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Resonator with a back deformable mirror for the formation of a beam with a given intensity distribution

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Abstract. **We present the results of a numerical-analytical study of formation of an output beam with a given intensity distribution for a resonator with a back deformable mirror (DM) and semitransparent spherical front mirror. Using the theory of inverse optical problems in the diffraction approximation, the basic characteristics of a laser resonator with a DM are considered as functions of the reference resonator configuration and the form of the distribution function, describing the given intensity. The laser beam formation in the reference resonator with plane-parallel and concentric configurations is compared. In the investigated resonators, the given intensity distribution is a fundamental mode; the selectivity (determined by the power losses of transverse modes) of the resonator with a DM is comparable with that of the reference resonator. It is shown that the formation of the given intensity distributions (uniform, super-Gaussian, or having several maxima) requires a DM with the amplitude of deformations of the optical surface of the order of λ and the number of control channels from 1 to nearly 10.**

Keywords: *laser beam, given intensity distribution, resonator with a back deformable mirror, inverse problem, diffraction.*

1. Introduction

A promising way for solving the problem of formation of laser beams with given parameters is associated with the use of the modalities of active (adaptive) optics [1, 2], e. g., deformable mirrors (DMs). However, the deformation amplitude of the reflecting DM surface is limited (usually within 1–10 μm), and the number of control channels (the number of DM attenuators) is about 10 [2]. The capabilities of DMs are greater in such multi-pass systems as interferometers, multi-pass telescopes, optical resonators. In these systems phase deformations accumulate, which makes it possible to vary not only the phase distribution of the output radiation, but also the distribution of its intensity within the system. Intracavity modalities of active optics are particularly promising. They are applicable to the solution of a wide scope of problems: correction of aberrations, introduced into the output radiation by the optical nonuniformity of the active medium and thermal deformations of the resonator mirrors [3–5]; maximisation of the output radiation power [6, 7]; formation of beams with the given intensity distribution (see, e.g., [8–14] and other papers); implementation of dynamic oscillation regimes [6, 7, 15].

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In the present paper, we consider the problem of formation of output laser radiation with the given intensity distribution. Using the theory of inverse optical problems [16] with the resonator specific features taken into account [12], the resonators with a back DM and semitransparent output mirror are investigated. Within the framework of the developed approach, using the diffraction approximation in relation to the configuration of the reference resonator, the basic characteristics of the system are studied, namely, the quality of formation of the given intensity distribution of the output radiation and its phase distribution, the resonator selectivity (with respect to transverse modes), the desired shape of the reflecting surface of the DM. Considerable attention is paid to the problem of optimising the reference resonator configuration in relation with diffraction effects.

2. Basic relations

Consider an inverse problem of intracavity formation of an output beam with a given intensity distribution under the following limitations imposed on the parameters of the resonator and the characteristics of laser radiation. We study a passive (without active medium) two-mirror resonator with a back DM. A beam with given intensity distribution $I(\boldsymbol{r})$ ($\boldsymbol{r} = (x, y)$ being the radius-vector of a point at the mirror aperture in the cylindrical coordinate system with the z axis coinciding with the optical axis of the resonator) is incident on the output semitransparent mirror (STM), the beam phase distribution being $\varphi_0(\boldsymbol{r})$. It is required to determine the shape of the reflecting surface of the DM, providing the given intensity distribution $I(\boldsymbol{r})$ in the aperture plane of the output STM reflector (as the field distribution in a resonator mode).

The mathematical formulation of the problem is reduced to the study of the solutions of the resonator equation [17]:

$$\int U(\boldsymbol{r}_1)\exp[\mathrm{i}\varphi_1(\boldsymbol{r}_1)]P_1(\boldsymbol{r}_1)K(\boldsymbol{r}_1,\boldsymbol{r}_2)P_2(\boldsymbol{r}_2) \times \exp[\mathrm{i}\varphi_2(\boldsymbol{r}_2)+\varphi_c(\boldsymbol{r}_2)]K(\boldsymbol{r}_2,\boldsymbol{r})\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2 = \gamma U(\boldsymbol{r}), \quad (1)$$

where $U(\boldsymbol{r})$ is the field distribution in the beam, incident on the STM reflector, corresponding to the distribution in the resonator mode; γ is the eigenvalue of Eqn (1); φ_1 and φ_2 are the additional phases, introduced into the phase distribution of the field by the reflection from the STM and DM, respectively; $\varphi_1(\boldsymbol{r}) = kr^2\rho_1$, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $\rho_1 = 1/R_1$ is the STM curvature, R_1 is the curvature radius; $\varphi_2(\boldsymbol{r}) = kr^2\rho_2$, $\rho_2 = 1/R_2$ is the curvature of the back DM in the absence of deformations, R_2 is the curvature radius; $R_j > 0$ ($R_j < 0$) – for convex (concave) mirror, respectively ($j = 1, 2$ for the STM and DM, respectively); $\varphi_c(\boldsymbol{r}) = 2kS_2(\boldsymbol{r})$

