

RELATIVISTIC-INTENSITY LASER FIELDS

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Electrons in a relativistic-intensity laser field: generation of zeptosecond electromagnetic pulses and energy spectrum of the accelerated electrons*

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Abstract. **We study the motion of an electron and emission of electromagnetic waves by an electron in the field of a relativistically intense laser pulse. The dynamics of the electron is described by the Newton equation with the Lorentz force in the right-hand side. It is shown that the electrons may be ejected from the interaction region with high energy. The energy spectrum of these electrons and the technique of using the spectrum to assess the maximal intensity in the focus are analysed. It is found that electromagnetic radiation of an electron moving in an intense laser field occurs within a small angle around the direction of the electron trajectory tangent. The tangent quickly changes its direction in space; therefore, electromagnetic radiation of the electron in the far-field zone in a certain direction in the vicinity of the tangent is a short pulse with a duration as short as zeptoseconds. The calculation of the temporary and spectral distribution of the radiation field is carried out.**

Keywords: *relativistic motion of an electron, ultrashort electromagnetic pulse generation.*

1. Introduction

Recently, $\sim 10^{22}$-W cm^{-2} intensities have been obtained by focusing ultrashort laser pulses. The study of interaction of such intense laser radiation with matter is of great interest for modern physics.

The motion of a charged particle (electron) in the laser radiation field is described by the Newton equation with the Lorentz force in the right-hand side. As a rule, in the case of short laser pulses it is impossible to use the ponderomotive force instead of the Lorentz force.

For short laser pulses the motion of the electron was analysed in [1–5]. The electron oscillates in the laser radiation field; at high intensities the oscillation amplitude becomes comparable with the wavelength. Earlier it was shown that the electron, initially located off the axis of a focused Gaussian beam, after a few oscillations is pushed out with a sufficiently high kinetic energy from the interaction volume at a certain angle. Such a process is interpreted as the electron scattering. The determination of the angular and energy spectra of the accelerated electrons is of great interest and may be used for diagnostics of the laser pulse parameters.

Of interest is also to study the generation of electromagnetic radiation by the electron, oscillating in the strong laser field. Some aspects of this process were investigated in Refs [6–11]. It was shown in [8] that the emission spectrum may be very broad, extending up to hard X-ray and even gamma region. This is an indirect evidence of generation of very short pulses by the electron. At the same time, we should note that no direct calculations of the parameters of the generated pulses, including direct comparison of their duration with spectral distribution, have ever been carried out.

The process of radiation generation by an electron driven by an external electromagnetic field is well-known and referred to as Thomson scattering. In the fields with a relativistic intensity this process becomes strongly nonlinear (multi-quantum), which drastically changes its character. For example, as shown in [12], for linear polarisation of the laser pulse the electromagnetic radiation of the electron into a certain small solid angle has the form of a few ultrashort pulses, each having the duration much shorter than the optical field cycle. Therefore, the widely used term 'nonlinear Thomson scattering' is not quite correct and, in our opinion, 'nonlinear generation of ultrashort electromagnetic pulses' is more adequate.

The goal of the present paper is to study the temporal and spectral characteristics of the electromagnetic radiation of an electron, oscillating in the field of pulsed laser radiation with a relativistic intensity, as well as the energy spectrum of accelerated electrons ejected from the interaction volume.

2. Equations of motion

To describe the dynamics of an electron in an intense electromagnetic field we used the equation of motion with the Lorentz force:

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -e\boldsymbol{E} - \frac{e}{c}[\boldsymbol{v}\boldsymbol{H}], \qquad (1)$$

where $e > 0$ is the absolute value of the electron charge; $\boldsymbol{p}$ is the electron momentum. The initial conditions for the velocity $\boldsymbol{v}$ and the position of the electron were the following: $\boldsymbol{v}(0) = \boldsymbol{v}_0$, $\boldsymbol{r}(0) = \boldsymbol{r}_0$. For the laser pulse, propagating along the z axis, the electric field $\boldsymbol{E}$ in the case of linear polarisation lies in the xz plane, and the magnetic field $\boldsymbol{H}$ lies in the yz plane.

We assume the laser radiation to be a focused beam with Gaussian transverse intensity distribution. In the caustic centre the phase front of the beam is plane, the longitudinal size of the caustic being twice the Rayleigh length $2z_{\mathrm{R}}$, where $z_{\mathrm{R}} = \pi\rho_0^2/\lambda$; ρ_0 and λ are the beam radius in the caustic centre and the radia-

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tion wavelength, respectively. For an arbitrary z the beam radius is given by $\rho(z) = \rho_0(1 + z^2/z_R^2)^{1/2}$.

