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Study of bifurcation processes in a multimode waveguide with statistical irregularities

A.A. Egorov

Abstract. **We consider the theoretical principles of the original investigation of irregular optical waveguides in the form of a dynamic dissipative system. The scattering of guided modes in an irregular optical waveguide is considered as a process of gradual transition of a dynamic dissipative system from an 'ordered' state into 'chaos.' The growth of scattering losses in an irregular optical waveguide is represented as an increase in chaos in the system under analysis. The phase retardation factor of a multimode waveguide is used as a control parameter of the process. The use of the methods of catastrophe theory can explain the behaviour of the dissipative system under study in the process of changing the control parameter. It is found that an increase in chaos in the system (an increase in losses due to scattering in an irregular waveguide under excitation of modes of increasingly higher order) can be explained by a sequence of direct bifurcations, i.e., the existence of stable cycles in the system. As a result, the irregular optical waveguide can be regarded as a system in which the energy of a regular process (the process of propagation of a guided mode) passes into the energy of a 'disordered' process, i.e., the energy of radiation modes.**

Keywords: *optical waveguide, statistical irregularities, waveguide scattering, dispersion relation, nonlinear equation, bifurcation phenomena, nonlinear dynamic system, one-dimensional mapping, numerical simulation, coupled modes, second harmonic generation, birefringence, dissipative system, bistability, noise.*

1. Introduction

Scattering of a guided mode in an irregular optical waveguide can be considered as a process of gradual transition of a dynamic dissipative (open) system from the 'ordered' state into 'chaos.' As a control parameter of the problem, use can be made of the natural physical parameter of the system – the phase retardation factor γ of a multimode waveguide whose temporal variation is accompanied by a change in the effective thickness of the waveguide.

The problems of propagation of guided modes in multimode optical waveguides were studied in many papers. For those who wish to read more widely in particular subjects, we suggest, for example, monographs and papers [1–16].

By the ordered state of the system is meant a state of an irregular optical waveguide*, in which some guided mode (the regular component) is maintained, and the contribution of radiation modes (the irregular component) to the total field distribution of the waveguide is negligible. Chaos, on the contrary, is characterised by a significant contribution of the radiation modes to the total field distribution.

The increase in chaos in the system, i.e., the growth of losses due to scattering in an irregular waveguide under excitation of modes of increasingly higher order, can be explained by a sequence of direct bifurcations. Bifurcation is the emergence of a new quality in the behaviour of a dynamic system when its parameters change; in the waveguide under study the irregularity does not change over time, i.e., stationary, and so instead of 'bifurcation' use can be made of the term 'catastrophe' [17–20].

The aim of this work is the consistent presentation of the theoretical principles of a new method for investigating irregular optical waveguides as dynamic dissipative systems. At present, we are not aware of any scientific publications that would use this method for investigating irregular optical waveguides. The proposed method can be, for example, promising for both qualitative and quantitative study of scattering in waveguides with a complex structure and various topology of elements, where the application of analytical and computational methods is impossible or requires significant computational resources.

2. Radiation losses in an optical irregular waveguide

Consider, as an example, a three-layer planar optical waveguide with an arbitrary deformation, for example, of one of the interfaces between the media of the waveguide (Fig. 1) [1–6]. We will approximate this violation of the regularity of the interface by a sequence of small stepwise changes. The same approach can be used in the case of an arbitrary change in the profile of the dielectric constant, for example, of the

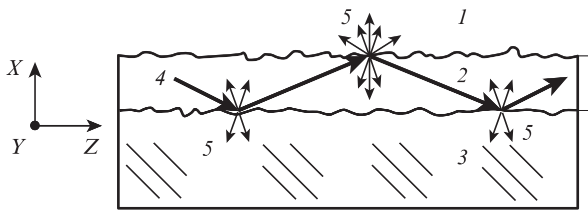

Figure 1. Three-layer planar optical waveguides with rough boundaries: (*1*) cladding layer (air; refractive index, n_c); (*2*) waveguide layer (refractive index, n_f); (*3*) substrate (refractive index, n_s); (*4*) propagation direction of the optical beam; (*5*) scattering of the optical beam at rough film/substrate and film/air interfaces; h is the thickness of the waveguide layer.

* A waveguide with rough interfaces between its constituent media and/or nonuniform (with respect to the refractive index) the structure of these media.

