

ACOUSTO-OPTICS

PACS numbers: 42.65.Re; 42.79.Jq
DOI: 10.1070/QE2011v041n12ABEH014720

Effect of group velocity mismatch on acousto-optic interaction of ultrashort laser pulses

K.B. Yushkov, V.Ya. Molchanov

Abstract. **Equations describing acousto-optic diffraction of ultrashort laser pulses in an anisotropic medium are derived, taking into account the group velocity mismatch of optical eigenmodes. It is shown that the solution of the modified coupled-mode equations taking into account the group delay is characterised by an increase in the pulse duration, a decrease in diffraction efficiency, a change in the shape of the wave packet envelope, as well as by an increase in the width of the transmission function.**

Keywords: *femtosecond laser systems, dispersive delay line, acousto-optic diffraction.*

Ultrashort-pulse laser systems are one of the fastest growing trends in modern photonics [1]. Acousto-optic (AO) delay lines allow an increase in laser power by influencing the spectral amplitudes and phases of the laser pulses [2 – 4].

Until now, AO diffraction of ultrashort pulses (USPs) has been described using the theory of quasi-stationary electromagnetic fields [5, 6]. However, this approximation is fundamentally inapplicable to ultrashort pulses because of the difference in group velocities of optical eigenmodes with respect to one another. Effect of group delay on mode coupling manifests itself, in particular, in second harmonic generation in nonlinear crystals [7, 8]. In this paper, we describe the diffraction of ultrashort pulses using the modified coupled-mode equations that take into account the relative group delay of eigenmodes in the AO crystal.

Consider an optical wave in the form of a wave packet propagating along the z axis. We assume that anisotropic AO diffraction takes place. The electromagnetic field vector is a sum of two fields of eigenmodes with unit polarisation vectors $\boldsymbol{e}_p$ and slowly varying amplitudes $A_p(t,z)$ [1]:

$$\boldsymbol{E}(t,z) = \sum_{p=0,1} \boldsymbol{e}_p A_p(t,z) \exp[\mathrm{i}(\omega t - k_p z)], \tag{1}$$

where the subscript p denotes the number of the diffraction order and the wave numbers k_p are defined by the dispersion equation $k_p = n_p\omega/c$ (c is the speed of light in vacuum, n_p are the refractive indices of the crystal). The group velocities of the waves are found from the expression $u_p = (\partial k_p/\partial\omega)^{-1}$, whereas the relative delay between the modes is $v = u_0u_1/(u_0 - u_1)$. For definiteness, we assume $u_0 > u_1$, which corresponds to an incident fast mode and a diffracted slow mode.

K.B. Yushkov, V.Ya. Molchanov National University of Science and Technology 'MISIS', Acoustooptical Research Center, Leninsky prosp. 4, 119049 Moscow, Russia;
e-mail: konstantin.yushkov@ misis.ru

Received 14 September 2011; revision received 28 October 2011
Kvantovaya Elektronika **41** (12) 1119–1120 (2011)
Translated by I.A. Ulitkin

The electromagnetic field obeys the wave equation

$$\frac{\partial^2 \boldsymbol{E}}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \hat{\varepsilon}(t,z)\boldsymbol{E}}{\partial t^2} = 0, \tag{2}$$

in which the tensor of the dielectric constant, $\hat{\varepsilon}(\omega;t,z) = \hat{\varepsilon}_{\mathrm{m}}(\omega) + \Delta\hat{\varepsilon}_{\mathrm{ac}}(t,z)$, corresponds to a homogeneous transparent medium with dispersion. Substituting expression (1) in equation (2) allows us to derive a generalised system of equations for the envelopes of wave packets, $A_p(t,z)$, in the presence of the first-order dispersion in a crystal with a periodic modulation of the refractive indices:

$$\begin{aligned} &\left(\frac{\partial}{\partial z} + \frac{1}{u_0}\frac{\partial}{\partial t}\right)A_0 = \frac{q}{2}\exp(-\mathrm{i}\Delta k z)A_1, \\ &\left(\frac{\partial}{\partial z} + \frac{1}{u_1}\frac{\partial}{\partial t}\right)A_1 = -\frac{q}{2}\exp(\mathrm{i}\Delta k z)A_0, \end{aligned} \tag{3}$$

where q is the coefficient of AO coupling, proportional to the perturbation amplitude $\Delta\hat{\varepsilon}_{\mathrm{ac}}$; Δk is the phase mismatch [5]. The initial conditions are set at $z = 0$, and the boundary of the interaction region corresponds to $z = L$.