The expressions for the fields with arbitrary elliptic polarisation have the form [12]

$$E_x = E_0(x,y,\xi)\sqrt{\frac{1+\alpha}{2}}\cos\varphi,$$

$$E_y = \mp E_0(x,y,\xi)\sqrt{\frac{1-\alpha}{2}}\sin\varphi,$$

$$E_z = -2E_0(x,y,\xi)\frac{\varepsilon}{\rho}\Big(\sqrt{\frac{1+\alpha}{2}}x\sin\tilde{\varphi} \pm \sqrt{\frac{1-\alpha}{2}}y\cos\tilde{\varphi}\Big), \quad (2)$$

$$H_x = -E_y, \ \ H_y = E_x,$$

$$H_z = 2E_0(x,y,\xi)\frac{\varepsilon}{\rho}\Big(-\sqrt{\frac{1+\alpha}{2}}y\sin\tilde{\varphi} \pm \sqrt{\frac{1-\alpha}{2}}x\cos\tilde{\varphi}\Big),$$

where $\varepsilon = \lambda/(2\pi\rho_0)$ is a small parameter; $\alpha = 1$ corresponds to linear polarisation along the x axis, $\alpha = -1$ corresponds to linear polarisation along the y axis, and $\alpha = 0$ corresponds to circular polarisation;

$$\varphi = \frac{2\pi c\xi}{\lambda} + \arctan\frac{z}{z_R} - \frac{zr^2}{z_R\rho^2} - \varphi_0;$$

$$\tilde{\varphi} = \varphi + \arctan\frac{z}{z_R};$$

φ_0 is the initial phase;

$$E_0(x,y,\xi) = \frac{E_m\rho_0}{\rho}\exp\Big[-\Big(\frac{\xi - z_d/c}{\tau}\Big)^q - \frac{x^2+y^2}{\rho^2}\Big]; \quad (3)$$

E_m is the maximal strength of the electric field; $\xi = t - z/c$; z_d is the initial distance between the laser pulse and the electron. The temporal profile of the beam in Eqn (3) is described by a super-Gaussian distribution with the parameter q and the 'duration' τ of the pulse. The validity conditions for Eqns (1) and (2) are $\lambda/\rho_0 \ll 2\pi$ and $\rho_0/c < \tau$ [13]. The beam intensity is expressed as

$$I(x,y,z,t) = \frac{c}{4\pi}\overline{[\boldsymbol{E}(x,y,z,t)\boldsymbol{H}(x,y,z,t)]}_z. \quad (4)$$

The maximal intensity $I_m = cE_m^2/(8\pi)$ is attained at the moment when the pulse maximum passes the caustic centre. At equal intensities of the laser radiation, the field amplitudes for linear and circular polarisations differ by the factor of $\sqrt{2}$.

The solutions of Eqn (1) are expressed below in dimensionless variables x/λ, y/λ, z/λ, ct/λ. The components of velocity v_x/c, v_y/c, v_z/c, the components of acceleration $\lambda v_x'/c^2$, $\lambda v_y'/c^2$, $\lambda v_z'/c^2$ and the kinetic energy $W/(mc^2)$ were calculated. The field amplitude was expressed via the dimensionless intensity I_m/I_r, where $I_r = m^2c^3\omega^2/(8\pi e^2) = 1.37\times10^{18}/\lambda^2$ is the relativistic intensity (in W cm^{-2}), λ is expressed in micrometers.

3. The motion of an electron in the laser field

We performed a series of calculations of the dynamics of the electron in the laser field. A short pulse ($c\tau/\lambda = 1.5$) with a Gaussian temporary profile ($q = 2$) was used. The intensity distribution was given by Eqn (3) under the condition of focusing radiation into a spot of very small size ($\rho_0/\lambda = 1$) with the intensity $I_m/I_r = 5000$ in it.

The character of motion of the electron, initially being at rest on the axis at the point $z = z_0$, essentially depends on its position with respect to the caustic centre of the beam. There is a certain value $z_0^* < 0$ for which after the interaction with the laser pulse the electron stops behind the focus at the distance $|z_0^*|$. In this case the trajectory of motion is symmetric with respect to the focus. At $z_0 < z_0^*$ after the interaction the electron moves with a considerable energy in the direction, opposite to that of the laser pulse propagation, at $z_0 > z_0^*$ it is accelerated in the forward direction. The kinetic energy of the electron oscillations is maximal in the case of the symmetric trajectory. This is valid for both linear and circular polarisation, although the corresponding values of z_0^* are somewhat different.