A.A. Egorov A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia; e-mail: yegorov@kapella.gpi.ru

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waveguide layer of a three-layer waveguide. The validity of this approximate approach has been confirmed both theoretically and experimentally (see, for example, [1]).

Consider a small (compared with the thickness of the waveguide h) solitary jump Δh or smooth narrowing/expansion in a certain section of the waveguide under study, which in both cases meet the condition $\Delta h/h \ll 1$. To calculate the integral power losses ΔP due to radiation in this section, we can use the expression [1–4]

$$\Delta P = P(1 - |C_t|^2 - |C_r|^2), \tag{1}$$

where P is the total power of the guided mode incident on the waveguide section under consideration; $\Delta P/P$ are the relative power losses; $\alpha = \Delta P/(Pl)$ is the attenuation coefficient of the guided mode on a section of length l; C_t and C_r are the amplitude (dimensionless) coefficients of transmission and reflection of the guided mode at the given section of the waveguide. It follows from (1) that $\Delta P/P$, C_t and C_r are dimensionless.

If the guided mode at the given section of the waveguide does not pass, then $C_t = 0$ and we deal only with its reflection. We assume for clarity that reflection is due to the transformation of the guided mode into the same mode, but travelling in another, for example, opposite direction. In this case, the analytic form of the coefficient C_r is known (see, for example, [1, 14–16]).

For further analysis, it is important that P and C_r presented in (1) are the functions of the effective thickness of the waveguide

$$h_{\text{eff}} = h + (\beta_0^2 - k_0^2 n_c^2)^{-1/2} + (\beta_0^2 - k_0^2 n_s^2)^{-1/2}, \tag{2}$$

where h is the thickness of the waveguide layer with a refractive index n_f; $\beta_0 = k_0\gamma$ is the propagation constant of the guided mode along the z axis; $k_0 = 2\pi/\lambda_0$ is the modulus of the wave vector $\boldsymbol{k}_0$; λ_0 is the wavelength of light in vacuum; n_c is the refractive index of the cladding layer (air); n_s is the refractive index of the substrate; in a symmetric waveguide, $n_f > n_s = n_c$, and in an asymmetric waveguide, $n_f > n_s > n_c$.

When the guided mode is diffracted or scattered at an irregular section of the waveguide, $C_r = C_r(h_{\text{eff}}, F)$, where F is determined by the statistics and the parameters of the corresponding irregularities. In the case of statistical irregularities, for example, irregularities (roughness) of the interfaces, it is necessary to take into account, at least, the dependence of the reflection coefficient of the guided mode on the interval (radius) of correlation r of the irregularities $C_r = C_r(h_{\text{eff}}, r)$.

We write formula (2) in the form

$$h_{\text{eff}} = h + k_0^{-1}[(\gamma^2 - n_c^2)^{-1/2} + (\gamma^2 - n_s^2)^{-1/2}], \tag{3}$$

which shows that h_{eff} is a function of the phase retardation factor (the effective refractive index of the waveguide) γ.

Using formula (3) and taking into account that $C_t = 0$, expression (1) can be rewritten in the form, well-known in the theory of nonlinear dynamic systems [17, 18]

$$\Delta P = P(1 - |C_r|^2) \to y = \mu(h_{\text{eff}})x(1 - x)$$
$$= \mu[h_{\text{eff}}(\gamma)]x(1 - x), \tag{4}$$

or in the form that is nonlinear with respect to the variable x:

$$y(x) = \mu x(1 - x). \tag{5}$$

To obtain expressions (4) and (5) (1) we made the following substitutions: $y = \Delta P/P_0$; $\mu(h_{\text{eff}}) = P(P_0|C_r|^2)^{-1} = P(C_{P_0})^{-1}$, where $C_{P_0} = P_0|C_r|^2$; $x = |C_r|^2$, with the range of changes in x and μ being limited in a natural way due to the definition of the reflection coefficient: $0 \leqslant x \leqslant 1$ and $0 \leqslant \mu < \infty$ $(P, P_r < P_0)$.

Equations (4) and (5) were obtained in dimensionless quantities. To do this, the powers ΔP and P were normalised to the maximum power P_0 of the guided mode incident on an irregular section of the waveguide (in principle, it can be the unit power: $P_0 = 1$ W).

