ρ_e is the equilibrium value of inversion in the absence of radiation; b_a is the active layer thickness; T_1 is the relaxation time of inversion of the active medium. The coupling coefficient of the partial waves in the case of a static 'checkerboard' Bragg grating is given by [10]

$$\alpha = \frac{4b_1 h l_z}{\pi^2}\frac{(\varepsilon - h^2/k^2)(1 + 1/\varepsilon^2)}{(h^2/\varepsilon^2 k^2 + h^2/k^2 + 1)b_0 + 2(h^2 - k^2)^{-1/2}}. \quad (9)$$

The coupling coefficient and the effective thickness of the waveguide b^{eff} [3] are written under the assumption that all partial waves are of TM type.

Assuming that the transverse relaxation time T_2 is small compared with other time scales, we will use the balanced approach by representing the components of the medium polarisation in the form

$$\hat{P}_z^{+} = \beta(2\hat{C}_z^{+}\hat{\rho}_0 + \hat{C}_z^{-}\hat{\rho}_{2z}^{*} + \hat{C}_x^{+}\hat{\rho}_{z-x} + \hat{C}_x^{-}\hat{\rho}_{z+x}^{*}),$$

$$\hat{P}_z^{-} = \beta(2\hat{C}_z^{-}\hat{\rho}_0 + \hat{C}_z^{+}\hat{\rho}_{2z} + \hat{C}_x^{+}\hat{\rho}_{z+x} + \hat{C}_x^{-}\hat{\rho}_{z-x}), \quad (10)$$

$$\hat{P}_x^{+} = \beta(2\hat{C}_x^{+}\hat{\rho}_0 + \hat{C}_x^{-}\hat{\rho}_{2x}^{*} + \hat{C}_z^{+}\hat{\rho}_{z-x} + \hat{C}_z^{-}\hat{\rho}_{z+x}^{*}),$$

$$\hat{P}_x^{-} = \beta(2\hat{C}_x^{-}\hat{\rho}_0 + \hat{C}_x^{+}\hat{\rho}_{2z} + \hat{C}_z^{+}\hat{\rho}_{z+x} + \hat{C}_z^{-}\hat{\rho}_{z-x}^{*}),$$

where

$$\beta = \frac{\pi\rho_e|\mu|^2 b_a c l_z T_2}{2h\omega_0 b^{eff}}$$

is the normalised pump intensity determining the gain of the active medium; μ is the dipole moment.

Note that in writing the system of equations (8)–(10) the space and time coordinates are normalised to the active medium length l_z. Therefore, the boundary conditions in the absence of external energy fluxes are expressed as

$$\hat{C}_x^{+}\big|_{X=0} = 0, \ \hat{C}_x^{-}\big|_{X=L_x} = 0, \ \hat{C}_z^{+}\big|_{Z=0} = 0, \ \hat{C}_z^{-}\big|_{Z=1} = 0, \quad (11)$$

where $L_x = l_x/l_z$.

As initial conditions we will use the seed small-amplitude noise field:

$$\hat{C}_{x,z}^{\pm}(X,Z,\tau = 0) = c_0\exp[-i\varphi_{x,z}^{\pm}(X,Z)], \quad (12)$$

where $\varphi_{x,z}^{\pm}(X,Z)$ are random functions.

The total output power can be expressed as a sum of powers of four partial wave fluxes, emitted in the directions $\pm z$ and $\pm x$ from the end faces of the active medium (see Fig. 1),

$$S = \frac{\rho_e h\omega_0 b_a l_z v_{gr}}{4}\hat{S}, \quad (13)$$

where

$$\hat{S} = \int_0^{L_x}(|\hat{C}_z^{+}(X,1)|^2 + |\hat{C}_z^{-}(X,0)|^2)\,dX$$

$$+\int_0^1 (|\hat{C}_x^+(L_x,Z)|^2 + |\hat{C}_x^-(0,Z)|^2)\,dZ. \qquad (14)$$

4. Small-signal approximation. Self-excitation conditions

Assuming that the field amplitudes of the partial waves are small, the system of equations (8), (10) can be linearised and reduced to the form

