

INTERFEROMETRY

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Enhancement of measurement sensitivity in the formation of shear interferograms of transparent plates with small residual wedging

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Abstract. **We have proposed a method for increasing the sensitivity of measurements of the wedge angle in transparent plates. The method is based on formation of the holographic shear interferograms using a combination of 180° rotation of the plate with by-turn adjustment of interferograms in its images to an infinitely wide fringe. The sensitivity enhancement is due to the increased number of interference fringes in the observed images of the wedged plate, which favours the reduction of the measurement error during optical processing of the obtained interferograms. Data on the experimental validation of the proposed method are presented.**

Keywords: *large lateral shear, 180° rotation of the wedged plate, interference pattern, sensitivity enhancement.*

1. Introduction

To determine small angles of wedged plates, the method of wedge angle measurement using a Fizeau interferometer is most widely applied in practice [1, 2] and recognised as the most sensitive and precise. The classical scheme of the Fizeau interferometer, aimed at measuring the wedge angle of transparent plates, has been improved many times [3, 4]. Two principal trends can be observed in the development of interferometric methods of small angle measurement, aimed at simplifying the optical schemes implementing the process of interference pattern formation and the wedge angle measurement procedure [3–6], as well as at improving the sensitivity and, hence, the accuracy of measurements, which is caused by increasing requirements to metrological characteristics of angle measures and quality of plane-parallel and wedged plates used in modern interferometers, laser and optoelectronic instrumentation [7].

According to [9 – 10], the enhancement of the interference method sensitivity leads to a proportional increase in the number of fringes in the observed interference pattern.

To increase the number of fringes in the interference pattern of the wedged plate, use can be made of a 180° rotation of the wedge with respect to its edge between the registrations of sequential interferograms. This method was implemented with a pair of interference images [2, 5], or with formation of an interference pattern in a reversal-shear interferometer [4, 6].

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The methods of shear interferometry proved to be most promising for enhancing the measurement sensitivity using the interference patterns from wedged plates. The registration of a pair of interference images of a wedged plate in the holographic lateral- or reversal-shear interferometry allows the reduction of the measurement error at the expense of doubling the number of interference fringes in the plate image [11, 12]. To reduce the error of the wedge angle measurements, it is necessary to use special techniques, providing the reduction of the interferometer sensitivity to external vibrations [13] and eliminating the systematic component of the error, associated with aberrations of the optical part of the device [12].

The high accuracy of the interferometric measurement methods is achieved when the high sensitivity of the interferometry in the phase-difference measurements is combined with the digital processing of interferograms. It should be noted that the minimal error of measurements is provided by achieving the maximal sensitivity of the parameter to be measured in the observed interference pattern.

In the present paper we propose a method for improving the sensitivity of measurements in holographic lateral-shear interferometry of wedged transparent plates in the from of a pair of interference images of the plate; the method is based on the combination of a 180° rotation of the plate with by-turn adjustment of interferograms in its images to an infinite-width fringe.

Note, that this method of wedging measurement is applicable to plates with the linear dimension in the direction of the shear not exceeding 1/3 of the probe light beam diameter. The reduction of the measurement error is ensured by an increase in the method sensitivity at the expense of increasing the number of interference fringes in the images of the investigated plate.

The large lateral-shear interferometry in the case of the linear dimensions of the investigated object not exceeding 1/3 of the probe light beam diameter allows formation of several interferometric images of the investigated object, modulated by interference fringes. In this case the description of the interference fringes is similar to that in the two-beam interferometry with a reference wave [10, 14].

2. Optical scheme implementing the method

Figure 1 shows the optical scheme of a holographic interferometer designed to implement the proposed method. The He–Ne laser (*1*), the plane mirror (*2*) and the telescopic system (*3, 4*) produce a collimated probe light beam of the appropriate size. The telescopic system (*6, 7*) is aimed at reducing (if necessary) the diameter of the probe beam and introducing it into the lateral-shear interferometer (*8*), which, beside the relative lateral shear between the interfering beams, should provide the control of the width and orientation of fringes in the interference pattern. The optimal version that satisfies these conditions is the interferometer based on the classical Mach–Zehnder scheme [15]. As mentioned above, the linear dimensions of the investigated wedged plate (*5*) in the direction of the lateral shear should not exceed 1/3 of the probe light beam diameter.