We call the parameter $\mu(\gamma)$ the control parameter of the dynamic dissipative system under consideration – irregular multimode optical waveguide. Because $C_{P_0} \leqslant P$ and $C_{P_0} \in [0, P]$, then $\mu(\gamma) \in [1, \infty)$.

Given that $\mu = \mu(\gamma)$ and $C_r = C_r(\gamma, r)$ in (5), we write the last equation, taking into account the explicit dependence of these quantities:

$$y(x) = \mu(\gamma)x(\gamma, r)[1 - x(\gamma, r)]. \tag{6}$$

Note that we have yet proceeded from the assumption that a solitary jump of the waveguide thickness satisfies the condition $\Delta h/h \ll 1$. In fact, fulfilment of this inequality makes it possible to meet the first approximation of the perturbation theory in the theory of waveguide scattering [1–8, 14–16], for which $\Delta P/P \ll 1$.

Accounting for small second-order quantities, which were neglected in (1), (4) and (5), allows one to write the nonlinear equation (6) in the form:

$$y(x) = \mu_1(\gamma)x(\gamma, r)[1 - x(\gamma, r)] + \mu_2(\gamma)[1 - x(\gamma, r)]^2, \tag{7}$$

where $\mu_1(\gamma) = \mu(\gamma)$, and $\mu_2(\gamma)$ is another control parameter of the dynamic dissipative system under study (taking into account the second order of smallness).

For concreteness, we consider the multimode optical waveguide with statistical irregularities at the interfaces between the media forming the waveguide [1, 3, 15, 16]. However, our conclusions will be quite valid for a multimode optical waveguide with other types of irregularities that meet the abovementioned limitations.

3. Study of bifurcation processes in an optical multimode waveguide with statistically rough boundaries

3.1. Basic concepts of the theory of nonlinear dynamical systems

For those who wish to get acquainted with the theory of bifurcation phenomena, we recommend book [17], and for those who wish to read more widely in the subject – monographs [19, 20]. Here we will mention only the basic concepts that facilitate the understanding of the material in section 3.2.

On a discrete set $x \in X$, where $x = x_1, x_2, ..., x_n$, we can write a simple one-dimensional equation describing some dynamic system:

$$x_{n+1} = ax_n(1 - x_n). \tag{8}$$

The sequence of values of x_n is called the orbit of mapping. The initial segment of the sequence is the transition regime and the rest is the steady state.

Equation (8) in the standard form is written as:

$$x_{n+1} = f(x_n), \tag{9}$$

where the function $f(x_n)$ is represented in the form

$$f(x) = 4\mu x(1 - x), \tag{10}$$

that is similar to equation (5). We emphasise that we should expect some differences between the dependences obtained in accordance with equations (6), (7) from the analogous dependences for expression (10) because the variable x depends on γ and r in (6), (7).

In analysing equation (10), use is made of the conditions $0 \leqslant x \leqslant 1$ и $0 \leqslant \mu \leqslant 1$. The first condition makes it possible to avoid unphysical situations, when $x_n > 1$, and $x_{n+1} < 0$.

The second condition allows one, in the case of an irregular optical waveguide, to perform *a fortiori* the first approximation of perturbation theory and to neglect the possible nonlinearity of the field problem. Indeed, if $\mu \to \infty$, then $P \to \infty$ (the case $P_{\rm r} \to 0$ is trivial), which means a sharp increase in the power of the waveguide mode and the need to consider the probable nonlinear optical effects in the waveguide which are related already to the field nonlinearity E. For example, for the SHG when the input TE mode with a frequency $\omega/2$ is converted at the output of the waveguide into the TE mode with a frequency ω, the complex amplitude of the polarisation, which describes the polarisation perturbation (in the right-hand side of the wave equation) in the medium, has the form $P_i^{(\omega)} = d_{ijk}^{(\omega)} E_j^{(\omega/2)} E_k^{(\omega/2)}$, where $d_{ijk}^{(\omega)}$ is the element of the nonlinear optical tensor.