The system of equations (3) is conveniently solved independently for each of the unknown functions A_0 and A_1; each of the partial differential second-order equations is considered in the accompanying reference frame: $\zeta_{\mathrm{p}}=(-1)^p z/L$, $\eta_p = (t - z/u_p)|v|/L$. The equations for the envelope of zero and first diffraction orders have the form

$$\frac{\partial^2 A_p}{\partial\zeta_p^2} + \frac{\partial^2 A_p}{\partial\eta_p\partial\zeta_p} + \mathrm{i}\pi H\frac{\partial A_p}{\partial\zeta_p} + \frac{\pi^2}{4}Q^2 A_p = 0, \tag{4}$$

where the dimensionless coefficients $Q = qL/\pi$ and $H = \Delta kL/\pi$ are introduced. In this case, the initial conditions at $\zeta = 0$ differ for the incident wave $[A_0(\eta_0, \zeta_0 = 0) = A_{\mathrm{in}}(\eta_0), \partial A_0/\partial\zeta_0 = 0)]$ and for the diffracted wave $[A_1(\eta_1,\zeta_1 = 0) = 0, \partial A_1/\partial\zeta_1 = -\frac{1}{2}\pi Q A_{\mathrm{in}}(\eta_1)]$.

In equation (4), the function $A_{\mathrm{in}}(\eta) = \mathrm{sech}(\eta/\eta_{\mathrm{in}})$ is selected as an initial form of the wave packet. In this function η_{i} characterises the duration of the USP at $\zeta_0 = 0$. The interaction region is limited by the interval $0 \le \zeta_p \le 1$ along the coordinate axes; the time interval $|\eta_p| \le 1.5$ provides zero boundary conditions for the functions $A_p(\eta_p, \zeta_p)$ and their derivatives $\partial A_p(\eta_p, \zeta_p)/\partial\eta_p$.

To solve equation (4), use was made of the method of finite differences. The initial pulse duration η_{in} was 0.2, which ensured the fulfilment of the boundary conditions and the group delay. Figure 1 presents the solution for the zero (a)

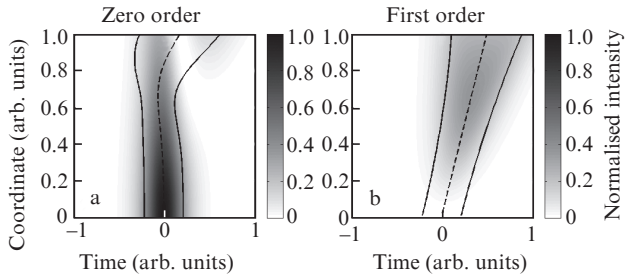

Figure 1. Temporal distribution of the intensity of the incident (a) and diffracted (b) pulses under acousto-optic diffraction.

and first (b) diffraction orders. Also, Fig. 1 shows the position of the pulse centre along the η_p axis (dashed line) and the boundaries of the pulse (solid lines), determined statistically [1].

As can be seen from the presented data, there is a substantial increase in the duration of ultrashort pulses both in the first and zero diffraction orders. An increase in the duration is due to the group delay between the interacting pulses. In our case, the group delay leads to a violation of spatiotemporal invariance of forward and backward scattering, which indicates different diffraction efficiencies of leading and trailing edges of the incident pulse. As a result, the group velocity of diffracted pulse propagation as well as the envelope shape changes. A similar nonreciprocity of the spatiotemporal relationship between the forward and backward scattering of light in a strong acoustic field was observed in [9].

These results demonstrate a significant reduction in the diffraction efficiency under the phase-matching condition ($H = 0$). Analysis of these data shows that the peak intensity of the diffracted beam is reached when $|\zeta_p| = 0.67$ and its relative value is 0.37. The maximum pulse energy of the first diffraction order is reached at $|\zeta_p| = 0.86$ and is 0.65 of the incident pulse energy.