is the additional phase, introduced by the deformation of the DM; $S_2(\boldsymbol{r})$ is the deviation of the DM reflecting surface shape from the reference spherical one; $P_{1,2}(\boldsymbol{r})$ is the aperture function of the STM and DM, respectively [$P_j(\boldsymbol{r}) = 1$ within the aperture and $P_j(\boldsymbol{r}) = 0$ beyond the aperture limits]; the diameter (width) of the STM (DM) reflector aperture is $2a_1$ ($2a_2$) and the corresponding Fresnel numbers are $N_{1,2} = a_{1,2}^2/(\lambda l)$; $K(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is the kernel of the propagation integral between the apertures of the reflectors. In the calculations we used the diffraction approximation:

$$K(\boldsymbol{r}_1, \boldsymbol{r}_2) = \exp(\mathrm{i}kl)\frac{k}{2\mathrm{i}\pi l}\exp\left[\frac{\mathrm{i}k(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{2l}\right],$$

where l is the resonator length.

Thus, the considered inverse problem is reduced to the following mathematical problem: to find a function $\varphi_c(\boldsymbol{r})$ such that the given field distribution function $U_0(\boldsymbol{r}) = \sqrt{I(\boldsymbol{r})}\exp[\mathrm{i}\varphi_0(\boldsymbol{r})]$ would be a solution of Eqn (1).

However, as known (see, e.g., [16, 18]), inverse optical problems do not always have a solution or may have an ambiguous solution. Numerical studies by the iterative method show that under the limitations introduced above, the considered inverse problem has no solution in the general case. Let us formulate the problem with softer conditions for the given field. We will seek for the solution in the general form $U(\boldsymbol{r}) = |U(\boldsymbol{r})|\exp[\mathrm{i}\varphi_0(\boldsymbol{r}) + \mathrm{i}\varphi(\boldsymbol{r})]$; where the function $|U(\boldsymbol{r})|$ should be close to $\sqrt{I(\boldsymbol{r})}$, and $\varphi(\boldsymbol{r})$ is the error of the given phase distribution formation. The characteristics of the given field distribution $U_0(\boldsymbol{r})$ are taken into account by choosing the function $\varphi_c(\boldsymbol{r})$. With this aim, we derive an expression for $\varphi_c(\boldsymbol{r})$ making use of the least-squares method to fit the left- and right-hand sides of relation (1) with the function $U_0(\boldsymbol{r})$ substituted into it instead of $U(\boldsymbol{r})$. In other words, we minimise the mean square modulus of the difference between the given relative field distribution at the output STM reflector and the relative distribution, produced by the complete (forth and back) resonator roundtrip. As a result, for the additional phase, introduced by the reflection from the DM (up to a constant inessential in the present case), we obtain the relation

$$\varphi_c(\boldsymbol{r}) = -P_2(\boldsymbol{r})\arg\{U_i(\boldsymbol{r})U_{r2}^*(\boldsymbol{r})\exp[\mathrm{i}\varphi_2(\boldsymbol{r})]\}, \tag{2}$$

where $U_i(\boldsymbol{r}) = \int U_0(\boldsymbol{r}_1)P_1(\boldsymbol{r}_1)\exp[\mathrm{i}\varphi_1(\boldsymbol{r}_1)]K(\boldsymbol{r}_1, \boldsymbol{r})\mathrm{d}\boldsymbol{r}_1$ and $U_{r2}(\boldsymbol{r}) = \int U_0(\boldsymbol{r}_1)K^*(\boldsymbol{r}_1, \boldsymbol{r})\mathrm{d}\boldsymbol{r}_1$ describe the field, incident on the DM and reflected from it (in the plane of the reflector aperture), respectively; $U_0(\boldsymbol{r}) = \sqrt{I(\boldsymbol{r})}\exp[\mathrm{i}\varphi_0(\boldsymbol{r})]$. Expression (2) is valid for arbitrary functions $\varphi_1(\boldsymbol{r})$ and $\varphi_2(\boldsymbol{r})$, which allows the study of the formation quality of the given intensity distribution depending on the configuration of the reference resonator. This expression agrees with the results of [12]; however, it explicitly takes into account the limitedness of the back mirror aperture.

