

TERAHERTZ LASER RADIATION

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Transmission characteristics of circular metallic waveguides for terahertz radiation

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Abstract. **Transmission characteristics of oversized circular metallic waveguides excited by linearly polarised Gaussian laser beams in the terahertz range (4 – 28 THz) are studied theoretically and experimentally. Calculating the transmission characteristics, we have determined the conditions of applicability of the method of the eigenoscillations in the approximations of a real metal by an ideal metal or dielectric, depending on the transmitted radiation frequency. The existence of the transition region is established in the behaviour of the electrodynamic properties of metallic waveguides in the frequency range of 7.5 – 15 THz.**

Keywords: *metallic waveguide, terahertz range, Gaussian beam, mode approach, ray-optics approach, transmission coefficient, polarisation.*

1. Introduction

In mastering the terahertz frequency range, scientists and developers encounter a number of scientific and engineering problems, including that of constructing effective transmission lines [1 – 3]. To solve this problem, photonic crystal waveguides [4], single-wire [5] and two-wire [6] waveguides of a surface wave, hollow metallic [7] and dielectric [8] waveguides have been recently proposed. In studying hollow waveguides, main efforts in most cases are aimed at developing various types of coatings applied to the inner surface of the waveguide to reduce the attenuation of radiation transmitted through them [9, 10]. However, the problems of electrodynamics of these waveguides, especially metallic ones, remain poorly studied. Despite quantitative estimates for the absorption of terahertz radiation in metallic waveguides, based on the theory of microwave waveguides in the approximation of an ideal metal [11 – 13], metals cannot be considered perfectly conducting in the high frequency part of this range.

The aim of this work is to study theoretically and experimentally transmission characteristics of Gaussian laser beams in circular hollow metallic waveguides in the terahertz range; to determine in their calculations the conditions of applicability of the method of the eigenoscillations in the approximations of a real metal by an ideal metal and dielectric, depending on the frequency of the transmitted radiation; and to develop recommendations on the use of this method in studying the transmission characteristics of metallic waveguides in terahertz transmission lines.

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2. Theoretical relations

2.1. Mode approach in the ideal-metal approximation

Let a linearly polarised axially symmetric Gaussian beam with the polarisation vector directed along the y axis be incident on the entrance end of a circular metallic waveguide of radius a, located along the z axis, and the waist be located at the waveguide input; then, the electric field $\boldsymbol{E}_0 = \boldsymbol{y}_0 E_{0y}(x, y, 0)$ in the source plane ($z = 0$) has the form:

$$E_{0y}(x,y,0) = A_0 \exp\left(-\frac{x^2+y^2}{2w^2}\right), \qquad (1)$$

where $\boldsymbol{y}_0$ is the unit vector of the Cartesian coordinates in the y direction; w is the beam radius measured at the e^{-1} level of its maximum intensity at the waist; A_0 is the field amplitude of the beam.

We take into account the influence of the entrance aperture of the waveguide on the incident radiation and consider 'weak' Gaussian beam diffraction. A Gaussian beam weakly diffracted by a circular aperture ($w \leqslant 0.7a$) can be approximated in the far field by another Gaussian beam with slightly different characteristics [14]. The relationship between the parameters of the incident beam and the beam passing through the entrance aperture of the waveguide, is determined by the expressions:

$$A_{0\mathrm{d}} = A_0 \frac{1-\exp(-a^2/w^2)}{1-\exp(-a^2/2w^2)},$$
$$w_{\mathrm{d}} = w \frac{1-\exp(-a^2/2w^2)}{[1-\exp(-a^2/w^2)]^{1/2}}, \qquad (2)$$

where $A_{0\mathrm{d}}$ and w_{d} are the field amplitude and the radius of the diffracted beam.