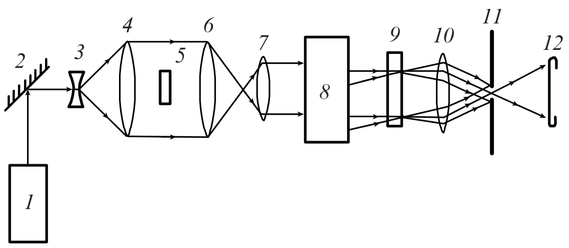


Figure 1. Optical scheme of the lateral-shear holographic interferometer: (*1*) He–Ne laser; (*2*) mirror; (*3, 4*) telescopic system; (*5*) wedged plate; (*6, 7*) telescopic system; (*8*) small-size lateral-shift interferometer; (*9*) reference hologram; (*10*) objective; (*11*) aperture; (*12*) the plane of observation of the interference pattern.

To eliminate the systematic error of measurements due to aberrations of the optical system, the interferometer is equipped with the reference hologram (*9*), which is recorded in the plane, optically conjugate with the zone of the location of the object under study (*5*). The system of spatial filtering consists of the objective (*10*) and the aperture (*11*) aimed at selecting the interfering waves, diffracted by the hologram (*9*) into the first and zero orders. The waves extracted as a result of filtering produce an interference pattern in the plane (*12*), optically conjugate with the hologram (*9*) and the plate (*5*). The aberrations in this process are compensated for in the same way as in the real-time holographic interferometry [16]. To perform the measurement operations visually, a matted screen is placed in the plane (*12*). For digital processing a CCD array is placed instead.

The method for improving the sensitivity of measurements in the holographic lateral-shear interferometry is implemented in several stages.

3. Recording a reference hologram of the wedged plate

At the first stage, the wedged plate (*5*) is placed into the central zone of the probe light beam (see the scheme in Fig. 1). The lateral shear s, exceeding the linear dimensions of the investigated plate (*5*), is set between the interfering beams at the output of the interferometer (*8*), and the reference hologram of the wedged plate is recorded in the plane (*9*).

The wedged plate (*5*) is placed so that the axis of the probe beam is perpendicular to its surface. The maximal shear s should not exceed $D/3$, where D is the diameter of the probe light beam between the objective lenses (*4*) and (*6*). In this case, the method can be applied to measure the wedging of plates having the linear dimensions along the shear direction up to $D/3$ inclusive.

The angle between the interfering beams for recording the hologram (*9*) is chosen so that the carrier frequency ξ of the hologram fringes is no less than 10 lines mm^{-1}. This condition is a practical recommendation [16], providing the spatial separation of the zero and first diffraction orders and, correspondingly, allowing the separation of appropriate diffracted beams by means of the aperture (*11*).

Let us choose the coordinate system xyz in the following way. The axes x and z are chosen so that the x axis coincides with the direction of the shear of the wave fronts in the interferometer (*8*), and the z axis coincides with the direction of propagation of the probe light beam. Figure 2a presents the contours of the probe light beam (circle) and the investigated wedged plate (polygon), and Fig. 2b shows the contours of the interfering light beams and the images of the wedged plate in the plane of the recording reference hologram (*9*).

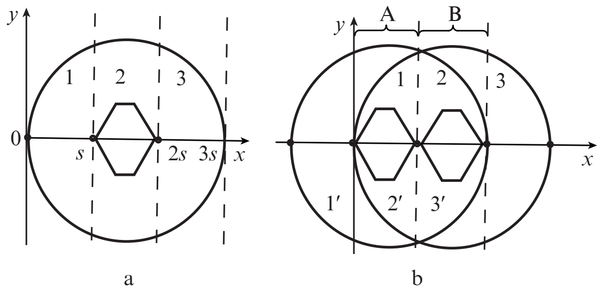


Figure 2. Image of the contours of the probe light beam (circle) and the investigated wedged plate (polygon) after passing the wedged plate (a) and in the planes of the reference hologram and of observation of the interference pattern (b).

