

# Generation of $K_\alpha$ radiation by high-efficiency laser targets

A.A. Andreev, K.Yu. Platonov

**Abstract.** The intensity of  $K_\alpha$  radiation emanating from transversely limited thin laser targets with a periodic relief superposed onto its front side was calculated. The relief parameters and the geometrical target dimensions were optimised with the help of an analytic model. The optimal target was shown to possess a nearly 100% absorption coefficient for laser radiation and a high (up to  $10^{-3}$ ) coefficient of laser radiation conversion to the X-ray  $K_\alpha$  radiation.

**Keywords:** laser-produced plasma,  $K_\alpha$  radiation, periodic relief.

## 1. Introduction

One of the tasks of laser plasma physics is to improve the coefficient of conversion of the energy of a laser pulse to the energy of X-ray  $K_\alpha$  radiation (a laser-driven source of hard monochromatic radiation). By varying the parameters of a laser target (the geometrical shape and dimensions, the chemical composition, the density, the relative position of chemical elements) it is possible to raise the absorption coefficient, obtain higher temperatures, and generate a higher number of the fast electrons in the laser-produced plasma, which in turn will raise the X-ray photon yield. Relatively recently Neely et al. [1] found that the coefficient of laser energy conversion to the thermal laser-produced plasma energy and the highest energy of laser-generated electrons are higher when use is made of thin (tens-to-hundreds of nanometres) metal foil targets limited in the transverse direction. Physically, the rise in electron energy is due to the fact that the hot electron spot on a limited target does not spread over the flat surface, and the spot electrons several times find themselves in the domain of laser field action to acquire additional energy.

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There also exists an optimal range for the target thickness, since too thin a target is transparent for laser radiation and exhibits a low absorptivity, while excessively thick targets are responsible for the spreading and cooling of the electron cloud. Both of the limiting cases are not optimal for maximising the fast-electron temperature. Several flat targets were optimised in geometrical characteristics, for instance, by Dong et al. [2] and by Buffechoux et al. [3]. The laser radiation absorption coefficient of these optimised targets is significant (0.5–0.6), but is not as high as unity. Lihua et al. [4] and Wang et al. [5] showed that superposing onto the facial laser target side a periodic microrelief with a height and period comparable with the wavelength of laser radiation is capable of raising to 90% [4] the coefficient of laser radiation conversion to fast electrons for relativistic (above  $10^{18}$  W cm $^{-2}$ ) intensities of laser pulses. A simple increase in the number of the fast electrons and in their energy up to relativistic values is insufficient for the subsequent fast electron-to-X-ray energy conversion. In particular, the logarithmic growth of the cross section for the  $K_\alpha$  photon emission for relativistic electron energies [6] in a massive laser target does not entail an increase in the laser-to- $K_\alpha$  radiation conversion coefficient: the X-ray radiation is ‘collected’ from the  $K_\alpha$  photon absorption length, while the relativistic electrons penetrate substantially greater depths, which the  $K_\alpha$  photons are unable to escape from. The logarithmic build up of the cross section and the high energy of the relativistic electrons can be taken advantage of to improve the  $K_\alpha$  photon yield in the case of a thin foil laser target of thickness of the order of or smaller than the  $K_\alpha$  photon absorption length. The fast electrons circulate through such a target, experiencing reflection from the electrostatic barrier at the foil edges. As this takes place, the X-ray radiation from the entire fast electron path is able to escape from the target, and the relativistic electron energy is used more efficiently [7]. Therefore, a thin laser target exhibits a higher coefficient of fast electron energy conversion to X-ray radiation.

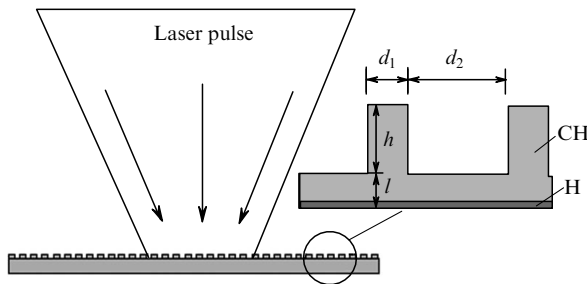
In this work we come up with the idea of combining the advantages of the high conversion coefficient inherent in thin limited targets with the additional improvement in laser radiation absorption due to the relief of the facial target side. By optimising the relief parameters and the geometrical dimensions of the main part of the target (of the substrate), for a given laser pulse it is possible to maximise the total hot electron energy and maximise the coefficient of laser energy conversion to the energy of X-ray  $K_\alpha$  line radiation.