Let us specify the parameters of the resonator and the beam. For practical reasons, the shape of the reflecting surface of the output mirror, as well as that of the back mirror in the absence of deformations, is chosen spherical; the corresponding resonator (the reference-configuration resonator) is assumed to be stable, so that $0 \leqslant g_1 g_2 \leqslant 1$ ($g_j = 1 + l\rho_j$; $j = 1, 2$). The output beam is completely within the aperture of the semitransparent reflector. It is assumed that the phase distribution and the relative amplitude distribution do not change when the beam passes through the STM. The result of the solution of the inverse problem under study essentially depends on the choice of the phase distribution function $\varphi_0(\boldsymbol{r})$. Let the phase distribution in the beam under formation coincide with that of the fundamental mode of the reference resonator (with infinite mirrors), $\varphi_0(\boldsymbol{r}) = -\varphi_1(\boldsymbol{r})/2$. Under these conditions, we have $U_i(\boldsymbol{r}) = U_{r2}^*(\boldsymbol{r})$ in the DM aperture plane, and Eqn (2) is reduced to the form:

$$\varphi_c(\boldsymbol{r}) = -2P_2(\boldsymbol{r})\arg\{U_i(\boldsymbol{r})\exp[\mathrm{i}\varphi_2(\boldsymbol{r})/2]\}, \tag{3}$$

where $U_i(\boldsymbol{r}) = \int\sqrt{I(\boldsymbol{r})}P_1(\boldsymbol{r}_1)\exp[\mathrm{i}\varphi_1(\boldsymbol{r}_1)/2]K(\boldsymbol{r}_1, \boldsymbol{r})\mathrm{d}\boldsymbol{r}_1$.

In correspondence with Eqn (3), the DM implements phase conjugation of the field. Note, that the similar result of Refs [11] and [13] relates only to the resonators with plane-parallel reference configurations.

When the DM aperture is unlimited, relation (3) yields the exact solution of the inverse problem. If the aperture of the mirror is limited, then a part of the beam $|U_i(\boldsymbol{r})|^2$, propagating beyond the edges of the DM, leaves the resonator, and Eqn (3) yields the solution with close-to-minimal difference between the given $U_0(\boldsymbol{r})$ and arisen $U(\boldsymbol{r})$ field distributions (in the sense of the quality criterion accepted above).

Equation (1) together with relation (3) allows one to determin the spectra of eigenfunctions U and eigenvalues γ and to investigate the quality of the given intensity distribution formation depending on the reference resonator configuration and the characteristics of the given distribution.

When use is made of a DM, the limitation of its deformation amplitude should be taken into account [2]. Keeping this in mind, the analysis of relation (3) for beams with a uniform amplitude distribution (for Fresnel numbers $N_1 > 1$) shows that it is preferable to use resonators with plane-parallel or concentric reference configurations ($g_1 g_2 = 1$). However, depending on the end use of the system and the given parameters of the output beam, resonators with other reference configurations may be also of practical interest (e.g., a semiconfocal reference resonator [19]).

In the numerical experiment significant attention is paid to the study of the role of radiation diffraction by the reflector edges in the formation of the given intensity distribution. The diffraction causes undesirable modulation in the intensity and phase distributions of the output radiation, leads to severe requirements to the DM, may reduce the laser efficiency due to the radiation losses because of mismatch between the dimensions of the beam and the active element in the resonator. The following methods for reducing the diffraction effect are considered: (i) increasing the size of the DM; (ii) using a resonator with a concentric (at $0 < g_1 < 1$) reference configuration (which allows reduction of the beam size at the back mirror because of the concavity of the output reflector); (iii) choosing $I(\boldsymbol{r})$ in the form of a function that decreases monotonically to zero or near-zero level when approaching the STM edges.

To estimate the role of diffraction in the choice of the reference resonator configuration, we will analyse the relation for the field, incident on the DM, taking the results of Ref. [17, pp 93–96] into account. Let a two-dimensional cylindrical beam with a uniform amplitude distribution be specified at the STM. The beam width increases in the course of propagation due to diffraction effects, and in the plane of the DM aperture, it becomes nearly $1 + 1/\sqrt{\pi g_1 N_1}$ times greater ($N_1 g_1 > 1$, $g_1 > 0$). At the same time the dimensions of the beam change by g_1 times due to the cylindrical shape of its wave front. Hence, in the plane of the DM reflector aperture, the beam half-width is

$$a_{02} \approx a_1 g_1 (1 + 1/\sqrt{\pi g_1 N_1}). \quad (4)$$

High-quality formation of the given distribution is provided if the DM reflector aperture half-width is not smaller than the beam half-width, i.e., $a_2 \geqslant a_{02}$. From this condition and relation (4), it is clear that in the case of a plane-parallel reference resonator ($g_1 = g_2 = 1$), it is reasonable to choose the DM aperture half-width equal to (or not smaller than)

