

LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

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Control of kinematics of neutrons in a coaxial magnetic trap

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Abstract. **We discuss the physical basis of how to control changes in basic kinematic parameters of neutrons (energy, velocity, density, etc.) through variation in the current of a coaxial magnetic trap containing neutrons. We present quantitative assessment of possible changes in the parameters by several orders of magnitude and outline potentials and prospects for application of the approach discussed.**

Keywords: *quantum nucleonics, acceleration and deceleration of neutrons, manipulation of neutron concentration, ultracold neutrons, nuclear reaction with neutrons, neutron pumping of gamma lasers.*

1. Introduction

Goal-directed control of kinematics of the neutrons, hindered by their electrical neutrality, is needed to solve problems in neutron optics, microscopy, interferometry, quantum nucleonics, etc. (see, for example, [1 – 3]). Typically, the neutron motion is controlled by using the interaction of its magnetic moment with the electromagnetic field having a finite gradient of the magnetic induction. This paper discusses the physical basis of the control of kinematic parameters of neutrons accumulated in a coaxial magnetic trap, which was successfully applied earlier to maintain the beam of neutral sodium atoms [4]. The method may help to solve problems in areas that are discussed in [1 – 3].

2. Neutrons in a coaxial magnetic trap

The basis of a coaxial trap is a straight conductor of diameter $2r_j$ with a current J, producing, at a sufficient distance from the ends of the conductor in a cylindrical coordinate system (z, r, φ), the magnetic field with induction

$$B = \frac{\mu_0}{4\pi}\frac{2J}{r} \qquad (r > r_j) \tag{1}$$

and circular lines of force, encompassing the current in the plane (r, φ) (μ_0 is the magnetic permeability of free space). In this field, the vector of the magnetic dipole moment with the modulus $m = 0.95 \times 10^{-20}$ J T^{-1} is set along a tangent to the line of force, and a neutron finds itself trapped in a potential well

$$U(r) = -\frac{\mu_0 m}{2\pi}\frac{J}{r}, \tag{2}$$

experiencing, at point $r \geqslant r_j$, the impact of a gradient-magnetic force with the modulus

$$F_B = \left| m\frac{\mathrm{d}B}{\mathrm{d}r} \right| = \frac{\mu_0 m}{4\pi}\frac{2J}{r^2}, \tag{3}$$

directed along the radius to $r = 0$..

To describe the behaviour of the neutron in a coaxial magnetic trap, which is a macroscopic device, the classical consideration is sufficient and the quantum-mechanical analysis is hardly necessary.

In fact, the solution of the Schrödinger equation for a neutron with mass M and energy ε is $\Psi^2(r, \varphi, z) = Y^2(r) \times \cos^2(N\varphi)\cos^2(Qz)$, where N is the integer orbital quantum number; $Q = (2M\varepsilon_z)^{1/2}/\hbar$; ε_z is the longitudinal component of the energy ε; $Y(r)$ is the integral of the equation

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}Y}{\mathrm{d}r} + \left[2M\hbar^{-2}(\varepsilon - \varepsilon_z) + \frac{\mu_0 mMJ}{\pi\hbar^2 r} - \left(\frac{N}{r}\right)^2\right]Y = 0.$$

Next, using the analysis of quantization conditions, for example, for orbital degree of freedom of the neutron [the factor $\cos(N\varphi)$], and estimating, in accordance with the Bohr rules, the orbital quantum number $N = (r/\hbar)(2M\varepsilon_\varphi)^{1/2}$ as the ratio of the circular orbit length with radius r to the orbital component of the de Broglie wavelength $\Lambda_\varphi = 2\pi\hbar(2M\varepsilon_\varphi)^{-1/2}$, where ε_φ is the orbital component of the energy, it is easy to see that the energies of the two neighbouring states with the numbers $N + 1$ and N differ by $\Delta\varepsilon_\varphi \approx (\hbar/r)(2\varepsilon_\varphi/M)^{1/2}$. If the difference $\Delta\varepsilon_\varphi$ is comparable to the homogeneous energy width of the orbital state $\delta\varepsilon$ ($\Delta\varepsilon_\varphi \leqslant \delta\varepsilon$), the spectrum of the states becomes continuous, and the quantum properties of the system are lost together with the discrete nature of the spectrum.