In our work the optimal target characteristics are determined by way of analytical estimates and numerical PIC target simulations with the target parameter set varied

in the vicinity of the analytically derived optimal values. We calculate the  $K_\alpha$  photon yield for the optimal target of the set considered and compare it with the yield from the relief-free plane unlimited foil target of the same thickness. The optimisation is shown to improve the X-ray photon yield by more than an order of magnitude (by about a factor of 13).

## 2. Numerical simulations of targets with different parameters of relief elements and substrate thicknesses

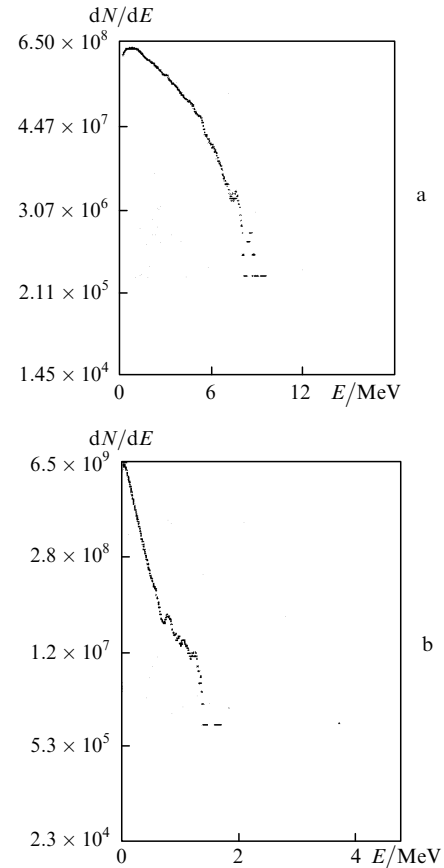
In the present work we investigated targets in the form of a substrate of thickness  $l = 200 - 400$  nm, which was a  $C^{+6}H^{+1}$  foil with a density of  $0.4 \text{ g cm}^{-3}$  with a superposed relief in the form of rectangular (Fig. 1) or sinusoidal asperities, whose period and height were the same in both cases. We considered both unlimited targets and those limited in the transverse direction. Invariably the laser pulse intensity was equal to  $10^{20} \text{ W cm}^{-2}$ , the duration was  $\tau_{\text{las}} = 15$  fs (Gaussian profile), and the focal spot diameter  $3 \mu\text{m}$  (Gaussian profile). The wavelength of laser radiation incident normally on the target was  $0.8 \mu\text{m}$ , the electric vector of the wave field was perpendicular to the relief of the facial side. In the course of calculations we varied the parameters of the rectangular microrelief: its width  $d_1$ , the microrelief spacing  $d_2$  (for a sinusoidal profile, the modulation period was varied instead of  $d_{1,2}$ ), the microrelief height  $h$ , and the foil thickness  $l$ ; also varied were the heterogeneity and homogeneity of the target (the mutual density distribution of  $C^{+6}$  and  $H^{+1}$ ). We emphasise that relief targets call for a high contrast ratio, since a long prepulse will destroy a 150-nm high relief. It is conceivable that targets with large values of  $d_1$ ,  $d_2$ , and  $h$  specific for each prepulse will be efficient for realistic contrast ratios. Our simulations were aimed at maximising the energy of fast electrons and maximising the radiation absorption in the target.



**Figure 1.** Schematic representation of a relief laser target with indication of the geometric parameters of its profile.