$$a_2 \approx a_1 (1 + 1/\sqrt{\pi N_1}). \quad (5)$$

Note also that the transverse dimension of the beam in the cross sections located closer to the back mirror appear to be greater than the given dimension $2a_1$ of the beam at the STM reflector. Therefore, when using a rod active element with the constant width $2a_1$ of cross sections along the optical axis, a part of the beam near the DM appears to be outside the active element. This leads to the reduction of the active medium efficiency and to lowering of the quality of the given distribution formation. This drawback is eliminated using a resonator with a concentric configuration, since if we assume that

$$g_1 \approx 1/(\sqrt{1 + 1/(4\pi N_1)} + 1/\sqrt{4\pi N_1})^2, \quad (6)$$

then, taking Eqn (4) into account, in all places inside the resonator the dimensions of the cross sections of the laser beam and the active element will become approximately the same ($a_1 \approx a_2 \approx a_{02}$). Besides, because the beam incident on the DM reflector (with diffraction broadening and geometrical convergence taken into account) is practically completely located within its aperture, the field formation quality provided is nearly the same as in the case of a resonator with a DM, plane-parallel reference configuration and increased aperture (5) of the back mirror.

Consider a plane-parallel resonator with the given field amplitude distribution, smoothed to the edges of the STM aperture. To estimate the optimal dimension of the near-edge smoothing area, we assume that the width $2a_{02}$ of the beam at the back mirror is equal to the width $2a_1$ of the given beam at the output mirror. With this aim let us approximate the smoothed field distribution with a rectangular one: within the smoothing region with the width Δa_1 at the edges of the aperture, the field is equal to zero, and in the centre of the aperture within the region with the half-width $a_1 - \Delta a_1$, the field has a uniform distribution. We evaluate the width of the smoothed beam at the DM using relation (5) by replacing a_1 with $a_1 - \Delta a_1$ in its right-hand side. Taking the equality $a_1 \approx a_2 \approx a_{02}$ into account, we get

$$\Delta a_1 \approx a_1/\sqrt{\pi N_1}, \quad (7)$$

i.e., the dimension of the boundary smoothing region is chosen to provide a look-ahead compensation of the diffraction beam divergence, occurring on the path from the output reflector to the DM. If the width of the smoothing region is smaller than that given by relation (7), then due to diffraction the quality of the given distribution formation becomes lower. If the width of the smoothing region is greater, then the beam concentrates near the resonator axis and the selectivity of the resonator (with respect to the power losses) becomes lower. Hence, the size of the smoothing region is an optimised parameter, depending on particular requirements to $I(\boldsymbol{r})$ and the characteristics of the laser resonator.

3. Results of the numerical experiment

In the numerical experiment, the additional phase (2) introduced by the DM was first calculated at given $I(\boldsymbol{r})$, g_1, g_2, $P_1(\boldsymbol{r})$, $P_2(\boldsymbol{r})$, N_1 and N_2. Then the integral equation (1) was solved. The mean square deviations (MSDs) of arising distributions of intensity (σ_{I}) and phase (σ_φ) from the given ones were accepted as the beam quality factors. The appropriate variances were calculated using the formulae

$$\sigma_{\mathrm{I}}^2 = 1 - \frac{\left(\int JI\,\mathrm{d}\boldsymbol{r}\right)^2}{\int J^2\mathrm{d}\boldsymbol{r}\int I^2\mathrm{d}\boldsymbol{r}}, \quad \sigma_\varphi^2 = \frac{\int\sqrt{I}\,\varphi^2\mathrm{d}\boldsymbol{r}}{\int\sqrt{I}\,\mathrm{d}\boldsymbol{r}} - \left(\frac{\int\sqrt{I}\,\varphi\mathrm{d}\boldsymbol{r}}{\int\sqrt{I}\,\mathrm{d}\boldsymbol{r}}\right)^2.$$

Here $J = |U^2|$ and the integration is performed within the limits of the output reflector aperture. At small MSDs ($\sigma_\varphi < 1$) due to phase distortions of the field [20], the Strehl number of the resulting beam is $Is \approx \exp(-\sigma_\varphi^2)$.

Numerical studies were carried out for the resonators with plane-parallel and concentric reference configurations. Let the mirrors have rectangular shape and let the given intensity distribution function allow separation of variables x and y: $I(\boldsymbol{r}) = I_x(x)I_y(y)$. Then, Eqn (1) is split into two independent equations. Finally, we get $\varphi_{\mathrm{c}}(\boldsymbol{r}) = \varphi_{x\mathrm{c}}(x) + \varphi_{y\mathrm{c}}(y)$, $\gamma = \gamma_x\gamma_y$. The function $\varphi_{x\mathrm{c}}(x)$ ($\varphi_{y\mathrm{c}}(y)$) and γ_x (γ_y)# are found for a two-dimensional (strip) resonator with the given intensity distribution $I_x(x)$ ($I_y(y)$).