$$\pm\frac{\partial \hat{C}_z^{\pm}}{\partial Z} + \frac{\partial \hat{C}_z^{\pm}}{\partial \tau} - 2\beta\hat{C}_z^{\pm} = (\beta\rho_g - i\alpha)(\hat{C}_x^+ + \hat{C}_x^-),$$
$$\pm\frac{\partial \hat{C}_x^{\pm}}{\partial X} + \frac{\partial \hat{C}_x^{\pm}}{\partial \tau} - 2\beta\hat{C}_x^{\pm} = (\beta\rho_g - i\alpha)(\hat{C}_z^+ + \hat{C}_z^-). \qquad (15)$$

In the case of a static two-dimensional Bragg grating, when the partial waves are coupled due to modulation of the thickness of the dielectric structure [see (1)], we can set $\rho_g = 0$ in equations (15). In the strong wave-coupling approximation ($\alpha \gg 1$), in the absence of the active medium ($\tilde{\beta} = 0$) the spectrum of two-dimensional Bragg resonator eigenmodes is found analytically [11]

$$\delta_{m,n} = \pm\left[2\alpha + \frac{\pi^2}{4\alpha}\left(n^2 + \frac{m^2}{L_x^2}\right)\right] + i\frac{\pi^2}{2\alpha^2}\left(n^2 + \frac{m^2}{L_x^3}\right), \qquad (16a)$$

$$\delta_{m,n} = \pm\frac{\pi^2 mn}{2\alpha L_x} + i\frac{\pi^2}{2\alpha^2 L_x}\left(n^2 + \frac{m^2}{L_x^2}\right), \qquad (16b)$$

where n, m are the mode indices; $\delta_{m,n} = (\omega_{m,n} - \omega_0)/v_{gr}$ is the complex detuning of eigenmode frequencies from the Bragg carrier frequency. The spectrum shows two groups of modes: the frequency of one group are in the vicinity of the exact Bragg resonance frequency (16b), and the frequencies of the other group are arranged symmetrically (16a) near the band-gap boundaries, $\delta = \pm 2\alpha$.

To remove the symmetry and its corresponding degeneracy, we assume that the resonator length is twice its width $l_z = 2l_x$ ($L_x = 0.5$). Then the highest-Q mode is the mode with indices $\{m = 0, n = 1\}$ of a group of modes (16b). The frequency of this mode coincides with the Bragg frequency ($\mathrm{Re}\,\delta_{0,1} = 0$) and the damping constant has the form

$$\mathrm{Im}\,\delta_{0,1} = \pi^2/\alpha^2. \qquad (17)$$

This decrement is at least half the damping constants of all other modes, which provides high selective properties of a two-dimensional Bragg resonator. Accordingly, in the presence of an active medium the lasing threshold is found from the expression

$$2\beta = \mathrm{Im}\,\delta_{0,1}. \qquad (18)$$

Figure 3 shows the dependence of the lasing threshold, given by expression (18) and found by solving numerically the corresponding system of characteristic equations. One can see that at large wave-coupling coefficients α this formula yields good approximation of the threshold conditions, but at $\alpha = 2$, which is used below for simulating the nonlinear dynamics, there is a noticeable difference in the values of β.

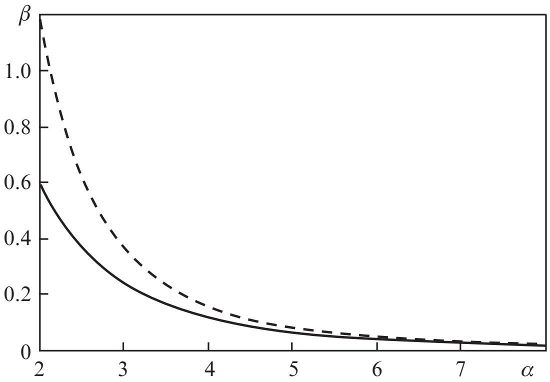

Figure 3. Dependence of the threshold value of the gain β on the wave-coupling coefficient α in a laser with a two-dimensional Bragg resonator. The dashed curve shows the approximation by expression (18).