Figure 1a shows a symmetric trajectory ($z_0/\lambda = -9.58 = z_0^*/\lambda$) of the electron, initially localised exactly on the beam axis. It is seen that the entire trajectory consists of quasi-rectilinear segments. The velocity of the electron at these segments is close to the speed of light. The central part of the trajectory is shown in Fig. 1b. At the ends of the rectilinear segments the electron executes a looping turn. Initial displacement of the electron from the axis leads to its ejection form the volume of interaction at a certain angle to the z axis. In Fig. 1c the electron trajectory at $x_0/\lambda = 0.1$ is presented. This trajectory also consists of quasi-rectilinear segments, and at the last segment the electron is ejected from the caustic with residual kinetic energy $W_r/(mc^2) = 128$.

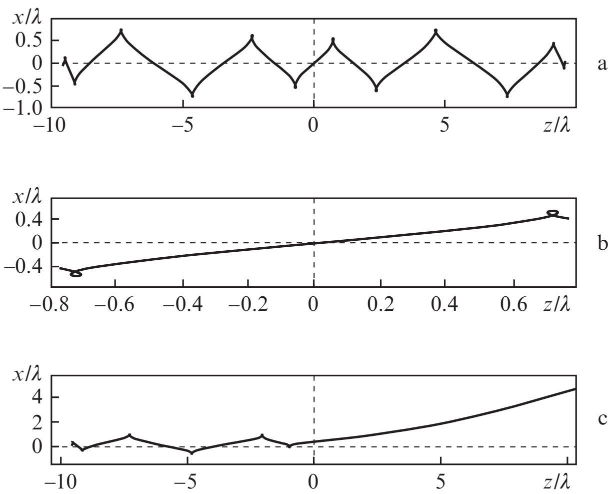

Figure 1. Trajectories of the electron motion in the case of linear polarisation: the symmetric trajectory for $z_0/\lambda = -9.58 = z_0^*/\lambda$ (a), the central part of the symmetric trajectory (b), and the trajectory for the initial displacement $x_0/\lambda = 0.1$ (c).

Figure 2a presents the symmetric trajectory ($z_0/\lambda = -6.79 = z_0^*/\lambda$) of the electron for circular polarisation. The parameters of the laser pulse and the focal spot diameter are the same as for linear polarisation. The trajectory of the electron is a spiral with the diameter strongly changing at the beginning and at the end of the pulse. The spiral has variable pitch, the kinetic energy smoothly changes having the maximal value in the centre of the spot $W_r/(mc^2) = 42$. At circular polarisation the trajectory has no rectilinear segments, but due to the partial trapping of the electron by the laser field the central part of the trajectory is substantially prolonged and the motion along it is close to rectilinear. Figure 2b presents the trajectory of the electron motion at the initial radial displacement by $r_0/\lambda = 0.1$. At the beginning, the trajectory is close to a spiral, then the electron is ejected from the interaction region with the energy $W_r/(mc^2) = 92$.

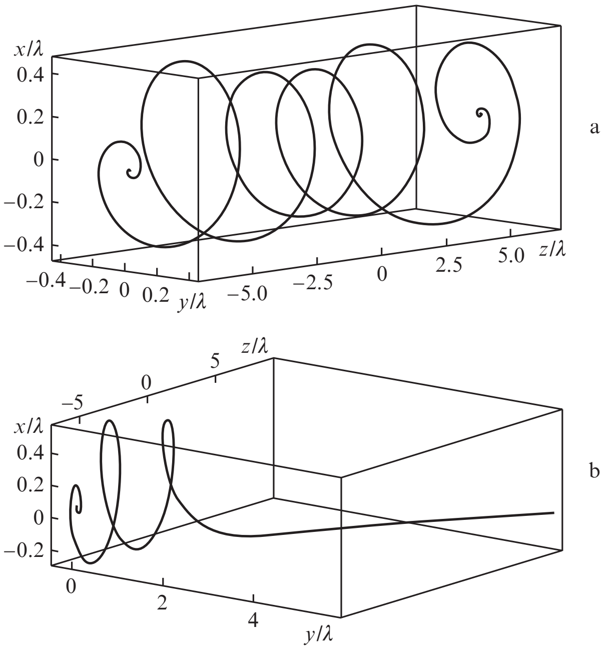

Figure 2. Trajectory of the electron motion in the case of circular polarisation: the symmetric trajectory for $z_0/\lambda = -6.79 = z_0^*/\lambda$ (a) and the trajectory at the initial displacement $r_0/\lambda = 0.1$ (b).