As is known [11], the transverse components of the electric field in the metallic waveguide can be represented as a series expansion in orthogonal waveguide TE and TM modes. We pass to polar coordinates (r, φ) and introduce the dimensionless parameters $\rho = r/a$, $w_0 = w/a$. With the polarisation of the input radiation beam specified in the form of (1), only TE_{1m} and TM_{1m} waves will be excited in the wavegiude, where the first subscript $n = 1$ is the azimuthal index, and the second subscript m is the radial index. The transverse electric field components for these waves [15] have the form

$$V_{1m}^{\rm TE}(\rho,\varphi) = \boldsymbol{x}_0 A_{1m} J_2(\chi_{1m}\rho)\sin 2\varphi + \boldsymbol{y}_0 A_{1m}[J_0(\chi_{1m}\rho) - J_2(\chi_{1m}\rho)\cos 2\varphi],$$

$$V_{1m}^{\rm TM}(\rho,\varphi) = -\boldsymbol{x}_0 B_{1m} J_2(\eta_{1m}\rho)\sin 2\varphi + \boldsymbol{y}_0 B_{1m}[J_0(\eta_{1m}\rho) + J_2(\eta_{1m}\rho)\cos 2\varphi],$$

where $A_{1m} = \{J_2(\chi_{1m})[2\pi(\chi_{1m}^2 - 1)]^{1/2}\}^{-1}$, $B_{1m} = [J_2(\eta_{1m})(2\pi)^{1/2}]^{-1}$ are normalisation factors; J_j is the jth-order Bessel function of the first kind; χ_{1m} is the mth root of the equation $J_1'(\chi) = 0$; η_{1m} is the mth root of the equation $J_1(\eta) = 0$.

Then, the field distribution in the waveguide cross section at a distance L from the entrance face has the form

$$\boldsymbol{E}(\rho,\varphi,L) = \sum_{m=1}^{M} B_m \boldsymbol{V}_{1m}^{\rm TE}(\rho,\varphi)\exp({\rm i}\gamma_{1m}^{\rm TE}L) + \sum_{m=1}^{M} C_m \boldsymbol{V}_{1m}^{\rm TM}(\rho,\varphi)\exp({\rm i}\gamma_{1m}^{\rm TM}L), \quad (3)$$

where the amplitudes of the B_m and C_m modes excited at the waveguide output are found from the relations

$$B_m = \iint \boldsymbol{E}_0 \boldsymbol{V}_{1m}^{\rm TE}{\rm d}S; \quad C_m = \iint \boldsymbol{E}_0 \boldsymbol{V}_{1m}^{\rm TM}{\rm d}S.$$

When the waveguide is excited by a Gaussian laser beam with a relative radius $w_0 \leqslant 0.7$, the coefficients B_m and C_m can be represented in the form [16]

$$B_m = \frac{w_0}{J_2(\chi_{1m})(\chi_{1m}^2 - 1)^{1/2}}\exp\left(-\frac{\chi_{1m}^2 w_0^2}{4}\right),$$

$$C_m = \frac{w_0}{J_2(\eta_{1m})}\exp\left(-\frac{\eta_{1m}^2 w_0^2}{4}\right).$$

The error in the calculations using these expressions does not exceed 10^{-4}. Calculation of B_m and C_m by numerical integration and all further calculations were performed with the error of the same order. The propagation constants for the TE_{1m} and TM_{1m} modes, $\gamma_{1m} = \beta_{1m} + {\rm i}\alpha_{1m}$, appearing in (3) are expressed as

$$\beta_{1m}^{\rm TE,TM} = 2\pi[\lambda^{-2} - (\lambda_k^{\rm TE,TM})^{-2}]^{1/2};$$

$$\alpha_{1m}^{\rm TM} = \frac{R_{\rm s}}{R_0 a}\left[1 - \left(\frac{\lambda}{\lambda_k^{\rm TM}}\right)^2\right]^{-1/2};$$

$$\alpha_{1m}^{\rm TE} = \frac{R_{\rm s}}{R_0 a}\left[\frac{1}{\chi_{1m}^2 - 1} + \left(\frac{\lambda}{\lambda_k^{\rm TE}}\right)^2\right]\left[1 - \left(\frac{\lambda}{\lambda_k^{\rm TE}}\right)^2\right]^{-1/2};$$

$\lambda_k^{\rm TE} = 2\pi a/\chi_{1m}$ is the critical wavelength for the TE_{1m} modes; $\lambda_k^{\rm TM} = 2\pi a/\eta_{1m}$ is the critical wavelength for the TM_{1m} modes; $R_0 = 376.73\ \Omega$ is the wave impedance of free space; $R_{\rm s}$ is the surface resistance of the waveguide material. The number M of the expansion terms in expression (3), specified by the required accuracy of calculations, is chosen to be 20.