The applicability of the theory in real conditions is limited, on the one hand, by the use of the first order approximation of the dispersion theory, limiting the width of the emission spectrum, and on the other hand – by the condition of short duration of the optical pulse compared with the delay time between the eigenmodes. Let us give quantitative estimates of the impact of these factors. The second-order dispersion can be neglected if the spectral width of radiation, $\Delta\omega$, satisfies the inequality $L\Delta\omega^2|\gamma| \ll 1$, where γ is the second-order dispersion coefficient. In this case, the quasi-static approximation is not applicable if the pulse duration is considerably smaller than the group delay that is equal to $L/|v|$. Numerical estimates show that ultra-short pulses with a central wavelength of 800 nm, typical of Ti : sapphire lasers, in a 60-mm-long paratellurite crystal are diffracted, the theory is applicable in the case when the width of the emission spectrum lies in the range from 0.1 to 1 nm, which corresponds to a transform-limited-pulse duration of 1 – 10 ps.

In the presence of the phase mismatch, the dependence of the diffraction efficiency on H determines the transmission function of the AO cell. The results of calculations of the normalised transmission function are shown in Fig. 2 in comparison with the conventional transmission function of the Bragg diffraction with the 100% efficiency. Group delay leads to broadening of the transmission function and disappearance of side lobes. Diffraction efficiency T is defined as the ratio of the pulse energy in the first diffraction order to the energy of the initial ultrashort pulse.

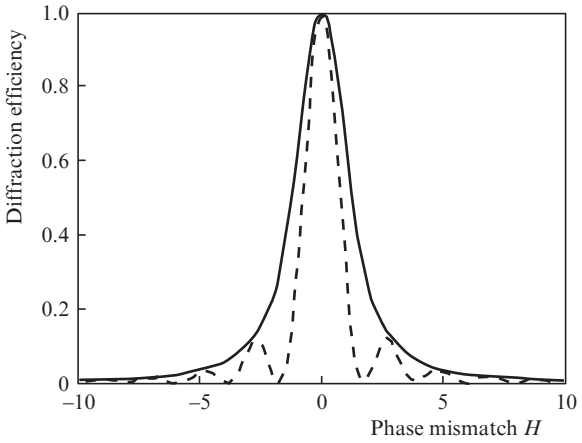

Figure 2. Normalised transmission function of the acousto-optic cell for ultrashort pulses (solid line); dashed line shows the transmission function of the cell for quasi-stationary electromagnetic waves.

These results indicate that the difference between the group velocities of the incident and diffracted waves in the case of AO interaction of ultrashort pulses in crystals leads to a qualitative change in the form of the solution to coupled-mode equations. As a result, the pulse duration increases, the diffraction efficiency decreases, and the pulse shape is markedly distorted. These phenomena are important in the development of AO devices controlling the parameters of ultrashort pulses.

Acknowledgements. This work was supported by the Program of Creation and Development of NUST 'MISIS' and by the Russian Foundation for Basic Research (Grant No. 09-07-00 365-a).

References

1. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optics of Femtosecond Laser Pulses* (New York: AIP, 1991; Moscow: Nauka, 1988).
2. Pozhar V.E., Pustoboit V.I. *Kvantovaya Elektron.*, **14**, 811 (1987) [*Sov. J. Quantum Electron.*, **17**, 509 (1987)].
3. Tournois P. *Opt. Commun.*, **140**, 245 (1997) .
4. Molchanov V.Ya., Chizhikov S.I., Makarov O.Yu., et al. *Appl. Opt.*, **48**, C118 (2009).
5. Parygin V.N., Chirkov L.E. *Kvantovaya Elektron.*, **2**, 318 (1975) [*Sov. J. Quantum Electron.*, **5**, 180 (1975)].
6. Verluise F., Laude V., Huignard J.-P., et al. *J. Opt. Soc. Am. B*, **17**, 138 (2000).
7. Sidick E., Knoesen A., Dienes A. *J. Opt. Soc. Am. B*, **12**, 1704 (1995).
8. Akhmanov S.A., Sukhorukov A.P., Chirkin A.S. *Zh. Eksp. Teor. Fiz.*, **55**, 1440 (1968).
9. Magdich L.N., Molchanov V.Ya. *Zh. Eksp. Teor. Fiz.*, **47**, 1068 (1977).