The nonlinear frequency conversion is effective under the condition that the overlap integral

$$I = \int E^{(\omega)} P^{(\omega)} \mathrm{d}x\mathrm{d}y \tag{11}$$

is large. Integration in (11) is performed over the cross section of the waveguide. It is known that for the conditions $I \to$ max to be met in (11), the spatial overlap of the field of the TE mode, $E^{(\omega)}$, and the amplitude of the nonlinear polarisation, $P^{(\omega)}$, in the transverse plane xy should be maximal. One can see from (11) that it is better to optimise I for weakly oscillating profiles of the transverse distribution of the field energy. This explains the preference for lower-order modes for the SHG.

In addition, to ensure effective coupling between the modes along the entire length of the interaction, it is necessary to fulfill the phase matching condition, i.e., to provide the equality of the wave vectors of the nonlinear polarisation and the pump mode:

$$\beta_{\rm p}^{(\omega)}(h) = 2\beta_{\rm m}^{(\omega/2)}(h). \tag{12}$$

In fact, in a planar waveguide the waveguide layer thickness h is a variable that plays the same role as the angle between the propagation direction of a plane wave and the position of the crystal in the bulk case.

It is important to note that in optical waveguides, due to the presence of modes with different polarisations (TE and TM) with respect to a given plane, even in an isotropic medium there already exists the 'splitting' of the curve $\beta(h)$ [or $\gamma(h)$] at a given frequency ω, or of the curve $\beta(\omega)$ at a given thickness h. However, this does not make it possible to establish phase matching for TE_0 and TM_0 modes at the maximum overlap integral, because their dispersion curves do not intersect. In an anisotropic waveguide, phase matching of the fundamental modes is possible. Here, the birefringence is sufficient to compensate for the dispersion of the modes of the same order at the fundamental frequency and second harmonic frequency [21].

Note an important advantage of the wave interactions in the case of the SHG in waveguide structures, compared with the classical bulk nonlinear media – the possibility of a significant (up to several orders of magnitude) increase in the frequency conversion efficiency.

If the phase-matching condition (12) is not satisfied or $I \to$ min in (11), we can neglect the possible field nonlinearity in a birefringent medium.

Let us return to equation (10). The function $f(x)$ transforms any point in the interval [0, 1] to some other point of the same interval, and, therefore, $f(x)$ is called one-dimensional mapping. Equation (9) is called standard mapping. The main properties of this mapping were studied by Feigenbaum (1978) and the detailed description can be found in the literature. We only mention some important properties.

With the growth of μ, the system gradually shifts from the periodic 'motion' (behaviour) to chaotic ($\mu_{\rm ch} \approx 0.892486$). At the same time, within the regions of chaos narrow windows of periodic 'motion' are observed. It is important to note that at μ slightly larger than 0.75, the only immobile point x^* (here, there is a cycle S^1, i.e., a cycle with a stable point) splits, i.e., there occurs bifurcation into two oscillating values – x_1^* and x_2^* (there appears a cycle S^2, i.e., a cycle with two stable points). A pair of these points forms a stable attractor. In the region of chaos ($\mu > \mu_{\rm ch}$) two nearby initial points diverge along different trajectories after several iterations.

To study the dynamic behaviour of such systems on a parameter μ, use is made of the graphical method of iteration $f(x)$. It consists of the following: we choose a point x_0, which is not fixed (for example, $x_0 \neq 0$), draw a vertical line from the point ($x = x_0$, $y = 0$) to its intersection with the curve $y = f(x)$ at the point $\{x_0, y_0 = f(x_0)\}$. Then, a horizontal line is drawn from the point (x_0, y_0) to its intersection with inclined line $y = x$ at the point (y_0, y_0). The value of x at the point of intersection is the first iteration $x_1 = y_0$. Similarly, we can find other iterations. The iterative process converges to the fixed point, which is called stable (stable attractor). To explain the stability, use is made of the stability criterion of the fixed point: the slope of the curve at this point should be less than unity (see below).

3.2. Stationary order and chaos in a multimode optical waveguide

We will illustrate the dynamics of transition of an irregular optical waveguide from the ordered state into chaos with the help of a wavenumber diagram, combined with the dependence of the attenuation coefficient of the guided modes α on the factor γ (Fig. 2). The relative power loss of the guided mode due to scattering in the irregular-waveguide section of unit length is related to the attenuation coefficient by a simple expression: $\Delta P/P = 2\alpha$.

We will show that the growth of chaos in the system (an increase in losses due to scattering in an irregular waveguide under excitation of modes of increasingly higher order) can be explained by a sequence of direct bifurcations, for example, by the presence of stable cycles S^1, S^2, S^3, S^4, etc., in the system.