Having calculated the radiation intensity at the observation point, $I(\rho,\varphi,L) = |E(\rho,\varphi,L)|^2$, we find the energy flux through the waveguide cross section at a distance L from its face end:

$$W_{\rm out}(L) = \int_0^{2\pi}{\rm d}\varphi\int_0^1 I(\rho,\varphi,L)\rho{\rm d}\rho. \quad (4)$$

The derived relations make it possible to determine the transmission coefficient of radiation in the waveguide, $T(L)$, and the degree of output radiation polarisation, $\Pi(L)$:

$$T(L) = \frac{W_{\rm out}(L)}{W_{\rm in}}, \quad \Pi(L) = \frac{I_y(L) - I_x(L)}{I_y(L) + I_x(L)}, \quad (5)$$

where $W_{\rm in} = \pi A_0^2 w^2$ is the input beam power;

$$I_{x,y}(L) = \int_0^{2\pi}{\rm d}\varphi\int_0^1 \rho{\rm d}\rho\,|E_{x,y}(\rho,\varphi,L)|^2.$$

2.2. Mode approach in the approximation of a real metal by a dielectric

The propagating types of oscillations of an oversized hollow metallic waveguide in the approximation of a real metal by a dielectric represent three sets of 'fast' modes [17]: hybrid – EH_{nm}, circular electric – TE_{0m}, and circular magnetic – TM_{0m}, where the first subscript is also the azimuthal index and the second subscript is the radial index. Similarly to (1), we assume that the initial beam is polarised along the y axis, $\boldsymbol{E}_0(\rho,0) = \boldsymbol{y}_0 E_0(\rho,0)$, and using (2), we take into account the influence of the entrance aperture of the waveguide on the incident radiation. Due to the beam polarisation specified in this manner, only EH_{1m} modes will be excited in the waveguide. They are characterised by the axial symmetry and linear polarisation of the field. At $ka \gg |v|u_{nm}$, they are also described by functions entering the complete orthogonal system, where $k = 2\pi/\lambda$ is the wave number; v is the refractive index of the waveguide walls; u_{nm} is the mth root of the equation $J_{n-1}(u_{nm}) = 0$. In the orthonormal form these functions are expressed as

$$V_m(\rho) = \frac{\sqrt{2}\,J_0(U_{0m}\rho)}{J_1(U_{0m})}. \quad (6)$$

We set below that $M < \sqrt{a/\lambda}$ [18], i.e., all $V_m(\rho)$, $m \in [1, M]$ describe sufficiently accurately the propagating EH_{1m} modes. Then, the complex field amplitude in the cross section $z = L$ can be written in the form

$$E(\rho,L) = \sum_{m=1}^{M} D_m V_m(\rho)\exp({\rm i}\gamma_{1m}L), \quad (7)$$

where $D_m = \int_0^1 E_{0y}(\rho)\,V_m(\rho)\rho{\rm d}\rho$;

$$\gamma_{1m} = \frac{2\pi}{\lambda}\left[1 - \frac{1}{2}\left(\frac{U_{0m}\lambda}{2\pi a}\right)^2\left(1 - \frac{{\rm i}v_{EH}\lambda}{\pi a}\right)\right]$$

are the propagation constants of the EH_{1m} modes [17];

$$v_{EH} = \frac{0.5(v^2 + 1)}{\sqrt{v^2 - 1}}.$$

In this case, the coefficients D_m at $w_0 \leqslant 0.7$ can be expresses as [16]:

$$D_m = \frac{\sqrt{2}\,w^2}{a^2 J_1(U_{0m})}\exp\left(-\frac{U_{0m}w^2}{2a^2}\right).$$

The presented expressions make it possible to determine the transmission characteristics of a waveguide segment. Expression (7) can be used to find the field amplitude and phase distribution at the waveguide output. In this case, the coefficients of radiation transmission in the waveguide, $T(L)$, and the degree of the output radiation polarisation, $\Pi(L)$, can be found similarly to (4), (5).