The results of calculations based on Eqns (1)–(7) for a two-dimensional resonator are presented in Figs 1–3. In all figures the normalised spatial coordinate $X = x/a_1$ is used. Figure 2 presents, for the investigated types of resonators, the dependences of the square modulus of eigenvalues on the number of the mode n ($n = 0, 1, \ldots$).

The obtained results of the numerical calculations agree with the analytical estimates of the effect of diffraction on the choice of configuration of the reference resonator presented above. Replacing the resonator having a DM and plane-parallel reference configuration with mirrors of similar size (Fig. 1a) by a resonator having an enlarged DM (Fig. 1b) or by a resonator with a concentric reference configuration (Fig. 1c) results in more precise formation of the given field. If a super-Gaussian intensity distribution, smoothed near the edges, is given (Fig. 1d), then, as one could naturally expect, the effect of diffraction on the field formation becomes smaller.

In all considered cases, the given intensity distribution is a fundamental mode. The selectivity of the synthesised resonator, as follows from the analysis of Fig. 2, appears to be close to that of a plane-parallel resonator. The phase distribution of the output radiation is close to the given one (uniform or cylindrical). In all calculations, the Strehl number $Is > 0.9$, which indicates the high quality of the obtained field.

The additional phase (path difference) introduced by the DM is comparable with the wavelength of the resonance radiation. The maximal value of the DM deformation, as follows from the analysis of Fig. 1, amounts to $\sim\lambda/10$, and the required