In a photo-induced DFB laser, the static Bragg grating is absent ($\alpha = 0$) and partial wave fluxes are coupled through periodic inversion gratings ρ_{x+z} and ρ_{z-x}. Assume that such dynamic gratings are fabricated by using pump radiation in the form of standing waves (4), (5), which corresponds to $\rho_g = 0.5$. For the chosen relation between length and width of the structure ($L_x = 0.5$), the system of linear equations (15) becomes a one-parameter system and takes the form

$$\pm\frac{\partial \hat{C}_z^{\pm}}{\partial Z} + \frac{\partial \hat{C}_z^{\pm}}{\partial \tau} - 2\beta\hat{C}_z^{+} = 0.5\beta(\hat{C}_x^+ + \hat{C}_x^-),$$
$$\pm\frac{\partial \hat{C}_x^{\pm}}{\partial X} + \frac{\partial \hat{C}_x^{\pm}}{\partial \tau} - 2\beta\hat{C}_x^{+} = 0.5\beta(\hat{C}_z^+ + \hat{C}_z^-). \qquad (19)$$

Numerical simulation of the linear stage of transition processes based on equations (19) makes it possible to find the threshold lasing condition $\beta > 1.36$. In this case the laser frequency coincides with the Bragg frequency.

5. Simulation of processes of radiation synchronisation in static and dynamic two-dimensional DFB lasers

In simulating the nonlinear dynamics of a two-dimensional DFB laser with the help of the system of equations (8), (10), the wave-coupling coefficient is different from zero in the case of the static Bragg resonator. To this end, the pumping is assumed spatially uniform: $\rho_g = 0$. In the subsequent simulation we use coefficient $\alpha = 2$, at which the most uniform spatial distribution of the fundamental mode fields of a two-dimensional Bragg resonator is achieved. The threshold lasing condition (18), as seen from Fig. 3, is as follows: $\beta = 0.6$. In the case of moderate excesses over the threshold ($\beta \leqslant 3$), stationary lasing is established (Fig. 4).

The process of synchronisation and establishment of stationary lasing is shown in Fig. 5, which plots the spatial distribution of field amplitudes of partial waves at successive instants of time (the amplitudes are normalised to maximum amplitudes at a given time and, therefore, the increasing average radiation intensity in the processes of generation development is not visible in this figure). At the initial stage ($\tau = 0$) initial random noises are shown, but already after several round trips of radiation through the cavity ($\tau = 2$), the characteristic

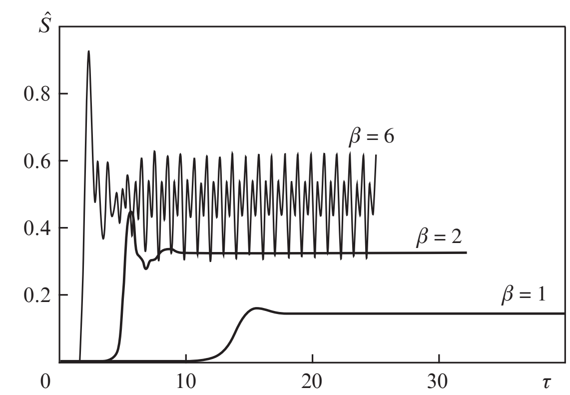

Figure 4. Time dependences of the normalised power in a laser with a two-dimensional Bragg resonator at different excesses over the threshold β; $\alpha = 2, \hat{T}_1 = 1$.