2.3. Ray-optics approach

Using the ray-optics representation, we assume that the beam consists of ray tubes or rays contained in an elementary solid angle, lying in the meridional planes of the waveguide and having the common origin – the centre of the beam. In this case, the rays propagate at small angles to the optical axis (paraxial approximation) in the range of angles $\theta(r_0) = r_0/(kw_{\rm d}^2) \ll 1$, where r_0 is the radial coordinate in the plane of the beam waist, and taking into account the waveguide material we assume that $|v| \gg 1$. Because the distance between two successive reflections of the beam from the waveguide walls is equal to $2a/\theta$, the total number of reflections of the beam in a waveguide is $N = L\theta/(2a) = Lr_0/(2akw_{\rm d}^2)$. Taking into account the approximations made in accordance with the Fresnel formulas [19] for the power reflection coefficients of the propagating beam from the waveguide wall, the beam being polarised parallel and perpendicular to the plane of the waveguide wall, we have

$$|r_{\parallel}|^2 = 1 - \frac{2\sqrt{2}\,\theta\,|v|}{1+\sqrt{2}\,\theta\,|v| + \theta^2|v|^2}, \quad |r_{\perp}|^2 = 1 - \frac{2\sqrt{2}\,\theta}{|v|}. \quad (8)$$

Then, using (2) and (8), we find the energy flux through the cross section of the waveguide at a distance L from its entrance face [20]:

$$W_{\rm out}(L) = 2\pi A_{0\rm d}^2 \int_0^{r_{\max}} \left(\frac{r_{\parallel}^{2N} + r_{\perp}^{2N}}{2}\right) \exp\left(-\frac{r_0^2}{w_{\rm d}^2}\right) r_0 {\rm d}r_0,$$

where $r_{\max}$ is a radial coordinate corresponding to the maximal value of $\theta(r_0)$;

$$r_{\perp}^{2N} = \exp\left(-\frac{\sqrt{2}}{|v|}\frac{L}{a}\frac{r_0^2}{k^2 w_{\rm d}^4}\right);$$

$$r_{\parallel}^{2N} = \begin{cases} \exp\left(-\dfrac{1+\sqrt{2}\,|v|}{2}\dfrac{L}{a}\dfrac{r_0^2}{k^2 w_{\rm d}^4}\right), & 0 \leqslant r_0 < \dfrac{kw_{\rm d}^2}{|v|}, \\ \exp\left(-\dfrac{1+\sqrt{2}}{2|v|}\dfrac{L}{a}\right), & r_0 \geqslant \dfrac{kw_{\rm d}^2}{|v|}. \end{cases}$$

Similarly to (5), we determine the coefficient of radiation transmission in the waveguide:

$$T(L) = \frac{1-\exp(-a^2/w^2)}{2} \times \left[\frac{1}{F_1} + \frac{1}{F_2} + \left(1 - \frac{1}{F_2}\right)\exp\left(-F_2\frac{k^2 w_{\rm d}^2}{|v|^2}\right)\right], \quad (9)$$

where

$$F_1 = 1 + \frac{\sqrt{2}}{|v|}\frac{L}{a}\frac{1}{k^2 w_{\rm d}^2}; \quad F_2 = 1 + \frac{1+\sqrt{2}\,|v|}{2}\frac{L}{a}\frac{1}{k^2 w_{\rm d}^2}.$$

3. Comparison of experimental and numerical results

Using the techniques described above we calculated the transmission coefficient and the degree of polarisation of the field in circular metallic copper waveguides excited by linearly polarised Gaussian beams of terahertz radiation with a field of type (1). The radiation frequency was varied from 4 to 28 THz. The studies were performed when the relative radius of the initial beam w_0 was varied in the range from 0.1 to 0.7 (in the region of its 'weak' diffraction [14]). For copper the surface resistance was $R_{\rm s} = 2.625\times10^{-7}\sqrt{c/\lambda}$, taking into account the specific conductivity of the metal at a direct current $\sigma_0 = 5.73\times10^7$ S m^{-1} [21]. The refractive index of copper for the corresponding radiation frequency is chosen according to [22].