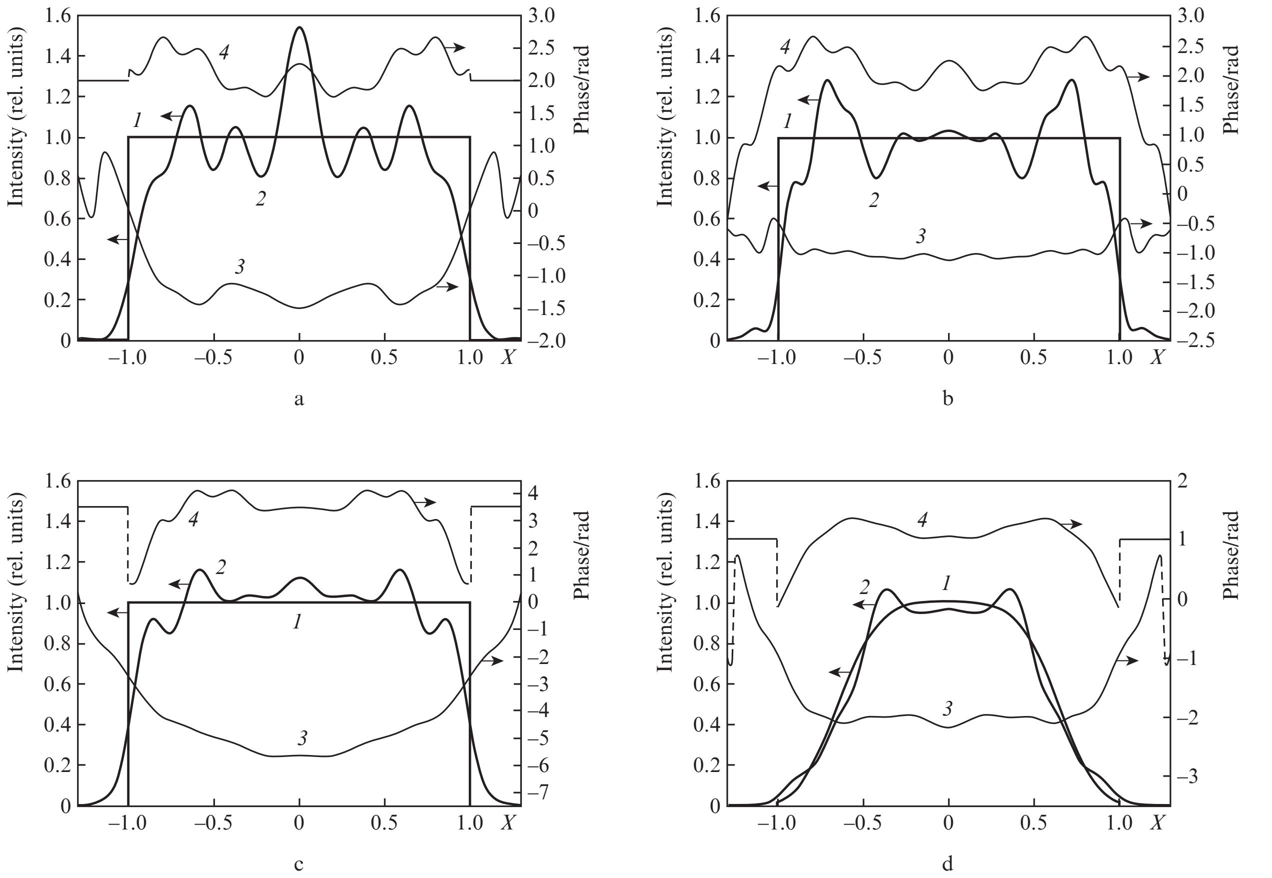

Figure 1. Intensity and phase distributions for the resonator with a DM and plane-parallel reference configuration ($N_1 = N_2 = 2.5$, $\sigma_I = 0.38$, $\sigma_\varphi = 0.28$) (a), the resonator with an enlarged DM and plane-parallel reference configuration ($N_1 = 2.5$, $N_1 = 4.6$, $\sigma_I = 0.13$, $\sigma_\varphi = 0.14$) (b), the resonator with a DM and concentric reference configuration ($N_1 = N_2 = 2.5$, $g_1 = 1/g_2 = 0.7$, $\sigma_I = 0.14$, $\sigma_\varphi = 0.11$) (c), and the resonator with a DM and plane-parallel reference configuration with the given super-Gaussian distribution ($N_1 = N_2 = 2.5$, $\sigma_I = 0.1$, $\sigma_\varphi = 0.17$) (d): (*1*) and (*2*) the given and the formed intensity distributions; (*3*) the phase distribution of the output radiation; (*4*) the additional phase introduced by the DM. Figures 1a–c correspond to the uniform distribution, $I_x(X) = 1$; Fig. 1d corresponds to the super-Gaussian distribution, $I_x(X) = \exp(-4X^4)$.

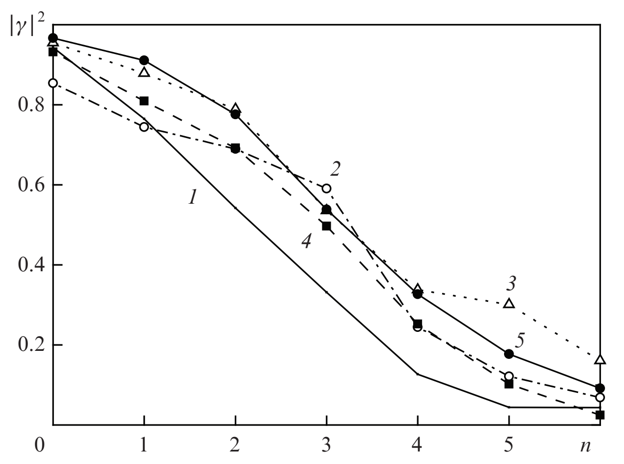

Figure 2. Square modulus of eigenvalues for different resonator configurations as functions of the mode number: (*1*) reference plane-parallel resonator ($N_1 = 2.5$, $N_2 = 4.6$); (*2,3*) resonators with a DM and plane-parallel reference configuration (with Fresnel numbers $N_1 = N_2 = 2.5$, and $N_1 = 2.5$, $N_2 = 4.6$, respectively); (*4*) resonator with a DM and concentric reference configuration ($N_1 = N_2 = 2.5$, $g_1 = 0.7$); (*5*) resonator with a super-Gaussian distribution ($N_1 = N_2 = 2.5$, $g_1 = g_2 = 1$, $\Delta a_1 \approx 0.35a_1$).

number of channels used to control the DM shape lies in the range from 2 to 7. Three-dimensional resonators are characterised by the increased deformation range $\lambda/5$–$\lambda/2$ and the increased number of control channels from 10 to 50. The technology of fabricating DMs with such characteristics is known [2].

The efficiency of formation of distributions, having several intensity maxima within the aperture limits, was also investigated (Fig. 3). Figure 3a corresponds to the distribution $\sqrt{I_x(X)} = \exp[-16(|X| - 0.5)^2]$ with two maxima, and Fig. 3b corresponds to the distribution $\sqrt{I_x(X)} = \exp[-35(|X| - 0.7)^2] + \exp[-35|X|^2]$ with three maxima.

It follows from the analysis of Fig. 3 that for the formation of a beam with several maxima, the shape of the DM is similar to the function of the intensity distribution. This may be treated as the presence of several local resonators, coupled via the field, inside the unique resonator. If we tentatively consider each intensity peak as the radiation from a separate laser, then the DM, external with respect to all these lasers, provides phase matching of the lasers.