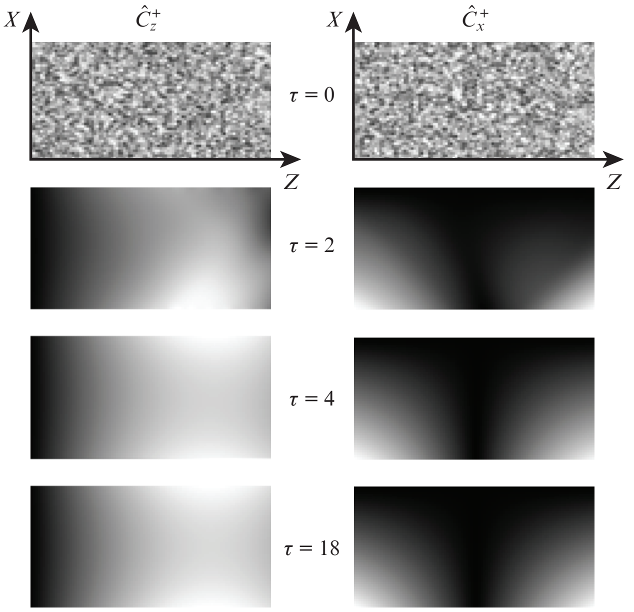

Figure 5. Evolution of the spatial distribution of the amplitudes of the partial waves $\hat{C}_x^+$ and $\hat{C}_z^+$ in the active region of the laser with a two-dimensional Bragg resonator; $\alpha = 1, \beta = 1, \hat{T}_1 = 1$.

scales of field inhomogeneities are of the order of the active region size. In this case, the phases of the fields at this stage become regular. At $\tau = 4$ the structure of the fields of the partial waves that is close to the structure of the fundamental mode of a two-dimensional Bragg resonator is formed at the linear stage of the transition process. At $\tau \geqslant 18$ the growth of the field amplitudes is limited by a drop in the average inversion of the active medium, resulting in a stationary lasing regime. In the simulated case of a relatively small excess over the threshold ($\beta = 1$), field distribution in the stationary regime is close to that of the fundamental mode. However, as the excess over the threshold ($\beta > 2$) increases, there is a noticeable distortion of the structure of the fields in the stationary regime compared to the fundamental mode, which is due to a nonuniform distribution of inversion. Analysis of the emission spectrum shows that in the stationary regime the laser frequency is close to the Bragg frequency. With an increase in the normalised pump intensity β, transition occurs to the regimes of periodic ($\beta > 3$), and then chaotic ($\beta > 7$) self-similarity, which is accompanied by the complication of the spectrum of the generated radiation.

Note that the stationary solutions of equations (8)–(10), realised at a fixed moderate excess over the threshold $\beta = \text{const}$, have the property of self-similarity. If the normalised relaxation time of inversion $\hat{T}_1$ is decreased, then the spatial distribution of the amplitudes of partial waves in the stationary lasing regime is preserved. At the same time, the normalised wave amplitude, polarisation components, and the normalised output power change according to the law

$$\left|\hat{C}^{\pm}_{x,z}\right|\sqrt{\hat{T}_1} = \text{const},\ \left|\hat{P}^{\pm}_{x,z}\right|\sqrt{\hat{T}_1} = \text{const},\ \hat{S}\hat{T}_1 = \text{const}. \quad (20)$$

Using the mentioned self-similarity of the stationary solutions we can formulate the laws of an increase in the size of the active medium and integrated output power. Because the parameters entering into equations (8)–(10) are normalised by the length l_z, at a fixed physical relaxation time T_1 a decrease in the dimensionless parameter $\hat{T}_1$ is achieved by increasing the size of the active region $l_{x,z}$. In this case, it is needed to reduce proportionally the equilibrium value of the population inversion ρ_e (for example, by decreasing the pump power density) and depth of corrugation b_1, keeping the products $l_{x,z}\rho_e = \text{const}$ and $l_{x,z}b_1 = \text{const}$. Then, while maintaining the ratio of geometric dimensions of the active region $l_x/l_z = \text{const}$ the integrated pump power $Q = h\omega_p\rho_e l_x l_z b_a/T_1$ increases according to the law $Q/l_{x,z} = \text{const}$. Similarly, the total integrated output power will increase: $S/l_{x,z} = \text{const}$.