In all considered cases the losses due to the bremsstrahlung are small. A separate publication will be devoted to the consideration of these losses.

4. Electromagnetic radiation of the electron

To study temporal and spectral characteristics of electromagnetic radiation of the electron, the parameters of its motion, obtained above, are used. The electric field is determined by the Lienard–Wiechert potentials and has the following form (for the electron)

$$\boldsymbol{E} = -e\frac{1 - v^2/c^2}{(R - \boldsymbol{R}\boldsymbol{v}/c)^3}\Big(\boldsymbol{R} - \frac{\boldsymbol{v}}{c}R\Big) - \frac{e}{c^2(R - \boldsymbol{R}\boldsymbol{v}/c)^3}\boldsymbol{R}\Big[\Big(\boldsymbol{R} - \frac{\boldsymbol{v}}{c}R\Big)\boldsymbol{v}'\Big], \qquad (5)$$

where the radius-vector $\boldsymbol{R}_0$ drawn from the origin of the coordinate frame determines the point, where the radiation is observed; $\boldsymbol{r}$ is the radius-vector of the electron; the vector $\boldsymbol{R}$ connects the electron with the observation point; $\boldsymbol{r}(t) + \boldsymbol{R}(t) = \boldsymbol{R}_0$; $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$; $\boldsymbol{v}' = \mathrm{d}^2\boldsymbol{r}/\mathrm{d}t^2$. All quantities in Eqn (5) are taken at the time t. They can be recalculated for the time t_r of arrival of radiation at the observation point. Taking the retardation into account, t and t_r are related as $t + R(t)/c = t_r$.

Let us choose the observation point of electromagnetic radiation of the moving electron in the xz plane at the distance $R_0/\lambda = 10000$. The angle φ between $\boldsymbol{R}_0$ and the z axis may vary in a wide range. To characterise the electric field we use the dimensionless vector $\boldsymbol{\Psi} = \boldsymbol{E}/(e/\lambda^2)$. At linear polarisation of the driving laser radiation the vector $\boldsymbol{\Psi}$ is perpendicular to $\boldsymbol{R}_0$ and lies in the xz plane. In the case of circular polarisation the vector $\boldsymbol{\Psi}$ has two components, namely, $\boldsymbol{\Psi}_{||}$ in the plane containing $\boldsymbol{R}_0$ and the z axis, and $\boldsymbol{\Psi}_{\perp}$ in the plane, perpendicular to it. The quantity Ψ_ω is the spectral distribution of $\boldsymbol{\Psi}$.

Figure 3 presents the results of calculations of the parameters of the electromagnetic field, radiated by the electron at the central part of the symmetric trajectory (Fig. 1a) at the angle $\varphi = 29°$ in the laser field with linear polarisation. The time dependence of the quantity Ψ has the form of two short symmetric heteropolar pulses (Fig. 3a). The left pulse is shown separately in Fig. 3b. Its duration at the wavelength $\lambda = 800$ nm (Ti : sapphire laser) is 3.5×10^{-21} s. The spectral distribution Ψ_ω is presented in Fig. 3c and, in a narrower frequency interval, in Fig. 3d. For this case the maximal dimensionless electric field force at the observation point is $\Psi = 1.5\times10^4$, which corresponds to the intensity of 3×10^8 W cm^{-2}.