To simplify the mathematical calculations, in describing the method we assume that the edge of the wedged plate is oriented parallel to the x axis. In this case, after passing the wedged plate (Fig. 2a), the phase of the probe light wave $\Phi_1(x, y)$ may be described as follows:

$$\Phi_1(x,y) = \begin{cases} 2\pi\eta y + \varepsilon(x,y) & \text{for zone 2,} \\ \varepsilon(x,y) & \text{for zones 1 and 3,} \end{cases} \tag{1}$$

where $\varepsilon(x, y)$ is the phase distortion by the aberrations of the optical system producing the probe light beam; $2\pi\eta y$ is the phase change caused by the wedged plate; $\eta = \alpha/\lambda$; α is the angle of the light beam deflection by the wedged plate (for the case of small wedge angles); λ is the wavelength of the light source. The quantity α is determined by the wedge angle γ and the refractive index n of the plate material [17]: $\alpha \approx \gamma(n-1)$.

In the interferometer (*8*) (Fig. 1), the probe light wave is amplitude-split into two waves. The superposition of these two waves in the plane of the recording reference hologram (*9*) yields an interference pattern (Fig. 2b) with the fringes, separated by the distance s from each other. In the case when the interference fringes are oriented parallel to the y axis, the phase of the interfering waves in the plane of the hologram (*9*) may be presented as follows:

$$\Phi_{11}(x,y) = \begin{cases} 2\pi(\xi x + \eta y) + \varepsilon(x,y) + \varepsilon_1(x,y) & \text{for zone 2,} \\ 2\pi\xi x + \varepsilon(x,y) + \varepsilon_1(x,y) & \text{for zones 1 and 3,} \end{cases} \tag{2}$$

$$\Phi_{12}(x,y) = \begin{cases} 2\pi\eta y + \varepsilon(x+s,y) + \varepsilon_2(x,y) & \text{for zone 2}', \\ \varepsilon(x+s,y) + \varepsilon_2(x,y) & \text{for zones 1}' \text{ and 3}', \end{cases} \tag{3}$$

where ξ is the carrier spatial frequency of the hologram fringes; $\varepsilon_1(x, y)$ and $\varepsilon_2(x, y)$ are the phase distortions of the first and second light beams acquired while travelling along different paths in the interferometer. The waves with the phases described by Eqns (2) and (3) overlap and produce a reference hologram of the wedged plate in the regions A and B (Fig. 2b), provided that the appropriate zones of the first and second interfering beams overlap. The spatial period of the interference fringes of the hologram is $1/\xi$. The amplitude transmission coefficient of the reference hologram of the wedged plate may be presented in the form:

$$\tau(x,y) \sim \begin{cases} 1+\cos[2\pi(\xi x+\eta y)+\varepsilon(x,y)-\varepsilon(x+s,y)+\varepsilon_1(x,y)-\varepsilon_2(x,y)] \text{ for region A}, \\ 1+\cos[2\pi(\xi x-\eta y)+\varepsilon(x,y)-\varepsilon(x+s,y)+\varepsilon_1(x,y)-\varepsilon_2(x,y)] \text{ for region B}. \end{cases} \quad (4)$$

The quantity ξ is determined by aberrations of the optical system, recorded in the reference hologram.

After recording and processing the reference hologram, it is placed back at the same place. The precision of installation is controlled using the coincidence of the pair of images of the wedged plate, produced by the objective lenses (*6*, *7*), with the plate images recorded in the hologram (*9*). One more feature that allows such control is the appearance of an infinitely wide fringe (uniform across the whole field of illumination) in the plane of observation of the interference pattern (*12*). This fringe appears when the reference hologram (*9*) is illuminated with the waves having the phases (2) and (3), and a pair of light waves is extracted by the aperture (*11*); the latter are focused by the objective (*10*) and propagate exactly along the z axis. If the reference hologram is installed correctly, an aberration-free interference pattern in the form of a uniformly illuminated field is observed in the plane (*12*). A minor deflection of any of the light beams, illuminating the hologram, leads to the appearance of finite-width fringes in the interference pattern [16].