Figure 1 shows the results of calculations of transmission characteristics of the copper waveguide with a diameter $2a = 3$ mm and length $L = 370$ mm, using ray-optics and mode approaches in the approximations of a real metal both by an ideal metal and dielectric. As a reference graph for the coefficient of the radiation transmission in the frequency range 4–28 THz we selected the curve calculated using ray-optics theory, because its calculation is not related to the existence of a specific set of waveguide modes, and only depends on the refractive index of the waveguide material at a given frequency.

We can distinguish three regions of the terahertz range, where the results obtained using the mode approach in calculating the transmission characteristics of metallic waveguides are different. They are the frequency region above 15 THz ($\lambda < 20$ μm), where more reliable results are obtained by using the mode approach in the approximation of a real metal by a dielectric (Fig. 1a); the frequency region below 7.5 THz ($\lambda \geqslant 40$ μm), where we observe the best agreement between the results of calculations with the help of the ray-optics and mode methods in the approximation of an ideal metal (Figs 1d, e, f); and the frequency region 7.5–15 THz (20 μm $\leqslant \lambda <$ 40 μm), where the choice of the calculation method is determined by the relative radius of the exciting beam w_0 (Figs 1b, c). If $w_0 < 0.3$ the best results are obtained by using mode technique in the ideal-metal approximation, and if $w_0 > 0.3$ – in the approximation of the metal by a dielectric. This can be explained by an increase in the refractive index of copper with decreasing frequency of the transmitted radiation. At a radiation frequency below 7.5 THz, the copper in its electrodynamic properties is closer to the ideal metal.

To measure the transmission coefficient and the degree of polarisation of radiation, a copper waveguide with a diameter of 3 mm and a length of 370 mm was excited by linearly polarised Gaussian beams with a field intensity of type (1) from an optically pumped CH_3OH laser ($f = 4.25$ THz, $\lambda = 70.5$ μm) and a CO_2 laser ($f = 28.3$ THz, $\lambda = 10.6$ μm). The experimental setup is similar to that described in [23].

Gaussian radiation beams of different radii with a plane phase front at a frequency $f = 4.25$ THz were produced by the mirror system that is similar to that described in [23]. In calculating the transmission coefficient of the waveguide, radiation attenuation in the atmosphere inside the waveguide was taken into account. At various days during the experimental studies, it varied in the range 3–3.4 dB m^{-1} depending on the air humidity in the laboratory. Experimental points in the dependence of the transmission coefficient on the exciting beam radius w_0 for the given frequency are presented in Fig. 1f. There is good agreement between experimental and calculated data obtained using the mode approach in the ideal-metal