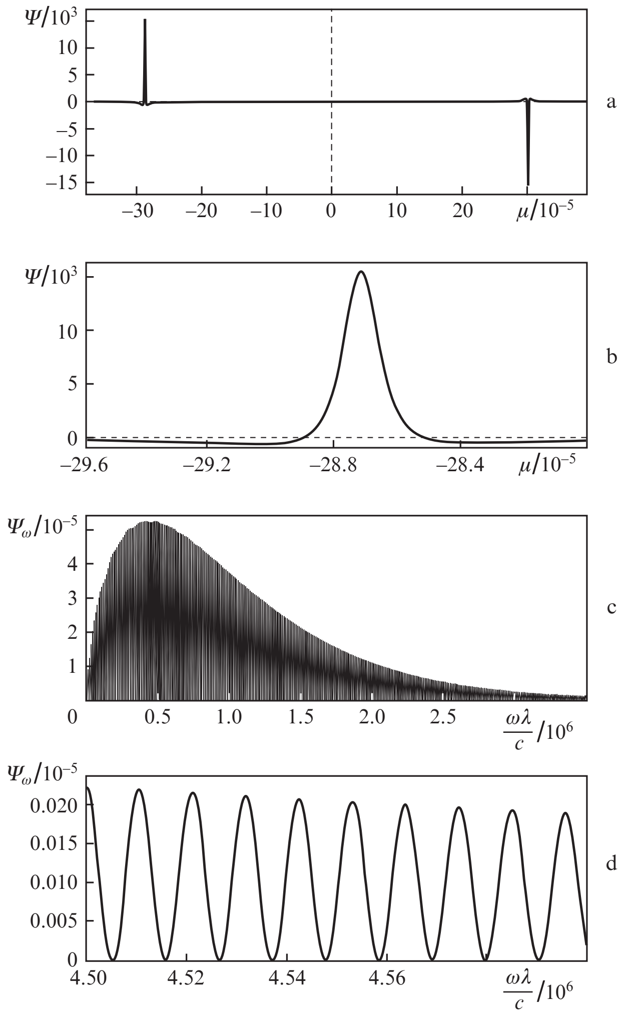

Figure 3. Parameters of the electromagnetic field for the angle $\varphi = 29°$: the electric field versus time $\mu = ct_r/\lambda - \mu_0$ (a), the left pulse in Fig. 3a in the expanded scale (b), spectra of the radiation pulses presented in Fig. 3a (c, d).

Figure 4 presents the results of calculations of the parameters of the electromagnetic field, radiated by the electron, initially displaced from the beam axis (Fig. 1c). We studied radiation propagating at the angle $\varphi = 14.25°$ at the final (before the ejection from the interaction region) part of the trajectory. Figure 4a shows the time dependence of the quantity Ψ, having the form of two short asymmetric heteropolar pulses. The right pulse, having greater intensity, is depicted separately in Fig. 4b. Its duration at $\lambda = 800$ nm amounts to 5.5×10^{-21} s. The structure of the spectral distribution (Fig. 4c) becomes more complicated due to the asymmetry of the pulses under study.

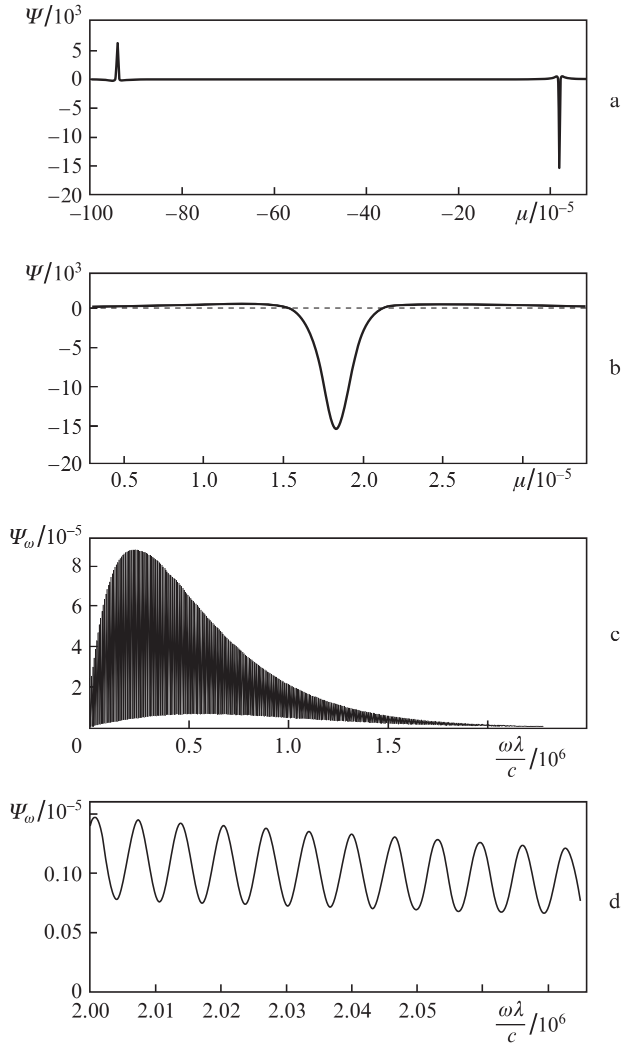

Figure 4. Parameters of the electromagnetic field for the angle $\varphi = 14.25°$: the electric field versus time $\mu = ct_r/\lambda - \mu_0$ (a), the right pulse in Fig. 4a in the expanded scale (b), spectra of the radiation pulses, presented in Fig. 4a (c, d).