The homogeneous width $\delta\varepsilon = 2\pi\hbar\tau_{\mathrm{n}}^{-1}$ is inversely proportional to the smallest of the lifetimes τ_{n} of the state (for example, to the storage time of a neutron in a trap, to the inverse probability of neutron collisions, etc., up to the maximum time – the lifetime of a β-radioactive neutron $\tau_\beta \approx 1300$ s), i.e., $\delta\varepsilon = 2\pi\hbar\tau_{\mathrm{n}}^{-1} \geqslant 2\pi\hbar\tau_\beta^{-1} \approx 3 \times 10^{-18}$ eV, with an overstoke, because usually $\tau_{\mathrm{n}} \ll \tau_\beta$, and the entire energy spectrum of the states is much more dense due to quantization of two other degrees of freedom of the neutron.

Thus, the need for a quantum approach is lost if

$$\varepsilon_\varphi r^{-2} < 2\pi^2 M\tau_\beta^{-2}\left(\frac{\tau_\beta}{\tau_{\mathrm{n}}}\right)^2 \approx 10^4\left(\frac{\tau_\beta}{\tau_{\mathrm{n}}}\right)^2 \ (\text{eV cm}^{-2}), \tag{4}$$

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i.e., at macroscopic parameters quite typical of the trap (e.g., at $r < 10^{-4}$ cm even for thermal neutrons with $\varepsilon \approx 25$ meV and without reservations due to $\tau_n \ll \tau_\beta$ and compression of the spectrum of states by other degrees of freedom).

As a result, for further consideration of the trap of macroscopic geometry, it is sufficient to resort to a cylindrical version of the Kepler model, restricting ourselves for definiteness, only to circular orbits.

3. Kepler model of a coaxial magnetic trap

In the Kepler model with cylindrical geometry, along with the gradient-magnetic force F_B (3) the centrifugal force with the modulus

$$F_c = 2\varepsilon_\varphi / r. \tag{5}$$

acts on the neutron.

The equality $F_c = F_B$ sets the stationary motion of the neutron along the circular orbit with the radius

$$r_0 = \frac{\mu_0 m}{4\pi}\frac{J}{\varepsilon_\varphi} \approx 6\times 10^{-13}\frac{J}{\varepsilon_\varphi}\ \text{(cm)} \tag{6}$$

and rotation frequency

$$\Omega = \frac{4\pi}{\mu_0 m J}\left(\frac{2\varepsilon_\varphi^3}{M}\right)^{1/2} \approx 2\times 10^{18}\frac{\varepsilon_\varphi^{3/2}}{J}\ (\text{s}^{-1}). \tag{7}$$

In this case, the product

$$r_0\varepsilon_\varphi = \frac{\mu_0 m}{4\pi}J \approx 6\times 10^{-13}J\ (\text{cm eV}^{-1}) \tag{8}$$

forms a family of hyperbolas with the current J as a parameter (here and below, in numerical expressions – cm, s, eV, A).

Because the force components tangential to the orbit of the neutron are absent, its angular momentum remains unchanged,

$$r(2M\varepsilon_\varphi)^{1/2} = \text{const}, \tag{9}$$

and, therefore, $F_c = \text{const}^2/(Mr^3)$. Thus, due to a rapid decrease in $F_c \sim r^{-2}$ as compared to $F_c \sim r^{-3}$ with increasing radius r, the equilibrium orbit of radius r_0 is stationary.