4. Enhancement of the measurement sensitivity due to 180° rotation of the plate

After recording the reference hologram and adjusting the interferometer until the infinite-width fringe appears in the interference pattern, the wedged plate (*5*) is rotated through the angle 180° around the normal (the z axis). In this case, after the probe light passes through the rotated wedged plate, the sign in Eqn (1) in front of the wave phase variation $2\pi\eta y$ will change to opposite, and the phases of the waves, illuminating the reference hologram (*9*), will take the form:

$$\Phi_{21}(x,y) = \begin{cases} 2\pi(\xi x-\eta y)+\varepsilon(x,y)+\varepsilon_1(x,y) \text{ for zone } 2, \\ 2\pi\xi x+\varepsilon(x,y)+\varepsilon_1(x,y) \text{ for zones 1 and 3}, \end{cases} \quad (5)$$

$$\Phi_{22}(x,y) = \begin{cases} -2\pi\eta y+\varepsilon(x+s,y)+\varepsilon_2(x,y) \text{ for zone } 2', \\ \varepsilon(x+s,y)+\varepsilon_2(x,y) \quad \text{for zones } 1' \text{ and } 3'. \end{cases} \quad (6)$$

When the hologram (*9*) with the transmission coefficient (4) is illuminated with light waves having the phases (5) and (6), two waves will propagate normally with respect to the hologram. The first wave is produced by the first-order diffraction from the wave with the phase (5), its phase being expressed as

$$\Phi_{31}(x,y) = \begin{cases} 2\pi\eta y+\varepsilon(x+s,y)+\varepsilon_2(x,y) \text{ for region A}, \\ -4\pi\eta y+\varepsilon(x+s,y)+\varepsilon_2(x,y) \text{ for region B}. \end{cases} \quad (7)$$

The second wave is produced by the zero-order diffraction of the wave having the phase (6). Because this wave is 'straight-passing', its phase is expressed by Eqn (6).

This pair of waves is focused by the objective (*10*), selected by the aperture (*11*), and in the plane (*12*) produces an infinite-width-fringe interference pattern, looking as a pair of the wedged plate images, modulated by the interference fringes. The intensity distribution is expressed as

$$I(x,y) \sim \begin{cases} 1+\cos(4\pi\eta y) \quad \text{for region A}, \\ 1+\cos(-4\pi\eta y) \textit{ for } \text{region B}. \end{cases} \quad (8)$$

The behaviour of fringes in the regions of A and B zones in the interference pattern of form (8) does not differ from that in the interference pattern obtained in a two-beam single-pass interferometer.

One can see from the comparison of Eqns (8) and (1) that the 180° rotation of the wedged plate increases the sensitivity of measurements by two times, when the aberration-free interference images of the wedged plate are produced in real time. This effect manifests itself in doubling of the number of fringes in the left (region A) and right (region B) images of the wedged plate and, consequently, in the reduction of the spatial period of the fringes by two times as compared with that in the case of using the methods of two-beam or shear interferometry. As follows from Eqn (8), the period of interference fringes, modulating the image of the wedged plate, will be expressed as

$$P = \frac{\lambda}{2\gamma(n-1)}. \quad (9)$$

In the experimental implementation of the proposed method for increasing the sensitivity of interference measurements by means of 180° rotation of the wedged plate, the plate (*5*) was installed in a special holder that allowed controllable rotation of the plate around the normal to its surface. The edge of the plate wedge was oriented horizontally. Figure 3 presents the interference pattern having the form of

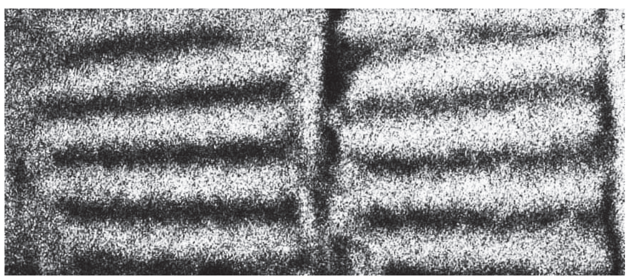


Figure 3. Interference image of the wedged plate in the case of the interferometer adjusted to infinite-width fringes, recorded after 180° rotation of the wedged plate.