In our case, the variable x, i.e., the attenuation coefficient α, depends both on the phase retardation factor γ and on the

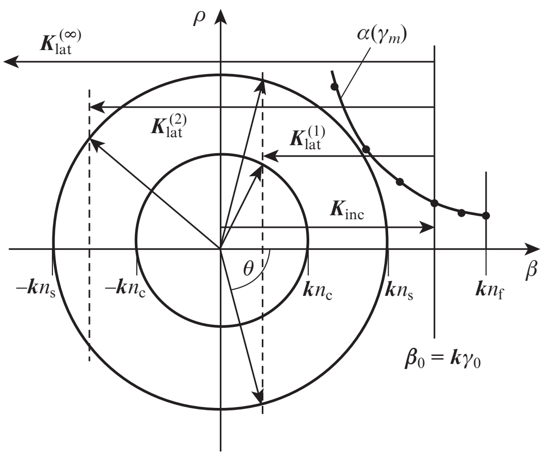

Figure 2. Wavenumber diagram combined with the dependence of the attenuation coefficient of the waveguide modes α on the phase retardation factor γ_m ($m = 0, 1, \ldots$).

correlation interval r of the waveguide irregularities. In the numerical simulation, we fix one parameter, for example γ, and consider the dependence of the function y [see equations (6), (7)] on the second parameter, i.e., on r.

It is found in computer simulations that in the cases described by nonlinear equations (6) and (7) it is possible to realise cycles S^1, S^2 and higher-order cycles by choosing appropriate control parameters. In the case of cycle S^2, the period-doubling bifurcation is observed: the scattering diagram exhibits the elements $\beta^{(a)}$ (here $a = 1, 2, 3, \ldots$), which are 'subharmonics' with respect to the propagation constant of the guided mode $\beta_{guid} = K_{inc}$: $\beta^{(1)} = \beta_{guid}/2$, $\beta^{(2)} = \beta_{guid}/4$, etc. [e.g., $\beta^{(1)} = \beta_{guid} - K_{lat}^{(1)}$, where $K_{lat}^{(1)} = 2\pi/\Lambda_{lat}^{(1)}$ and $\Lambda_{lat}^{(1)}$ is a period of some lattice in the spectrum of the waveguide irregularities].

As an example, Fig. 3 (an asymmetric waveguide) and Fig. 4 (a symmetric waveguide) show cycles S^1, possible in these cases. In calculating the dependences in Figs 3 and 4, we used the formulas for the relative power loss due to the integral scattering in a statistically irregular symmetric or asymmetric optical waveguide (see, for example, [1, 3, 15, 16]). Figure 3 shows the dependence of the relative power loss caused by radiation on the correlation interval r_{rel} at a fixed γ, and Fig. 4 shows the dependence of the relative power loss caused by radiation on γ_{rel} at $r = 0.1$ μm.

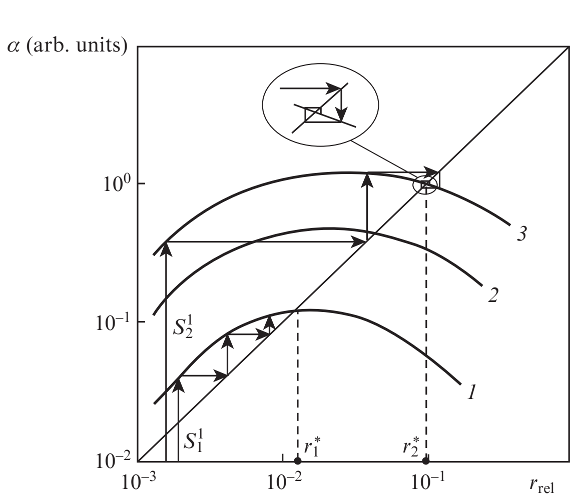

Figure 3. Dependences of the relative power loss caused by radiation on the correlation radius normalised by $r_{max} = 10$, which characterise the bifurcation process (cycles S^1) in the asymmetric planar optical waveguide ($n_c = 1.000$, $n_f = 1.590$, $n_s = 1.460$). The phase retardation factor is $\gamma = 1.570$ (*1*), 1.500 (*2*) and 1.470 (*3*).