Among unlimited targets, the highest absorption and the maximum total electron energy were exhibited by a rectangular target and a harmonically profiled target with the same period and height:  $d_1 = 0.15 \text{ nm}$ ,  $d_2 = 0.35 \text{ nm}$ ,  $h = 0.4 \text{ nm}$ . The highest electron energy was  $\sim 8 \text{ MeV}$ , and the absorption coefficients of these targets were hardly different. That is why for the subsequent optimisation we selected the target with a rectangular profile (as being practically more feasible). This target was limited to  $4 \times 4 \mu\text{m}$  in the transverse direction: in a limited target

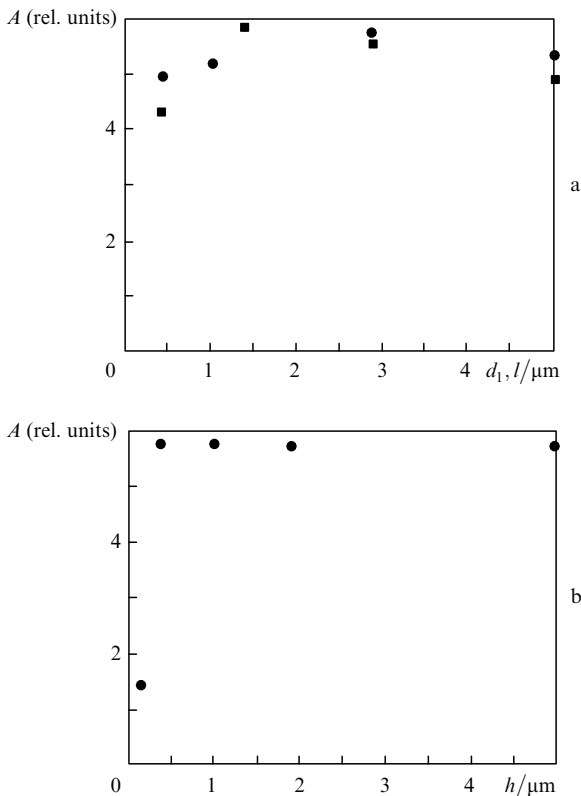
the electrons do not spread over the surface and retain their energy for a longer time [3]. Figure 2a depicts the electron distribution functions for this target immediately after the cessation of the laser pulse and at the end of the computation time (0.8 ps). Shown for comparison in Fig. 2b are these distribution functions in the case of a plane unlimited foil target. The total energy of the hot (above 10 keV) electrons in Fig. 2a, which number  $1.6 \times 10^9$ , is  $3 \times 10^{15} \text{ eV}$ . For the unlimited target represented in Fig. 2b, the number of the electrons is virtually the same,  $2 \times 10^9$ , but their total energy is substantially lower ( $5 \times 10^{14} \text{ eV}$ ).



**Figure 2.** Electron distribution functions upon cessation of the laser pulse for a limited profiled target (a) and a plane unlimited foil of the same thickness (b).

An analysis of the distributions in Fig. 2 suggests that the limitation of the transverse dimension and application of the optimal relief on the target raises the total energy of fast electrons by a factor of  $\sim 6$  and the highest electron energy by a factor of  $\sim 5$  in comparison with the plane foil target of the same thickness. In this case, limiting the transverse target dimension has only a minor effect on the total and highest electron energies immediately after the cessation of the laser pulse. Specifically, the highest electron energies in the limited and unlimited target cases are equal to  $\sim 9 \text{ MeV}$ . The total electron energies are also slightly different. However, the subsequent electron cooling leads to highly significant differences between the limited and unlimited targets. By the instant of termination (0.8 ps) of numerical simulations the maximum electron energy amounts to  $\sim 3 \text{ MeV}$  for the limited target and to only  $0.5 \text{ MeV}$  for