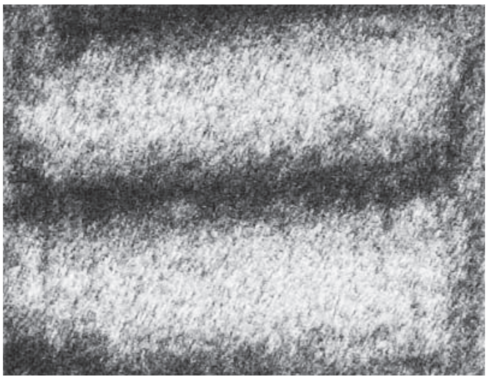


Figure 4. Interference image of the wedged plate in the case of the interferometer adjusted to infinite-width fringes, recorded in the two-beam Mach–Zehnder interferometer with a reference wave.

a pair of plate images, modulated with horizontal fringes, observed in the plane (*12*) after rotation of the wedged plate (*5*) through the angle 180°. Figure 4 shows the interference image of the same plate, obtained in the two-beam Mach–Zehnder interferometer with a reference wave.

5. Enhancement of the sensitivity by changing the fringe adjustment

Since the phase variations in Eqn (8) have opposite signs, use can be made of the approach, proposed in Refs [10, 11], to further enhance the sensitivity of measurements using the pair of produced interference images of the wedged plate. The essence of the approach consists in by-turn adjustment of the fringes in regions A and B, bounded by the contours of the investigated wedged plate images, until the infinitely wide fringe is obtained. Therefore, for optical processing two interferograms in the form of pair images of the wedged plate are produced by turn.

To get an infinitely wide fringe in region A when obtaining the first interferogram and in the region B when obtaining the second one (see Fig. 2b) one can change only the propagation direction of the first wave, illuminating the hologram (*9*), the phase of which is described by Eqn (5), leaving unchanged the propagation direction of the second wave with the phase, described by Eqn (6). It may be shown [11], that in this case, the first interferogram is produced with the intensity distribution

$$I_1(x,y) \sim \begin{cases} \text{const} & \text{for region } A, \\ 1 + \cos(-8\pi\eta y) & \text{for region B.} \end{cases} \tag{10}$$

Having changed appropriately the propagation direction of the first wave, illuminating the hologram (*9*), whose phase is described by Eqn (5), and having left the propagation direction of the second wave with the phase described by Eqn (6) unchanged, we can also form the second interferogram, adjusted to produce the infinite-width fringe in the region B:

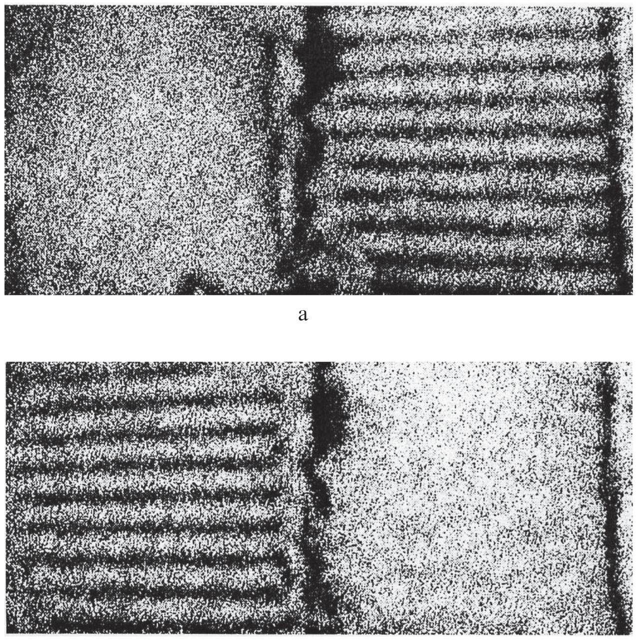
$$I_2(x,y) \sim \begin{cases} 1 + \cos(8\pi\eta y) & \text{for region } A, \\ \text{const} & \text{for region B.} \end{cases} \tag{11}$$

One can see from Eqns (10) and (11) that when the infinite-wide interference fringes are achieved one after the other in region A of the plate image for the first interferogram (10) and in the region B for the second interferogram (11), in other regions of the interferogram the image of the plate is produced with a doubled number of fringes compared to the interferogram (8). The spatial period of the fringes in this case is determined as

$$P_{1,2} = \frac{\lambda}{4\gamma(n-1)}. \tag{12}$$

To measure the wedge angle, the interference patterns (10) and (11) are obtained in turn and the fringe periods P_1 and P_2 are determined. As shown in Ref. [11], independent of the technique (visual or digital) used to process the interferograms (10) and (11), the increase in the number of the interference fringes leads to the reduction of the relative error of the wedge angle measurement.