The results of numerical simulations of equations (8)–(10) show that the above scaling of the parameters does lead to the establishment of stationary lasing with the integrated output power increasing with decreasing normalised longitudinal relaxation time $\hat{T}_1$ (Fig. 6). Note that the dynamics of the transient processes also depends on $\hat{T}_1$, tending to simplification with decreasing $\hat{T}_1$. In the case $\hat{T}_1 \to 0$, corresponding to an ultimately rapid relaxation of inversion within the scales of changes in the amplitudes of electromagnetic fields, the equations for the harmonics of the medium inversion in system (8) are reduced to algebraic.

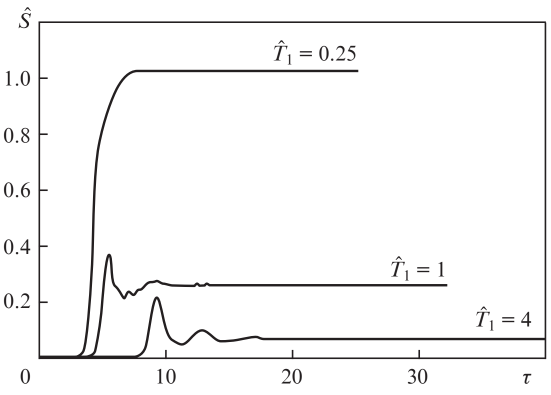

Figure 6. Time dependences of the normalised power under conditions of self-similar scaling of the parameters in a laser with a two-dimensional Bragg resonator at different $\hat{T}_1$ and $\alpha = 2, \beta = 2$.

In a photo-induced DFB laser, the partial wave fluxes are coupled on periodic gratings of inversion ρ_{x+z} and ρ_{z-x}. As in the analysis of the initial conditions, we assume that dynamic gratings can be fabricated by using the pump radiation in the form of standing waves (4), for which $\rho_g = 0.5$. Simulation of

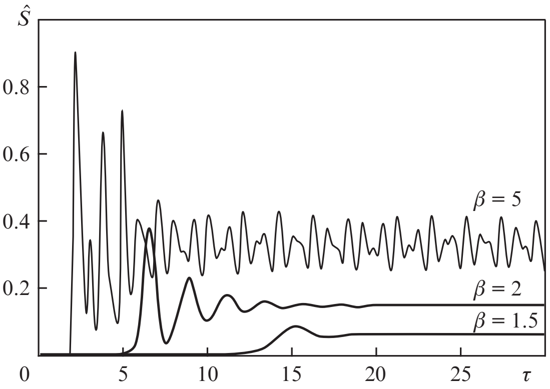

Figure 7. Time dependences of the normalised power in a two-dimensional photo-induced DFB laser at different excesses over the threshold β; $\rho_g = 0.5, \hat{T}_1 = 1$.

the nonlinear dynamics shows (Fig. 7) that, similarly to the two-dimensional Bragg resonator, the two-dimensional gain grating also allows for the establishment of stationary generation in a laser with a spatially extended active medium, whose frequency coincides with the Bragg frequency. In the case of a substantial excess over the lasing threshold, similarly to the static Bragg structures, self-modulation generation regimes with a complex multi-frequency radiation spectrum are realised. At the same time while maintaining the level of β constant in a two-dimensional photo-induced DFB laser, the above-described self-similarity of stationary solutions also takes place. As in the case of static Bragg structures, it is possible to increase the integrated output power with increasing geometric dimensions of the system and simultaneously decreasing proportionally pump intensity.

6. Conclusions

The analysis shows that the two-dimensional distributed feedback can be effectively used to synchronise the radiation of spatially extended laser active media. The proposed feedback mechanism can be implemented on the basis of both static two-dimensional Bragg structures formed by dielectric plates with a doubly periodic modulation of the thickness and dynamic gain gratings induced by the interference of several pump waves. Lasers with a static and dynamic two-dimensional distributed feedback produce four coupled wave fluxes propagating in mutually perpendicular directions, and providing for coherent radiation through the total volume of the active medium. In this paper, we have found threshold lasing conditions within the framework of the semiclassical approximation. Based on the numerical simulations, we have investigated the dynamics of transient processes and identified characteristics of stationary lasing, including the self-similarity conditions.