the unlimited one. Since the  $K_\alpha$  radiation is generated up to the point in time when the electron energy becomes equal to the  $K_\alpha$  photon energy, limited micrometer-sized targets with the optimal relief turn out to be more efficient from the standpoint of longer radiation duration and greater amount of generated photons. The target thicknesses employed in our simulations are substantially smaller than the  $K_\alpha$  photon absorption length, and the X-ray radiation is therefore collected from the entire path length of a fast electron in its circulation through the target. The limited target with the optimal relief, which exhibited the highest total energy of the fast electrons, was additionally investigated for the absorption coefficient normalised to the absorption coefficient of a plane target by varying the parameters  $d_1$ ,  $d_2$ ,  $h$ , and  $l$ . One can see that the optimal target parameters employed in the PIC simulations do lie in the region of the local absorption maximum attained under variation of the relief parameters. In this case, the maximum is explicitly traced under variation of substrate thickness and microrelief width, while the relief height may be raised up to values whereby the increase in target volume would be responsible for a fast electron cooling. Below we give analytical estimates employed to determine the optimal parameters of the target relief used in the numerical simulations described above. The adequacy of these estimates is borne out by Fig. 3.



**Figure 3.** Absorption coefficient  $A$  of a limited relief target normalised to the absorption coefficient of a plane  $0.3\text{-}\mu\text{m}$  thick unlimited target as a function of target relief parameters in relation to substrate thickness  $l$  ( $\bullet$ ,  $d_1 = 0.15 \mu\text{m}$ ) and relief asperity width  $d_1$  ( $\blacksquare$ ,  $l = 0.3 \mu\text{m}$ ) for  $d_2 = 0.35 \mu\text{m}$ ,  $h = 0.4 \mu\text{m}$  (a), as well as to relief height  $h$  for  $d_1 = 0.15 \mu\text{m}$ ,  $d_2 = 0.35 \mu\text{m}$ ,  $l = 0.3 \mu\text{m}$  (b).

### 3. Analytical model for laser radiation absorption by a thin relief target

We show that the existence of relief increases the number and energy of fast electrons, which next accelerate protons. In the transverse motion in the laser field and in the absence of longitudinal motion (for instance, in the foil skin layer), the electron energy is

$$\varepsilon_{\text{ch}1} \approx m_e c^2 [\sqrt{1 + I_{18} (\lambda_{\text{las}}/1.2 \mu\text{m})^2} - 1],$$

with the inclusion of longitudinal motion, the electron energy in vacuum in an electromagnetic wave is [8]

$$\varepsilon_{\text{ch}2} \approx 0.7 m_e c^2 I_{18},$$

where  $I_{18}$  is the laser intensity in  $10^{18} \text{ W cm}^{-2}$  units. That is why the electrons gain energy in vacuum with a higher efficiency than in medium. The characteristic size of electron orbit [8] is

$$\begin{aligned} r_{\text{ch}} &= e E_0 \left( m_e \omega^2 \sqrt{1 + \frac{e^2 E_0^2}{2 m_e^2 \omega^2 c^2}} \right)^{-1} \\ &= \frac{\lambda_{\text{las}}}{2\pi} \sqrt{\frac{1.37 I_{18}}{1 + 0.7 I_{18}}}, \quad \omega = \frac{2\pi c}{\lambda_{\text{las}}}. \end{aligned}$$

In the in-vacuum motion, the electron occupies in the wave the transverse space  $2r_{\text{ch}} \approx 0.5 \lambda_{\text{las}}$  ( $I_{18} \gg 1$ ), and therefore the optimal scale length is as follows:  $d_2 \sim \lambda_{\text{las}}/2$  ( $0.4\text{--}0.5 \mu\text{m}$ ). A smaller scale length is disadvantageous, because the orbit is disturbed (the motion becomes lateral), while for a larger scale length the laser spot area is improperly used for relief placement. The intensity dependence of  $d_2$  for the fixed remaining parameters of the laser pulse (see the parameters of the PIC simulation) is given in Fig. 4.

The field  $E_0$ , which acts on the side surface of the relief, is capable of extracting electrons into vacuum from a depth no greater than

$$l_{\text{extr}} = \frac{E_0}{en_e} = 4\pi \frac{c}{\omega_{\text{pe}}} \frac{\omega}{\omega_{\text{pe}}} \sqrt{1.37 I_{18}} = 4\pi l_s \frac{\omega}{\omega_{\text{pe}}} \sqrt{1.37 I_{18}}$$

( $\omega_{\text{pe}}$  is the electron plasma frequency). That is why the thickness  $d_1$  of a relief element should not exceed  $2l_{\text{extr}}$ . For a larger value the laser spot area will be used inefficiently, while for a lower value the number of vacuum electrons will be smaller. The intensity dependence of  $d_1$  for the fixed remaining parameters of the laser pulse (see the parameters of the PIC simulation) is also plotted in Fig. 4.