Note that in the expression for the force F_B (3), we made an assumption about parallelism of the dipole moment vector $\boldsymbol{m}$ and the vector $\boldsymbol{B}$, because at the precession frequency of the neutron, significantly higher than the frequency Ω (7), the vector $\boldsymbol{m}$ has time to follow the direction of the vector $\boldsymbol{B}$. For this reason, the orbital quantum number N also includes changes of Berry's phase, which occurs due to rotation of the vector $\boldsymbol{m}$, following the vector $\boldsymbol{B}$ in its orbital motion.

As a result, the neutron trajectory is a spiral with radius r_0 and step $\Delta z = 2\pi v_z/\Omega$, and a set of trajectories forms a circular cylinder of radius r_0 (6), moving with a velocity $v_z = (2\varepsilon_z/M)^{1/2}$ along the z axis and rotating around it with a frequency Ω (7).

The fact that the present non-monokineticity of the neutron flux is of purely thermodynamic origin raises doubts, because the energy dispersion of the neutron, $\Delta\varepsilon$, to a greater extent is due to the technological features of loading of neutrons in the trap. However, it is convenient to ascribe to neutrons a conventional parameter with the temperature dimension

$$T^* \equiv \Delta\varepsilon / k_B \tag{10}$$

(k_B is the Boltzmann constant).

A non-monokinetic neutron flux fills a cylindrical layer with an average radius r_0 (6), a relative radial thickness

$$\Delta r_0 / r_0 = k_B T^* / \varepsilon_\varphi \tag{11}$$

and a volume per unit length of the trap with a total length L along the z axis

$$V = 4\pi r_0 \Delta r_0 = 4\pi r_0^2 \frac{k_B T^*}{\varepsilon_\varphi} = \frac{(\mu_0 m J)^2}{4\pi\varepsilon_\varphi^3} k_B T^*. \tag{12}$$

The neutron ensemble in the volume VL can be assumed collisionless if the probability of collisions during the storage time of the neutron in the trap is small,

$$\sigma n L (k_B T^* / \varepsilon_z)^{1/2} \ll 1; \tag{13}$$

otherwise, the collision should be taken into account (σ is the collision cross section, n is the concentration of neutrons).

4. Neutrons in a coaxial magnetic trap (transient processes)

The change in the orbital states of the neutron in a coaxial trap proceeds under the variation of the current J. If J changes sufficiently slowly, i.e., if the relative change in the equilibrium radius, occurring during the time $2\pi/\Omega$, is small ($|\delta r_0|/r_0 \ll 1$), the transient process can be viewed as a sequence of stationary states, to which we can apply the results of the previous section. This limits the rate of change in the current by the inequality

$$\frac{dJ}{dt} \ll \frac{(2\varepsilon_\varphi)^{3/2}}{\mu_0 m M^{1/2}} \approx 3\times 10^{17}\varepsilon_\varphi^{3/2}\ (\text{A/c}), \tag{14}$$

which can hardly result in noticeable difficulties in experiments even with ultracold neutrons.

Due to the constancy of the orbital angular momentum (9) and fulfilment of (14), the change in the current from J_1 to J_2 leads the transition from the steady state 1 to state 2 with the following relationship between the initial and final parameters:

$$\varepsilon_2/\varepsilon_1 = (J_2/J_1)^2, \tag{15}$$

$$\Delta\varepsilon_2/\Delta\varepsilon_1 = T_2^*/T_1^* = (J_2/J_1)^2, \tag{16}$$

$$r_{02}/r_{01} = J_1/J_2, \tag{17}$$

$$V_2/V_1 = (J_1/J_2)^2, \tag{18}$$

$$n_2/n_1 = (J_2/J_1)^2, \tag{19}$$

$$\varepsilon_2 n_2/\varepsilon_1 n_1 = (J_2/J_1)^4, \tag{20}$$

$$\Omega_2/\Omega_1 = (J_2/J_1)^2. \tag{21}$$

A few necessary remarks.