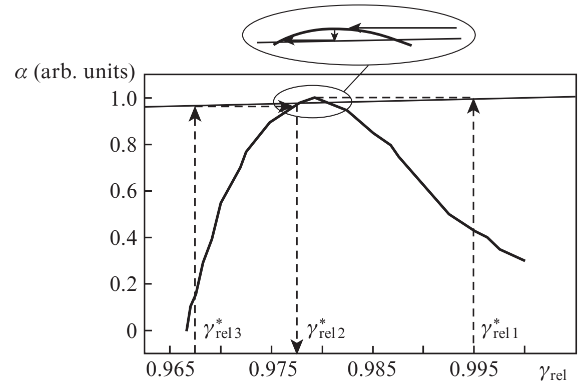

Figure 4. Dependence of the relative power loss caused by radiation on $\gamma_{rel} = \gamma/1.567$, which characterises the bifurcation process (cycles S^1) in the symmetric planar optical waveguide ($n_c = n_s = 1.460$, $n_f = 1.590$). The correlation radius r of the substrate roughness is equal to 0.1 μm.

In both cases, we can observe the convergence of the process from some arbitrary initial point to some constant values of independent variables (stable immobile points). In the first case, we deal with convergence to the points r_1^* or r_2^*, which are some characteristic parameters of the statistical irregularity of the dynamical system under study at given values of γ. Thus, if we assume independence of the random components of the irregularity, the result obtained here is consistent with the known conclusion that the sum of a large number of these terms, in accordance with the central limit theorem, has a Gaussian distribution with the correlation radius and the rms height of the profile, defined as ensemble-averaged values (and typical of the laser radiation wavelength) that are close to r^*.

In the second case (Fig. 4), we deal with the convergence to one characteristic value of the control parameter (in the terminology of catastrophe theory) $\gamma_2^* \approx 1.532$ ($\gamma_{rel\,2} = 0.977$) that is close to the inflection point on the dispersion curve, and to $\gamma_{opt} \approx 1.54$, in the vicinity of which the maximum radiation loss of the guided mode is observed. In the vicinity of the point $\gamma_2^* \approx \gamma_{opt}$, the system is most informative: here the signal-to-noise ratio (radiation scattered into the surrounding 3D space) reaches a maximum.

Numerical modelling in accordance with (6), (7) showed that in a multimode planar symmetric waveguide with statistical irregularities (roughness) at the interface, expression (6) more accurately describes the dependence of the attenuation coefficient α on γ (r is assumed fixed) if the correlation interval of the roughness is $r \leqslant \lambda/10$, and the expression (7) – if $r \geqslant \lambda$.

Using the methods of catastrophe theory [17–20], we can explain the behaviour of the dissipative systems under study as a function of the control parameter γ. To this end, we should investigate the first derivative of unsmoothed dependence of the relative power loss caused by radiation. This function, by analogy with catastrophe theory, can be regarded as a potential function of the system, which has minima (local and global). To determine the interval of values of the function where the modulus of first derivative does not exceed unity, horizontal lines +1 and –1 are drawn. In this interval, we observe the condition for the stability of the singular point $\gamma^*_{rel\,2} \approx 0.978$: $|\mathrm{d}\langle\Delta P/P\rangle/\mathrm{d}\gamma| \leqslant 1$.

The resulting dependence of the first derivative corresponds to the well-known case of bifurcation of the equilibrium state (a particle in a potential well with a barrier or shelf). At a high signal-to-noise ratio only one equilibrium state of the system is possible – when $\gamma \approx \gamma^*_{\rm rel\,2} \approx 0.978$. We can use here the concept of a phase transition at which the system changes qualitatively. For example, when $\gamma < \gamma^*_{\rm rel\,2}$ or $\gamma > \gamma^*_{\rm rel\,2}$, the system exhibits minimal losses and the waveguide does not virtually radiate (closed system), i.e., radiation is absent in the medium surrounding the waveguide (the track of the waveguide mode is not visible), and when $\gamma \approx \gamma^*_{\rm rel\,2}$, the system transforms into a qualitatively new state: the waveguide radiates (open system) and the track of the mode can be observed.

Thus, the irregular optical waveguide [1–6, 14–16, 23, 24] can be considered as a dissipative system, where the energy of the ordered process (propagation of the guided mode) transforms into the energy of the disordered process (scattering), i.e., in the energy of radiation modes, and, finally, into the heat.