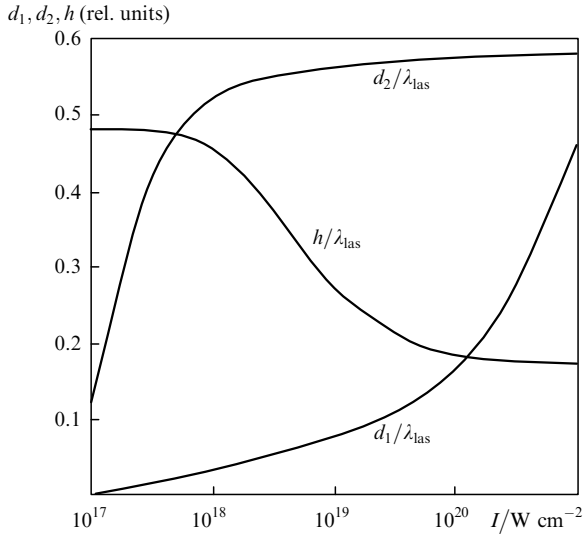
The field  $E_0$ , which acts on the side relief surface, is capable of extracting  $dN_e$  electrons from a unit relief surface area into vacuum:

$$\frac{dN_e}{dS} = n_e l_{\text{extr}} = \frac{E_0}{e}.$$

For a profile height  $h$ , the overall side surface of electron escape is

$$S = h d_{\text{las}} \frac{2d_{\text{las}}}{d_1 + d_2},$$

where  $d_{\text{las}}$  is the laser spot diameter and the factor 2 is due to the existence of two relief sides.



**Figure 4.** Optimal parameters of laser target relief as functions of laser intensity.

Accordingly the total energy of electrons in vacuum is

$$\varepsilon_{\text{tot}} = \frac{hd_{\text{las}}}{e} \frac{2d_{\text{las}}E_0}{d_1 + d_2} \varepsilon_{\text{eh}2}.$$

In the optimum this energy must be equal to the energy of the laser pulse. In this case, the absorption coefficient is assumed to be equal to unity. By equating the in-vacuum electron energy to the energy of the laser pulse we obtain the equation for the lower bound of relief height  $h$ :

$$h \geq \frac{e}{2} \frac{d_1 + d_2}{d_{\text{las}}^2 E_0} \frac{\varepsilon_{\text{las}}}{\varepsilon_{\text{eh}2}} \approx 0.05(d_1 + d_2) \frac{\omega\tau_{\text{las}}}{\sqrt{I_{18}}}. \quad (1)$$

The optimal height increases with lengthening the period  $d_1 + d_2$ ; this is clear, since it is possible to obtain a high absorption with only one relief element by directing the laser pulse along the long surface. This situation is optimal from the absorption standpoint; however, in this case the hot electrons remain with relief elements and do not transit to the substrate. Figure 4 shows the intensity dependence of the minimal  $h$  value for the fixed remaining parameters of the laser pulse (see the PIC simulation parameters). The target is most densely filled by fast electrons at precisely the smallest  $h$ . Therefore, the most efficient target will have the minimal possible values of  $d_1 + d_2$ , which corresponds to  $d_1 + d_2 = l_{\text{extr}} + 2r_e$ . In this case, the profile height is defined by formula (1). We emphasise that the minimal height should not be smaller than a half of the longitudinal dimension  $h^*$  of the ‘figure of eight’ in the electron motion in electromagnetic wave [8]. Otherwise in the derivation of formula for  $h$  there is no way of using the energy  $\varepsilon_{\text{eh}2}$ . In accordance with Landau and Lifshits [8],

$$h^* = \frac{\lambda_{\text{las}}}{16\pi} \frac{1.37I_{18}}{1 + 0.7I_{18}} \sim 0.05\lambda_{\text{las}}.$$