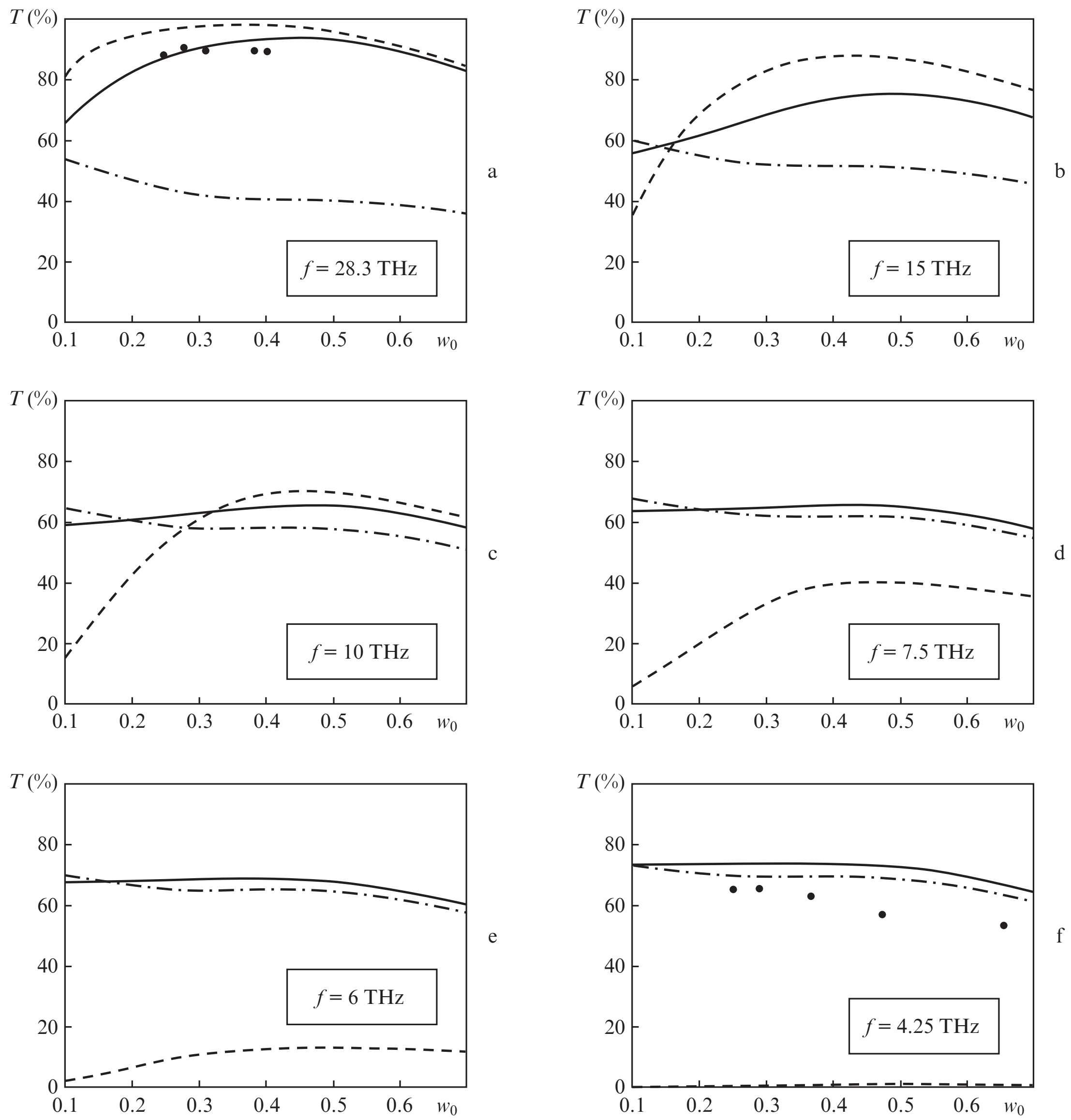

Figure 1. Calculated (curves) and experimental (points) dependences of the radiation transmission coefficient T on the relative radius w_0 of the exciting beam in a copper waveguide with a diameter $2a$ = 3 mm and length L = 370 mm at different radiation frequencies f. Solid curves were calculated using the ray-optics method, dashed curves – the mode method in the approximation of the metal by a dielectric, dash-and-dot curves – the mode method in the ideal-metal approximation.

approximation. Propagation of laser radiation through the waveguide is accompanied by its depolarisation. The measured degree of polarisation at different w_0 ranged from 10% to 50%. This indicates that in a metallic waveguide excited by a linearly polarised Gaussian beam at a given frequency, TE_{1n} and TM_{1n} waves with the field polarisation differing from linear one are excited.

When measuring the transmission characteristics at a frequency f = 28.3 THz (λ = 10.6 μm) we used a stabilised LG-74 CO_2 laser as a radiation source. Gaussian beams with a plane phase front were produced by the NaCl lenses with different focal lengths. The experimental results shown in Fig. 1a are in good agreement with calculations based on the mode method using the approximation of the metal by a dielectric at a given frequency. It is known that hybrid EH_{1m} modes are linearly polarised. The measured degree of polarisation of radiation passing through the waveguide excited by a linearly polarised beam with a relative radius w_0 = 0.5 was equal to 97.3%. For a similar segment of a dielectric waveguide the measured degree of the output radiation polarisation is 97.8% for the same parameters of the exciting beam. In measuring the degree of polarisation, use was made of a BaF_2 grating polariser with a polarisability of 98%. This experiment allows us to assert that in the investigated copper waveguide EH_{1m} hybrid modes are excited at a given frequency.