Figure 5 presents the first (Fig. 5a) and second (Fig. 5b) interferograms, obtained following the technique considered, which confirm the effect of increased sensitivity of the measurements in forming the interference patterns due to readjustment of the fringes in the images of the wedged plate. The presented interferograms are obtained in the plane (*12*) (see Fig. 1) after readjustment of the interference pattern, presented in Fig. 3.



a

b

Figure 5. Interference patterns with the adjustment to the infinitely wide fringe in the regions of the left (a) and right (b) images of the wedged plate.

6. Conclusions

Thus, the proposed method for increasing the measurement sensitivity in the holographic lateral-shear interferometry with producing interferograms of wedged transparent plates in the form of a pair of interference images of the plate (the method is based on the combination of 180° rotation of the plate with by-turn adjustment of interferograms in the plate images to infinite-width fringe) allows one, independently of the interferogram processing technique, to reduce the measurement error in comparison with the measurement errors inherent in the traditional interferometric methods. It should be noted that the method may be used also in the lateral-shear interferometers with double passing of the light beam through the wedged plate.

References

1. Puryaeva D.T. (Ed.) *Opticheskie izmereniya* (Optical Measurements) (Moscow: Mashinostroyeniye, 1987).
2. Malacara D. (Ed.) *Optical Shop Testing* (New York: Wiley, 1977; Moscow: Mashinostroenie, 1985).
3. Bhattacharyya D., Ray A., Dutta B.K., Ghosh P.N. *Opt. Las. Technol.*, **34**, 93(2002).
4. Chatterjee S. *Opt. Eng.*, **42**, 3235 (2003).
5. Moshkin B.E., Majorov B.S. Opt. Zh., **74**, 24 (2007) [*J. Opt. Techn.*, **74** (2), 94 (2007)].
6. Chatterjee S., Kumar Y.P. *Opt. Las. Technol.*, **39**, 662 (2007).
7. Senik B.N., Bel'skii A.B., Potelov V.V. *Opt. Zh.*, **76** (5) 2009) [*J. Opt. Techn.*, **76**, 455 (2009)].
8. Ostrovskii Yu.I., Butusov M.M., Ostrovskaya G.V. *Golograficheskaya interferometriya* (Holographic Interferometry) (Moscow: Nauka, 1977).
9. Zeilikovich I.S., Lyalikov A.M. *Usp. Fiz. Nauk*, 161, 143 (1991) [*Sov. Phys. Usp.*, **34**, 74 (1991)].
10. Lyalikov A.M. *Vysokochuvstvitel'naya gologroficheskaya interferometriya fazovykh ob'ektov* (High-Sensitivity Holographic Interferometry of Phase Objects) (Grodno: Izdatelstvo GrSU, 2010).
11. Lyalikov A.M. *Pis'ma Zh. Tekh. Fiz.*, **36**, 87 (2010) [*Tech. Phys. Lett.,* **36**, 969 (2010)].
12. But' A.I., Lyalikov A.M. *Opt. Spektrosk.*, **109**, 696 (2010) [*Opt. Spectrosk.*, **109**, 641 (2010)].
13. Chatterjee S, Kumar Y.P. *Opt. Commun.*, **284**, 57 (2011).
14. Lyalikov A.M. *Opt. Spektrosk.*, **99**, 151 (2005) [*Opt. Spectrosk.*, **99**, 141 (2005)].
15. Kolomiytsev Yu.V. *Interferometry. Osnovy inzhenernoy teorii. Primeneiye* (Interferometers. Fundamentals of Engineering Theory. Application) (Leningrad: Mashinostroyeniye, 1976).
16. Beketova A.K., Belozerov A.F., Berezkin A.N., et al. *Gologroficheskaya interferometriya fazovykh ob'ektov* (Holographic Interferometry of Phase Objects) (Leningrad: Nauka, 1979).
17. Begunov B.N. *Geometricheskaya optika* (Geometric Optics) (Moscow: Izd-vo MGU, 1966).