Usually the value of  $h$  obtained from expression (1) is significantly greater than  $h^*$ . We estimate the optimal relief parameters for the PIC simulation parameters we employed:  $\lambda_{\text{las}} = 0.8 \mu\text{m}$ ,  $I_{18} = 100$ ,  $\tau_{\text{las}} = 15 \text{ fs}$ . Then,

$d_2 \sim 0.4 \mu\text{m}$ ,  $d_1 \sim 0.15 \mu\text{m}$ ,  $h \sim 0.2 \mu\text{m}$ . Therefore, the parameters of numerical simulation were close to the optimal ones, with the exception of relief height. We note that making the relief height greater than the optimal one will not change the absorption of laser radiation and the total energy of fast electrons, but this energy will be distributed over a larger volume. For a velocity close to the speed of light, an electron traverses  $1 \mu\text{m}$  in 3.3 fs, and therefore the fast electrons will rapidly fill the target and make an electron Debye sphere around it. Targets with the same absorption coefficients possess the same total energy of fast electrons. The volume of one structure period  $V = (d_1 + d_2)sl + hld_1$ , where  $s$  is the lateral target dimension. In the case of a plane foil, the volume is  $(d_1 + d_2)sl$ ; if the targets have different relief heights  $h_{1,2}$ , the volume ratio is

$$\frac{V_1}{V_2} = \frac{(1 + d_2/d_1)s + h_1}{(1 + d_2/d_1)s + h_2}.$$

For equal absorption coefficients (total electron energies), the electron energy density will become  $V_1/V_2$  times lower and the maximum electron temperature upon expansion to the entire target volume will become  $(V_1/V_2)^{\gamma-1}$  times lower (here,  $\gamma$  is the adiabatic exponent). Therefore, to raise the  $K_\alpha$  photon yield requires not only increasing the absorption by way of making the relief elements higher, but also minimising the relief volume in comparison with the substrate volume. Targets with minimal  $d_1$ , maximal  $d_2$ , and minimal  $h$  from the relief parameter range optimised from the standpoint of absorption turn out to be the targets of choice from the standpoint of X-ray generation efficiency. We now address ourselves to the question of the number of X-ray photons generated by the optimal target and to determining the laser-to- $K_\alpha$ -photon energy conversion coefficient.

#### 4. $K_\alpha$ -photon generation by fast electrons

With the knowledge of the fast-electron distribution function  $f_e(E; t)$  (see, for instance, Fig. 2), its temporal evolution, and the  $K_\alpha$ -photon generation cross section, it is possible to write a simple formula for the total radiated-photon number  $N_{\text{ph}}$ :

$$N_{\text{ph}} = \frac{1}{T} \int_0^T dt \frac{\Omega}{4\pi} N_{\text{eh}} \int_{E_z}^{\infty} dE \varepsilon_z \sigma_z(E) \times \int_0^l dx n_a f_e(E_{e0}; t) \exp\left(-\frac{x}{\lambda_{\text{ph}}}\right), \quad (2)$$

where  $T$  is the period during which the electron energy lowers to the threshold of  $K_\alpha$  photon generation and  $N_{\text{eh}}$  is the total number of hot electrons. The  $K_\alpha$ -photon generation cross section correct in the range of relativistic incident electron energies was obtained in Refs [9, 10]:

$$\sigma_z(E) \approx R(u) 5m \left(\frac{Ry}{E_z}\right)^2 \frac{\ln(u+1)}{u+1} \times [1 - 0.8 \exp(-0.5u)] \pi r_B^2, \quad u = \frac{E}{E_z} - 1. \quad (3)$$