The discrepancy between the calculated and experimental data is explained by the irregular cross section, surface roughness and possible difference in the calculated material constants for the waveguides used in the study.

For quantitative evaluation of the appropriateness of the mode approach in the terahertz range, we calculated the dependence of the normalised average absolute deviation Δ of the transmission coefficients on the relative beam radius w_0, found using the ray-optics (G) and mode (R) approaches [24]:

$$\Delta(w_0) = |G(w_0) - R(w_0)|.$$

The results presented in Fig. 2 are confirmed by the presence of changes in the electrodynamic properties of metallic waveguides in the frequency range 7.5–15 THz. In this frequency range, the key parameter to assess the applicability of mode approaches is the relative radius of the radiation beam.

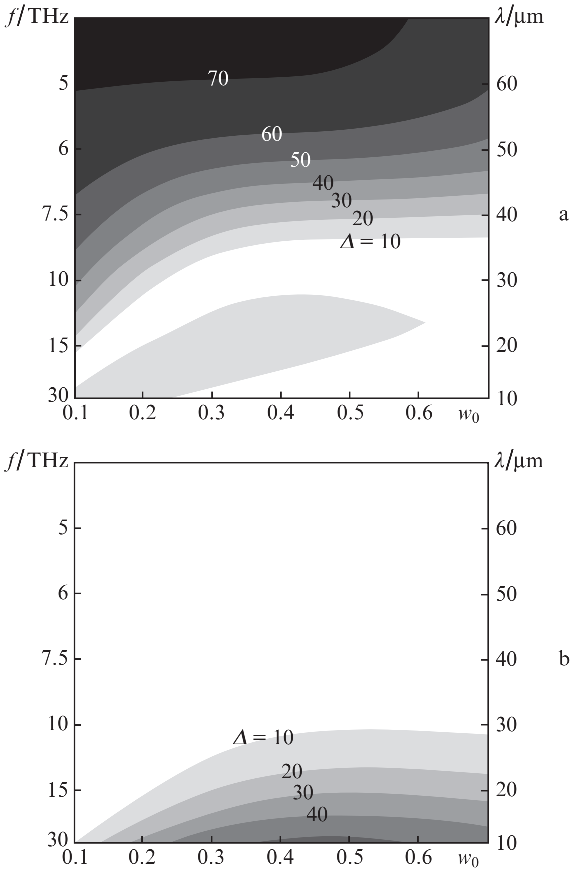

Figure 2. Calculated dependences of the deviation Δ of the transmission coefficient on the radiation frequency f (wavelength λ) and the relative radius w_0 of the exciting beam in a copper waveguide with a diameter $2a = 3$ mm and length $L = 370$ mm. Calculations were performed using the ray-optics and mode methods in the approximation of the metal by a dielectric (a), as well as using ray-optics and mode methods in the ideal-metal approximation (b).

4. Conclusions

We have studied theoretically and experimentally the transmission characteristics of linearly polarised Gaussian laser beams in hollow circular metallic waveguides in the terahertz range (4–28 THz). It has been shown for the first time that in the low-frequency region of this range (less than 7.5 THz), where the conductivity of metal is high, calculations of transmission characteristics of the metallic waveguide should be based on the mode approach in the ideal-metal approximation. In the high-frequency region of the terahertz range (above 15 THz), where the conductivity of metal is significantly low, calculations should be based on the mode approach in the approximation of a real metal by a dielectric.

We have established the existence of a transition region in the behaviour of the electrodynamic properties of metallic waveguides in the frequency range 7.5–15 THz. In this range the key parameter of the assessment of the mode approach applicability is the ratio of the radius of the exciting beam to the radius of the waveguide. In the case of narrow exciting beams ($w_0 < 0.3$) it is reasonable to calculate the transmission characteristics with the help of the ideal-metal approximation. When broad radiation beams are transmitted through metallic waveguides more accurate results are obtained using the approximation of a real metal by a dielectric.

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