The results of calculation of the parameters of the electromagnetic field, radiated by the electron moving along the trajectory, corresponding to Fig. 2a, in the circularly polarised laser field, are shown in Fig. 5. In the case of circular polarisation it is sufficient to analyse the radiation in one plane, e.g., xz plane. In the middle of the trajectory in this plane the tangent is directed at the angle $\varphi = 48.1°$. For this angle the time dependence of $\Psi_\perp$ is presented in Fig 5a. The radiation is a

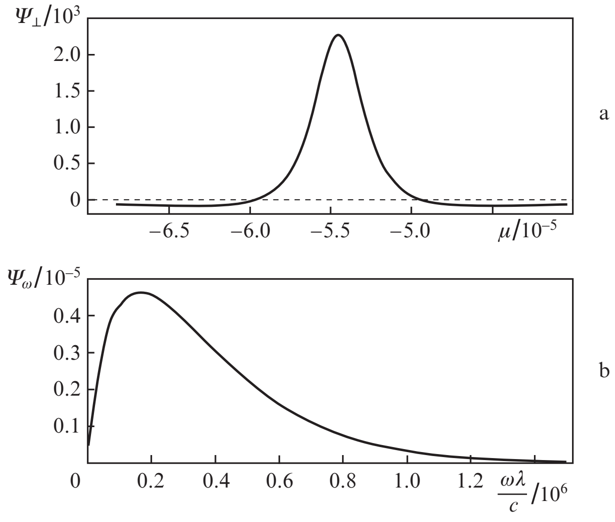

Figure 5. Parameters of the electromagnetic field for the angle $\varphi = 48.1°$: the electric field versus time $\mu = ct_r/\lambda - \mu_0$ (a) and the corresponding emission spectrum (b).

single short pulse with the duration 9.2×10^{-21} s at $\lambda = 800$ nm. Its spectral distribution is presented in Fig. 5b. The component $\Psi_{||}$ is also present in the radiation of the electron in the circularly polarised field; however, it is less intense and has a greater duration.

At a relativistic laser pulse intensity, radiation of electromagnetic waves by the electron occurs in the vicinity of the tangent to the trajectory. At rectilinear motion of the electron with the near-light velocity the fields, generated by the electron at different moments of time, arrive at the observation point practically at the same moment of time. This leads to formation of a very short electromagnetic pulse. In the case of linear polarisation the trajectory of the electron consists of a set of quasi-rectilinear segments. Since at each segment the trajectory still considerably differs from rectilinear, and the tangent changes its direction, only the motion over relatively small pieces of the trajectory segment, at which the motion can be considered as rectilinear, contributes to the electromagnetic radiation of the electron in the far-field zone at a certain moment of time and in a certain selected direction (in the vicinity of the tangent). As a rule, two different pieces of the same quasi-rectilinear segment correspond to a given angle φ. Two pulses of the opposite polarity are generated, because the sign of acceleration changes when passing the central point of a quasi-rectilinear segment. If several pieces of different quasi-rectilinear segments correspond to the same angle φ, there may be several pulses, and the spectrum appears to be modulated. The character of modulation and the total spectral width are determined by the parameters of the pulses. In the simplest case of two symmetric pulses the width of the spectrum is inversely proportional to the pulse duration, and the modulation frequency is inversely proportional to the temporal shift of the pulses. At the initial displacement of the electron perpendicular to the beam axis its trajectory also has quasi-rectilinear segment, the radiation at which is similar to that of the electrons without initial displacement. This is demonstrated by comparison of Figs 3 and 4.

In the case of circular polarisation the electron, moving along a spiral, radiates more uniformly, the tangent direction describing a spiral, too. In each direction in the vicinity of the tangent short pulses are radiated. At circular polarisation the length of the segments, at which the motion may be treated as rectilinear, is much smaller than at linear polarisation. The intensity of pulses, generated in a definite direction, in this case is smaller nearly by an order of magnitude (Figs 3 and 5).

The phases of different spectral components of the scattered radiation are determined by the initial laser beam and the parameters of the electron motion. The interference of these components leads to generation of ultrashort pulses.

The method for calculating the spectra used in the literature, in which the procedure of calculation of the Lienard–Wiechert potentials (5) is combined with simultaneous calculation of their Fourier transform, may lead to incorrect results. The sequential calculation of the radiation parameters and the parameters of the spectrum, used in the present paper, provides elimination of such errors.