4. 'Bistability' properties of an optical irregular waveguide

The behaviour of the system, when the control parameter γ changes with time, can be conventionally represented as a work of a bistable element having two stable states (especially in case of such an irregularity as a diffraction grating): radiation is absent or present. The first variant: grating is absent – radiation is off, grating is present – radiation is on; the second variant: the grating is always present, but the angle of the mode incidence on the grating changes. When 'switching', the power cost per information bit can be equal to approximately 10^{-9} W bit^{-1} for a symmetric silica fibre at SNR $\approx$ 10 and 10%–15% efficiency of He–Ne-laser radiation coupling into the waveguide.

Evaluation of the information volume, I_Σ, in the scattering diagram can be obtained using the formula from [22]: $I_\Sigma = N\log_2\mathcal{J} = (2\beta_3 L + 1)\log_2\sqrt{(P_{\rm S} + P_{\rm N})/P_{\rm N}}$, where N is the total number of symbols in a message (a continuous signal is replaced by a discrete sequence of counts); $\mathcal{J}$ is the number of different letters in the alphabet, which is, at the average power of the noise $P_{\rm N}$ and signal $P_{\rm S}$, is equal to $\sqrt{(P_{\rm S} + P_{\rm N})/P_{\rm N}}$. When SNR $\approx$ 1, the power cost per information bit is approximately 3.3×10^{-9} W bit^{-1}. In the first and second variants, $\mathcal{J} \approx 3$ and 1, respectively. These estimates are valid in the case of lack of correlation between the signal samples (i.e., their mutual independence) and additivity of signal and noise.

Comparison of I_Σ, obtained for the system with a 'bistable' element and for other systems with well-known switching elements, shows that this system is superior to many electronic and optical devices and can be compared with a neuron in the power cost per information bit [18].

Note also that the problem of influence of periodic irregularities on propagation of guided modes was addressed in many papers (see, for example, [1, 2, 4, 5, 8, 9]), which considered such important issues as transformation of the fields of the modes, mode coupling and synchronisation, change in the mode spectrum, etc. The use of optical fibres in a fibre-optic communication lines, fibre lasers and fibre sensors was studied in particular in [4, 8, 9, 11, 12]. Within the framework of our new theoretical method, analysis of such aspects of application of optical waveguides is possible in principle, but lies beyond the scope of this publication.

5. Conclusions

Transition from the integrated optical waveguide with such an irregularity as harmonic ripple to the waveguide with statistical irregularities (i.e., transition from a system with long-range order to a system with short-range order) can be illustrated in the phase plane as a transition from a system characterised by the usual attractor (stable focus in a system with losses) to a system characterised by the stochastic (strange) attractor (an attracting set of unstable trajectories in the space of states of a dissipative system). This transition is similar to the transition from an ordered phase state to the disordered state. Here, we can speak of a certain analogy with the loss of stability in the crystal when the restoring force, removing distortions of the crystal lattice, disappears at the phase transition temperature. A similar phenomenon takes place during heating of the waveguide film made of crystalline (polycrystalline) material.

The problems of transition of systems from an ordered state to the disordered state are encountered in different fields of physics, biology, chemistry, economics, politics, sociology, etc. Using a sequence of cycles S^1, S^2, S^4, ..., we can show that the random process arises as the limiting one for more complex structures (cycles S^{2p}). The emergence of a strange attractor can be explained with the help of cycle S^∞. Here, with the course of 'time' the paths of two close points diverge rapidly, and the behaviour of the system cannot be predicted (chaos becomes more random).

This paper presents the theoretical principles of a new method for investigating irregular optical waveguides as dynamic dissipative systems, which may be particularly promising for the qualitative analysis of scattering in a waveguide with a complex chemical structure and three-dimensional topology of elements, where the application of analytical and computational methods is impossible or requires substantial computational resources. Of fundamental and applied interest is the application of this method in studying the phenomenon of anomalous light scattering near phase transitions in liquid-crystal and polycrystalline waveguide layers. Of particular interest are studies in the field of the waveguide rainbow, where the fine structure of the spectrum of the light scattered in the waveguide can be observed with high resolution. Obviously, such research will open new opportunities for specialists in the field of materials science, as well as for biologists, physicians, and chemists.

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