Annular beams are also of practical interest. One of their advantages is the efficient propagation in nonlinear media [21]. Figure 4 presents the given annular distribution of intensity $I(\boldsymbol{r}) = \exp[-32(r/a_1 - 0.5)^2]$ and the distribution of the required additional phase, introduced by the DM.

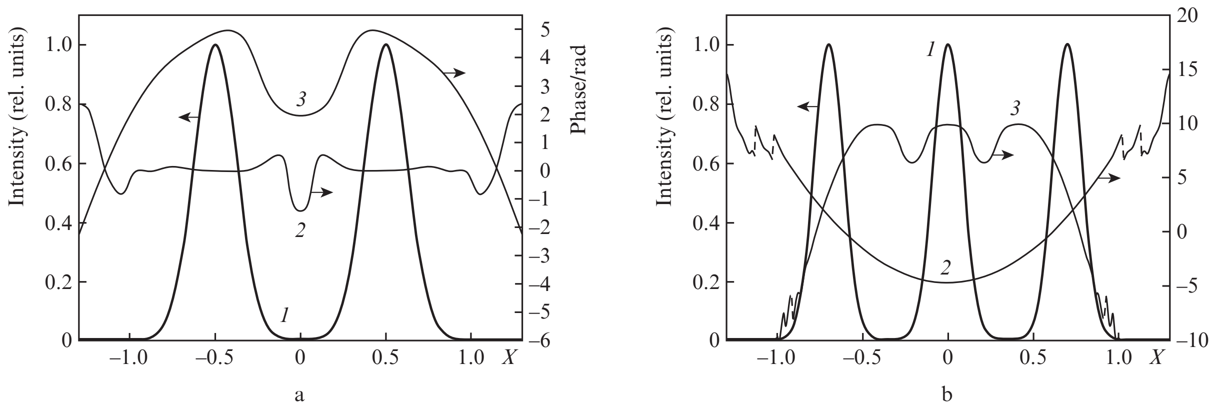

Figure 3. Intensity and phase distributions for the resonator with a DM and plane-parallel reference configuration ($N_1 = 2$, $N_2 = 3.4$, $\sigma_I = 0.26$, $\sigma_\varphi = 0.14$) (a) and the resonator with a DM and concentric reference configuration ($N_1 = N_2 = 10$, $g_1 = 0.6$, $\sigma_I = 0.03$, $\sigma_\varphi = 0.01$) (b): (*1*) the given intensity distribution; (*2*) phase distribution of the output radiation; (*3*) additional phase introduced by the DM.

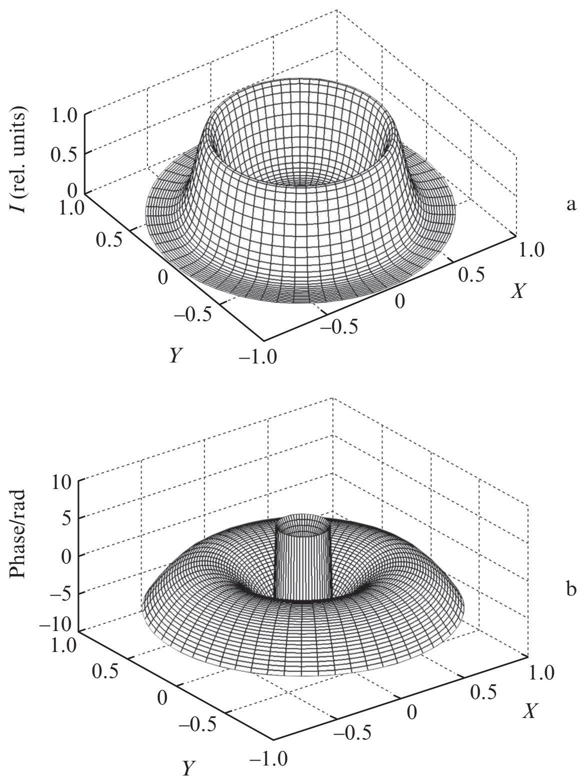

Figure 4. The given intensity distribution (a) and the additional phase $\varphi_c(\boldsymbol{r})$ (b) in the case of a plane-parallel reference resonator with similar round mirrors ($N_1 = N_2 = 10$).

In analogy with the two-dimensional case, the additional phase distribution function is similar to the distribution function of the given intensity distribution. The maximal mirror deformation, required to get an annular intensity distribution, amounts to $\sim\lambda/2$. The required shape of the DM may be obtained by applying an axially symmetric annual stress to the back surface of the DM (one control channel with annual drive).