1. The ratios between the neutron concentration (19) and the density of the orbital energy component (20) are valid under the assumption that the variation of the current J and

Tabl.1

Example No., state	J/A	r_0/cm	ε/eV	T_ε^*/K	n/cm^{-3}	εn/eV cm^{-3}
№ 1						
State 1	10	10^{-3}	6×10^{-13}	10^{-3}	10^5	6×10^{-8}
State 2	1	10^{-2}	6×10^{-15}	10^{-5}	10^3	6×10^{-12}
№ 2						
State 1	1	10^{-2}	6×10^{-11}	10^{-1}	10^5	6×10^{-6}
State 2	10	10^{-3}	6×10^{-9}	10	10^7	6×10^{-2}

the corresponding change in the volume V (18) occur without changing the number of neutrons: Vn = const.

2. Strictly speaking, in the collisionless neutron ensemble, subject to inequality (13), changes in the energy ε (15) and in the conventional temperature T^* (16) are directly related only to their orbital components; however, in the case of the opposite sign of inequality (13), neutron collisions can extend these changes to all degrees of freedom of the neutron.

3. Difficulties associated with large values of the current J and magnetic field strength can be eliminated by at least two ways: by using short current pulses, J, and/or by using the available superconducting wires with a high critical magnetic field (see, for example, [5]).

Parameters of the initial (state 1) and final (state 2) states of the transient processes can be illustrated by two numerical examples with a decrease (No. 1, $J_2 < J_1$) and an increase (No. 2, $J_2 > J_1$) in the current J (Table 1) which indicate the ability to manipulate all basic kinematic parameters of the neutrons that are listed in (15)–(21), by varying the current J.

5. Conclusions and brief discussion of some experimental situations

The list of the experimental tasks given below, does not contain any final solutions, but is useful from the point of view of some possible options for the application and development of this method of control of the neutron kinematics in the coaxial magnetic trap.

1. The change in the energy ε and the conventional temperature T^*, including the production of ultracold neutrons with $T_\varepsilon \sim 10^{-3}$ K (and even the hypothetically extreme cold neutrons with $T_\varepsilon \ll 10^{-3}$ K).

2. The change in the neutron density and energy density $n\varepsilon$.

3. Excitation of nuclear reactions by neutrons with optimised parameters – the energy ε, the concentration n, etc., for which the exposed nuclei are placed in a coaxial cylindrical layer with a finite radius r_{02} [for example, the reaction of radiative neutron capture (n, γ)].

4. Pumping of nuclear gamma-ray laser by neutrons with optimised parameters [both according to the Mössbauer scheme with a solid-state matrix and according to the scheme with hidden inversion of free nuclei [3], in particular, the radiation neutron capture (n, γ)] with the placement of the nuclei in an extended final coaxial layer in accordance with item 3.

5. Initiation of fission reaction of nuclei located in the final coaxial layer by neutrons with optimised parameters (the energy ε, the concentration n, etc.) in accordance with item 3.

6. Generation of a binary neutron–electron (ion) beam in a modified coaxial magnetic trap, where the central current-carrying conductor is replaced by a straight beam of free charged particles (e.g., electrons), which produces the necessary radial-gradient magnetic field of the form (1). The result of this modification is producing a mobile coaxial magnetic trap in the form of the unity of two coaxial beams of particles, moving in free space with a velocity equal to the lesser of the two transport velocities of the particles (it is important to emphasise an essential requirement for the inequality of the latter, which is needed to produce a magnetic field!). Such a moving coaxial magnetic trap (of course, if its long enough existence is established) offers the prospect of remote realisation of processes as listed in items 1–5.

In general, the above list indicates the attractive prospect of constructing a coaxial neutron magnetic trap with a variable current intensity as a tool for controlling kinematic parameters of neutrons (with a change by several orders of magnitude) in different areas of experimental physics.

In conclusion, it is worth mentioning that a coaxial magnetic trap [4] capable of varying the current significantly expands the range of its applications in atomic experiments, transforming it from a simple accumulator of atoms into the device controlling their kinematic parameters.

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