In real media a huge number of electrons are born within the caustic of the laser beam, and the pulses, generated by them, are not synchronised in time. The entire radiation from the caustic region is a pulse with a very broad spectrum and the duration equal to the time of laser pulse passing the caustic length. The analysis of the parameters of the spectrum may be used to identify the broadening mechanism. The problems of synchronisation of electromagnetic pulses, radiated by individual electrons, are beyond the scope of this paper.

5. Energy spectrum of electrons

When the relativistic-intensity laser pulse propagates through a gas, the total ionisation occurs at its leading edge, producing electrons with the concentration n_e in the focal region. To calculate their energy spectrum, let us divide the caustic region into parts by planes, perpendicular to the beam axis. Each part of the caustic, characterised by certain z_0, is then divided into cylindrical layers. It is assumed that the electrons, ejected from each layer, possess the same residual kinetic energy. The total energy spectrum of electrons is a sum of spectra produced by these layers.

The calculations show that the trajectory of an electron in the field of a linearly polarised laser pulse lies in the plane, containing the z axis and the point of the initial electron position. To simplify the expressions below, let us introduce the parameter $s \equiv z_0$. The residual kinetic energy $W_r(h,s)$ of the electron depends on its displacement from the axis $h = (x_0^2 + y_0^2)^{1/2}$ and the initial axial coordinate s both for linear and for circular polarisation of the laser beam. The total number of electrons, ejected from the interaction region with the length L, filled with ionised gas, is

$$N = \iiint_V n_e \mathrm{d}V = 2\pi \int_L \mathrm{d}s \int_0^\infty n_e h \mathrm{d}h.$$

On the other hand, the total number of electrons N may be presented as an integral over the energy spectrum:

$$N = \int_0^\infty P(W_r)\mathrm{d}W_r,$$

where $P(W_r)$ is the number of electrons with the residual energies from W_r to $W_r + \mathrm{d}W_r$.

The inverse function $h(W_r, s)$ exists at each fixed value of s. Transforming the integration variables from h, s to W_r, s (using the expression for the Jacobian $\partial(h,s)/\partial(W_r,s) = |\partial h/\partial W_r|$), we get

$$P(W_r) = 2\pi n_e \int_L h \left|\frac{\partial h}{\partial W_r}\right| \mathrm{d}s = \pi n_e \int_L \left|\frac{\partial h^2}{\partial W_r}\right| \mathrm{d}s. \quad (6)$$

In the case of the multiple-valued function $W_r(h,s)$ in Eqn (6) the summation should be performed over the branches of the solution. To normalise the spectrum, the constant $P_0 = \pi[n_e\lambda^3/(mc^2)](L/\lambda)$ is used. Hence, the calculation of the spectrum (6) should be performed using the function $h^2(W_r)$, obtained from the dependence $W_r(h)$.

The maximal residual kinetic energy W_r of the electron, ejected from the region of interaction, depends on the maximal energy of oscillations in the laser field. Therefore, measurements of high-energy part of the spectrum of electrons allow the assessment of the maximal energy of the electron, oscillations in the laser field.

It is shown in [14] that the high-energy part of the spectrum is described by the expression $P/P_0 \sim \exp[-W/(kT)]$, where T is a certain parameter, the 'temperature'. The dimensionless 'temperature' is expressed as $\theta = kT/(mc^2)$. The measurement of T is basic for the diagnostics of the maximal laser radiation intensity in the focal region.

The results of calculating the basic characteristics of $W_r(h)$ for a Gaussian pulse (λ = 800 nm, $c\tau/\lambda = 11$, $\rho_0/\lambda = 8.5$, $n_e = 2.67\times10^{16}$ cm^{-3}) with the intensity I_m/I_r = 10, 30, and 100 are presented in Fig. 6a.