4. Conclusion

Using the theory of inverse optical problems, we have studied the quality of formation of an output laser beam with the given intensity distribution as a function of the form of the given distribution function and the configuration of the stable reference resonator. It is proposed to use concentric reference resonators to provide matching of the dimensions of the given beam, which is broadened due to diffraction when approaching the DM, with those of the active element. It is shown that the resonators with plane-parallel and concentric reference configurations provide formation of the given intensity distribution of the output beam at back mirror deformation amplitudes of the order of λ with the number of degrees of freedom (number of DM drives) from 1 to nearly 10. If the given intensity distribution possesses several maxima, then the synthesised resonator can be tentatively considered as several coupled local resonators, each responsible for the corresponding intensity maximum. The investigated resonators at optimal choice of their parameters provide the selectivity (with respect to power loss for transverse modes), comparable with that of a plane-parallel resonator, and the matched volumes of the mode and the active medium. The error of the given intensity distribution formation is $\sim 5\%$, the Strehl number exceeds 0.9.

The results of the present work may be used in the studies of phase matching of an array of lasers, in particular, an array of laser diodes.

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References

1. Vorontsov M.A., Shmal'gauzen A.V. *Printsipy adaptivnoy optiki* (Principles of Adaptive Optics) (Moscow: Nauka, 1985).
2. Taranenko V.G., Shanin O.I. *Adaptivnaya optika v priborakh i ustroystvakh* (Adaptive Optics in Instruments and Devices) (Moscow: TSNIIATOMINFORM, 2005) p. 416.
3. Spinhirne J.M., Anafi D., Freeman R.H., Garsia H.R. *Appl. Opt.*, **20** (6), 976 (1981); Anafi D., Spinhirne J.M., Freeman R.H., Oughstun K.E. *Appl. Opt.*, **20** (11), 1926 (1981); Spinhirne J.M., Anafi D., Freeman R.H. *Appl. Opt.*, **21** (21), 3969 (1982); Oughstun K.E., Spinhirne J.M., Anafi D. *Appl. Opt.*, **23** (10), 1529 (1984).
4. Kislov V.I., Taranenko V.G. *Radiotekh. Elektron.*, **32**, 287 (1987).
5. Wittrock U., Welp P. *Proc. SPIE Int. Soc. Opt. Eng.*, **6113**, 61130C (2006).
6. Apollonov V.V., Vdovin G.V., Kislov V.I., Prokhorov A.M., Chetkin S.A. *Kvantovaya Elektron.*, **18**, 358 (1991) [*Sov. J. Quantum Electron.*, **21**, 325 (1991)].
7. Vinevich B.S., Evdokimovich L.N., Safronov A.G., Smirnov S.N. *Kvantovaya Elektron.*, **34**, 333 (2004) [*Quantum Electron.*, **34**, 333 (2004)].

8. Maslov V.P., Vorob'yov V.N. *Dokl. Akad. Nauk SSSR*, **179**, 558 (1968).
9. Averbakh V.S., Vlasov S.N. *Radiotekh. Elektron*., **14**, 1709 (1969).
10. Ischenko E.F., Reshetin E.F. *Opt. Spektrosk.*, **51**, 1050 (1981) [*Opt. Spectrosc.*, **51**, 581 (1981)].
11. Leger J.R., Chen D., Wang Z. *Opt. Lett.*, **19**, 108 (1994).
12. Apollonov V.V., Artemov D.V., Kislov V.I., Prokhorov A.M. *Kvantovaya Elektron.*, **20**, 1203 (1993) [*Quantum Electron*., **23**, 1045 (1993)].
13. Belanger P.A., Pare C. *Opt. Lett*., **16**, 1057 (1991).
14. Gerber M., Graf T. *J. Quantum Electron.*, **40** (6), 741 (2004).
15. Vdovin G., Kiyko V. *Opt. Lett.*, **26** (11), 798 (2001).
16. Bolts G.P. (Ed.) *Inverse Problems in Optics* (New York: Springer-Verlag, 1980).
17. Anan'ev Yu.A. *Opticheskiye resonatory i lasernye puchki* (Optical Resonators and Laser Beams) (Moscow: Nauka, 1990) p. 264.
18. Egorov A.A. *Zh. Radioelektron*., **4**, 19 (2009).
19. Apollonov V.V., Kislov V.I., Prokhorov A.M. *Kvantovaya Elektron.*, **23**, 1081 (1996) [*Quantum Electron*., **26**, 1051 (1996)].
20. Born M., Wolf E. *Principles of Optics* (Cambridge: Camrbridge University Press, 1997).
21. Azimov B.S., Platonenko V.T., Sagatov M.M. *Kvantovaya Elektron*., **18**, 323 (1991) [*Sov. J. Quantum Electron.*, **21**, 291 (1991)].