The correction factor  $R(u)$  in formula (3) has two

asymptotic limits:  $R \sim 1$  for nonrelativistic electron energies and  $R \sim u$  for relativistic ones. It is evident that the cross section grows logarithmically with electron energy in the relativistic limit under our investigation. In formulas (2), (3),  $r_B$  is the Bohr radius;  $n_a$  is the density of atoms in the target;  $Ry$  is the Rydberg constant;  $\Omega/4\pi$  is the solid angle of  $K_\alpha$ -photon emission;  $\varepsilon_z(Z) \approx Z^4/(a_z + Z^4)$ ;  $a_z \approx 10^6$  is the  $K_\alpha$  photon factor;  $\lambda_{ph} = |\cos \alpha|/(n_a \sigma_{ph})$  is the photon free path, which depends on the emission angle;  $\sigma_{ph}(Z, \hbar\omega_k) \approx 8.3 \times 10^{-18} Z^{-2} [E_z/(\hbar\omega_k)]^3$  is the  $K_\alpha$  photon photoabsorption cross section [11]; and  $Z$  is the nuclear charge.

In formula (2) the energy  $E_{e0}$  of an electron at the target surface and its energy at depth  $x$  are related by the Bethe–Bloch deceleration law:

$$E_{e0} \approx \sqrt{2 \int_0^x A_c dx} + E^2, \quad A_c \approx 2\pi e^4 n_i Z_{av} \ln(E_z/E_{av}),$$

where  $n_i$  is the target ion density;  $E_z \approx \frac{3}{4} Ry Z^2$  is the K-shell ionisation energy;  $E_{av} \approx 0.8 E_z$ ; and  $Z_{av}$  is the degree of ionisation of the target atoms.

The temporal evolution of the hot-electron distribution function  $f_e(E_{e0}; t)$  in formula (2) was determined from the data of numerical simulations. To this end the distribution function was approximated by an exponential function at the points in time 0.2, 0.4, ..., 0.8 ps (see Fig. 2), whence we determined the time dependence of the effective temperature. Numerical simulations were not performed for a period greater than 0.8 ps, and the temporal dependence of the temperature was extrapolated. Then, by formula (2) we determined the number  $N_{ph}$  of carbon  $K_\alpha$  photons for limited profiled and plane unlimited CH targets. The laser-to-carbon- $K_\alpha$  radiation conversion coefficient  $\kappa = N_{ph} \hbar\omega_{K_\alpha} / \varepsilon_{las}$ , where  $\varepsilon_{las} = 0.1$  J is the energy of the laser pulse and  $\hbar\omega_{K_\alpha} = 0.284$  keV is the carbon  $K_\alpha$  photon energy. For the optimal limited profiled target,  $\kappa \sim 10^{-3}$ , for a plane unlimited target of the same thickness  $\kappa = 6 \times 10^{-5}$ . Therefore, the optimal relief of the laser target improves the efficiency of laser radiation conversion to the X-ray line radiation by about a factor of 13. It is pertinent to note that the finiteness of the laser pulse contrast ratio (the existence of a prepulse) may result in relief destruction and impair the results indicated above. A higher relief and a thicker target may turn out to be optimal in view of the finiteness of the contrast ratio. To every value of the contrast ratio there will correspond optimal geometrical parameters. In our subsequent works we plan to calculate the optimal target in the presence of a laser prepulse.

## 5. Conclusions

The following conclusions can be drawn from the investigation into the  $K_\alpha$  radiation from limited thin relief laser targets executed in the present work.

1. Superposing relief raises the absorption of laser radiation by the target (both limited and unlimited) to maximum possible values close to 100 % and improves the X-ray  $K_\alpha$  photon yield. The absorption and the X-ray yield depend only slightly on the specific profile microrelief (rectangular, harmonic).

2. To attain the highest efficiency of the  $K_\alpha$  photon source, the volume of the relief elements of the limited target

should be smaller than the volume of the underlying foil. Otherwise the absorption of laser radiation will be high, but the subsequent spreading of the hot electron cloud over the large relief volume will result in temperature lowering and reduction of the X-ray photon yield.

3. For a target thickness not exceeding the  $K_\alpha$  photon free path, by employing the effect of circulation of a hot electron in the target it is possible to collect the radiation from the entire path of the hot electron.

4. An optimally thick and laterally limited target with the optimal relief exhibits the presently highest absorption and highest laser-to-X-ray  $K_\alpha$  radiation conversion coefficient (up to  $10^{-3}$  in the simulation outlined).

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