As practical applications of the investigated mechanism of the distributed feedback, we can single out quantum-well heterolasers. To date, heterostructures have been realised, in which under pumping at the radiation wavelength of the order of one micron, the lateral (transverse) size reaches up to several hundreds of microns and there is a technological possibility of its further increase to thousands of microns [14–15].

Acknowledgements. The authors thank A.S. Sergeev and E.R. Kocharovskaya for fruitful discussions and V.Ya. Aleshkin and V.A. Kukushkin for firsthand acquaintance with the results of the studies on lasers based on spatially extended heterostructures. This work was supported by the Federal Targeted Program 'Scientific and Scientific-Pedagogical Personnel of Innovative Russia' and Dynasty Foundation.

References

1. Yariv A. *Quantum Electronics* (New York: Wiley, 1975).
2. Kogelnik H., Shank C.V. *J. Appl. Phys.*, **43**, 2327, (1972).
3. Kogelnik H. *Theory of Dielectric Waveguides*, in *Integrated Optics* (Berlin–Heidelberg: Springer, 1979) Vol. 7.
4. Luk'yanov V.N., Semenov A.T., Shelkov N.V., Yakubovich S.D. *Kvantovaya Elektron.*, **2**, 2373 (1975) [*Sov. J. Quantum Electron.*, **5**, 1293 (1975)].
5. Rubinov A.N., Efendiev T.Sh. *Kvantovaya Elektron.*, **9**, 2359 (1982) [*Sov. J. Quantum Electron.*, **12**, 1539 (1982)].
6. Grigor'ev A.M. *Kvantovaya Elektron.*, **38**, 685 (2008) [*Quantum Electron.*, **38**, 685 (2008)].
7. Morthier G., Baets R. *J. Lightwave Technol.*, **9**, 1305 (1991).
8. Afanas'ev A.A., Mikhnevich S.Yu. *Kvantovaya Elektron.*, **34**, 315 (2004) [*Quantum Electron.*, **34**, 315 (2004)].
9. Ghafouri–Shiraz H. *Distributed Feedback Laser Diodes and Optical Tunable Filters* (New York: Wiley, 2003).
10. Baryshev V.R., Ginzburg N.S., Malkin A.M., Sergeev A.S. *Opt. Commun.*, **281**, 4879 (2008).
11. Baryshev V.R., Ginzburg N.S., Zaslavskii V.Yu., Malkin A.M., Sergeev A.S., Thumm M. *Kvantovaya Elektron.*, **39**, 463 (2009) [*Quantum Electron.*, **39**, 463 (2009)].
12. Baryshev V.R., Ginzburg N.S., Malkin A.M., Sergeev A.S. *Kvantovaya Elektron.*, **39**, 1159 (2009) [*Quantum Electron.*, **39**, 1159 (2009)].
13. Andreev A.V. *Usp. Fiz. Nauk*, **160**, 1 (1990) [*Sov. Phys. Usp.*, **33**, 997 (1990)].
14. Nekorkin S.M., Biryukov A.A., Demina P.B., Semenov N.N., Zvonkov B.N., Aleshkin V.Ya., Dubinov A.A., Gavrilenko V.I., Maremyanin K.V., Morozov S.V., Belyanin A.A., Kocharovsky V.V., Kocharovsky Vl.V. *Appl. Phys. Lett.*, **90**, 171106 (2007).
15. Aleshkin V.Ya., Babushkina T.S., Biryukov A.A., Dubinov A.A., Zvonkkov B.N., Kolesnikov M.N., Nekorkin S.M. *Kvantovaya Elektron.*, **40**, 855 (2010) [*Quantum Electronics*, **40**, 855 (2010)].