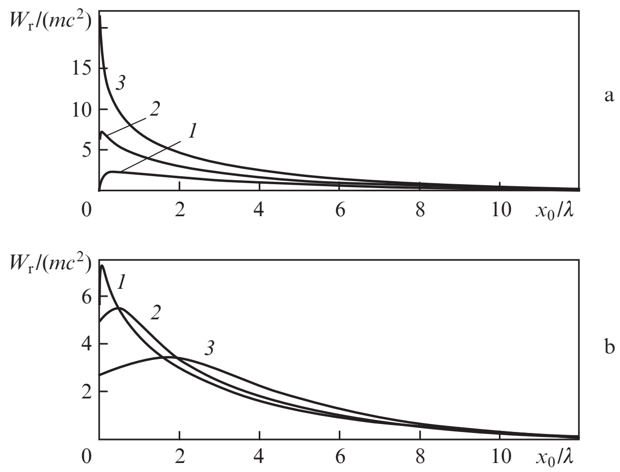

Figure 6. Dependences of the residual kinetic energy of an electron on its initial position x_0/λ ($y_0 = 0$, $z_0 = 0$) at I_m/I_r = 10 (*1*), 30 (*2*) и 100 (*3*) (a) and at the ionisation threshold 0 (*1*), 5.0×10^{15} (*2*), and 1.5×10^{17} W cm^{-2} (*3*) for I_m/I_r = 30 (b).

The dynamics of a single electron, considered above, implies its existence before the appearance of the time-dependent laser field. If the electron is born at the moment of time, when the intensity of laser radiation at a given point reaches the threshold values for multiphoton or tunnel ionisation, it experiences a strong dynamical impact at the creation moment. This may change its velocity and, correspondingly, the residual kinetic energy W_r. In turn, it will change the dependences $W_r(h,s)$ (Fig. 6b) and, consequently, the spectrum (6). Let us estimate these changes. In Fig. 6b curve (*1*) corresponds to the electrons, initially present in the caustic region. Curve (*2*) in Fig. 6b corresponds to the electrons, created when the intensity becomes equal to that of the ionisation threshold for a certain gas (e.g., for helium this threshold is $\sim5\times10^{15}$ W cm^{-2} [15]).

The estimation of the 'temperature' θ from the dependences, shown in Fig. 6b, yields the discrepancy of $\sim$3%. Even when using a gas with the ionisation threshold $\sim 1.5\times 10^{17}$ W cm^{-2} [curve (*3*) in Fig. 6b], the error does not exceed 10%.

The energy, measured with a detector, may differ from the residual electron energy, calculated in the previous Section. First, the positive charge of the ion cloud acts on the electrons and reduces their kinetic energy. Second, the energy of the electrons may change due to the plasma effects, e.g., due to the action of the wake wave. Third, the dynamics of the electron ensemble is affected by the Coulomb and magnetic (pinch effect) interactions within the cloud of ejected electrons. The second of the mentioned factors is essential for plasma, i.e., when ρ_0 is comparable with the Debye radius $r_D = [kT/(4\pi n_e e^2)]^{1/2}$. If $\rho_0 \ll r_D$, the plasma effects may be neglected. The contribution of the pinch effect at low electron concentrations is also small. The magnetic interaction can change the direction of the electron motion, but not the energy.

Let us estimate the effect of the Coulomb interaction of the electrons, ejected from the region of laser pulse action, with the remaining positive charge. This may be done as follows. Let the total number of free electrons, produced in the caustic region, be N_f. Then in this region the positive charge $Q = eN_f$ is produced. The caustic volume may be left only by N_f^* electrons, whose kinetic energy is greater or equal to the attraction energy of the positive charge. Taking into account the expression for the energy spectrum of the electrons, the following relation is obtained: $N_f^* = N_f \exp[-3N_f^* e^2/(2\rho_0 kT)]$. At $n_e \sim 10^{16}$ cm^{-3} we have $N_f^* \sim N_f$. This means that almost all electrons leave the caustic region, and the Coulomb interaction with the ion cloud slightly changes their residual energy.

From the analysis carried out it follows that the proposed method of diagnostics of maximal intensity of laser radiation implies low density of the gas in the focal region.

6. Conclusions

(i) Radiation, emitted by an electron in the field of a short relativistic-intensity laser pulse at a certain given angle to the axis of the beam, consists of electromagnetic pulses having the duration essentially shorter than the period of optical oscillations. The major mechanism of generation of these ultrashort pulses is coherent combining of radiation from the entire length of a quasi-rectilinear segment, present in the trajectory and oriented at a definite angle, into a single ultrashort pulse. Since the length of these quasi-rectilinear segments at linear polarisation is greater than at circular one, the intensity of the radiated pulses in the first case is higher and the duration is shorter. Broad spectral distributions (extending to the gamma range) correspond to the generated pulses. The asymptotic behaviour of the spectra may be used to specify particular mechanisms of their broadening.

(ii) It is shown that the technique for determining the maximal intensity of the laser radiation in the focus with the help of the energy spectrum of the accelerated electrons may be used both when focusing the radiation into the preformed plasma and when the gas ionisation is produced by the laser radiation.

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