Quantum Electronics **42** (2) 181–184 (2012)

CRYSTAL-LIKE STRUCTURES

PACS numbers: 14.20.Dh; 61.44.–n
DOI: 10.1070/QE2012v042n02ABEH014593

On associations of noninteracting particles (crystal-like neutron structures)

L.A. Rivlin

Abstract. **We discuss the physical feasibility of association of particles noninteracting with each other, which arises in accordance with the uncertainty relation under the 'corporate' spatial confinement of the particle ensemble as a whole. Investigation is conducted by the example of an ensemble of ultracold neutrons placed in a common potential well of infinite depth. We present quantitative estimates and indicate the expected properties of the arising crystal-like spatially periodic structures.**

Keywords: *quantum nucleonics, ultracold neutrons, laser methods of production of ultracold neutrons, neutron associations, and neutrons in the potential well of infinite depth.*

1. Introduction

Association of particles noninteracting with each other is usually assumed unrealisable. Formation of atomic associations – from the simplest hydrogen molecule H_2 to complex crystals – is accompanied by the indispensable interaction of neighbouring atoms, their electron 'collectivization', emergence of energy bands, etc. At the nuclear level the association in the form of a bound state of two neutrons, a so-called bineutron [1, 2], is also unrealisable. The attempt to build something like a stable planetary system of two neutrons orbiting around their common centre of mass along an orbit, where the magnetic-dipole attractive force of neutrons is equal to the centrifugal force, is also doomed to failure. Thus, to form neutron associations, neither short-range intranuclear forces nor long-range forces responsible for the existence of conventional atomic–molecular structures are apparently sufficient*.

The aim of this paper is to discuss, by the example of ultracold neutrons (UCNs), the possibility of forced association of particles noninteracting with each other (or rather, negligibly weakly interacting – see the negligibility estimate in Section 3), the existence of such associations being caused only by a common extraneous force action on the entire ensemble of particles.

Such a 'corporate' situation arises, for example, by placing many identical particles in a common potential well. In this case, retention of particles and their spatial arrangement is, in essence, a direct consequence of the Heisenberg uncertainty relation $\Delta p \Delta z \approx \hbar$ for the momentum p and coordinates z of the particle, just as 'the resistance to atomic compression is a quantum-mechanical effect and not a classical effect' [3]. In fact, the corresponding ground state energy of the particle $E = p^2/2M = \hbar^2/[2M(\Delta z)^2]$ with mass M is equal to the work produced by force $F = -\mathrm{d}E/\mathrm{d}z = -\hbar^2/M(\Delta z)^3 = -2E/\Delta z$ to compress initially unrestrictedly distributed Ψ-function of particles ($-\infty < \Delta z < \infty$) in a limited interval z, as shown by direct quantum-mechanical calculations [4] (a similar calculation for a photon is given in [5]). It is the 'Heisenberg' force F that keeps the particle within a spatial interval Δz. Besides, when there are several particles in a common potential well, the 'Heisenberg' force determines their spatial arrangement.

2. Model of a crystal-like structure of noninteracting neutrons

Construction of a simple potential well for neutrons is based on the ability of UCNs with temperature $T \approx 10^{-3}$ K and de Broglie wavelength $\Lambda_{\mathrm{dB}} \approx 10^{-6}$ cm to be reflected from the surface of a condensed substance according to the laws of electromagnetic optics [6–8], which allows one to store UCNs in macroscopic 3D potential wells of infinite depth, similar to hollow electromagnetic microwave resonators (see, for example, [9]). The walls of the potential well with zero boundary conditions exert common ('corporate') influence on the neutron ensemble as a whole at its borders and serve as the source of the extraneous force, which is responsible for the existence of a neutron crystal-like periodic structure (CLPS).

The solutions of the Schrödinger equation in such 'resonators' form spatially ordered probability density functions, which creates prerequisites for the formation of spatially periodic associations consisting exclusively of neutrons noninteracting with each other. Since the purpose of this consideration is merely to illustrate the physical feasibility of neutron CLPSs mentioned in Section 1, it is sufficient, without going into excessive mathematical generality, to restrict ourselves here to a simple model of the potential well in the form of an elongated hollow parallelepiped of square cross section with side a and length $L \gg a$; in this case,

$$2^{-1/2}\Lambda_{\mathrm{dB}} < a < (4/5)^{-1/2}\Lambda_{\mathrm{dB}}, \tag{1}$$

*By the way, if we place a neutron in the periodic table and denote it by $^1_0$Nt, then it will take the place of the zero term in the row of nuclei before hydrogen (proton) $^1_1$H. Then the nonexistent bineutron and neutron 'planet' should be considered respectively as a heavy isotope $^2_0$Nt and a neutron molecule Nt_2 – an analogue of a hydrogen molecules H_2.

L.A. Rivlin Applied Physics Laboratory, Moscow State Institute of Radioengineering, Electronics and Automation (MSTU – MIREA), prosp. Vernadskogo 78, 119454 Moscow, Russia;
e-mail: lev.rivlin@gmail.com

Received 27 March 2011; revision received 6 December 2011
Kvantovaya Elektronika **42** (2) 181–184 (2012)
Translated by I.A. Ulitkin

From the distribution of the probability density in such a well

$$|\Psi(x,y,z)|^2 = A\cos^2(\pi x/a)\cos^2(\pi y/a)$$

$$\times[\sin^2(2\pi z/\Lambda_{\rm dB})(1-\Lambda_{\rm dB}^2/2a^2)^{1/2}] \quad (2)$$

(the lowest transverse mode with $-a/2 \leqslant x \leqslant a/2$, $-a/2 \leqslant y \leqslant a/2$ and $0 \leqslant z \leqslant L$) it follows that under the resonance condition

$$L = \frac{N}{2\Lambda_{\rm dB}}\left(1-\frac{\Lambda_{\rm dB}^2}{2a^2}\right)^{-1/2} \quad (3)$$

all the space inside the well is divided into a line of $N \gg 1$ cells with dimensions $a\times a\times(\Lambda_{\rm dB}/2)\left[1-\frac{1}{2}(\Lambda_{\rm dB}/a)^2\right]^{-1/2}$, with zero probability density $|\Psi_k|^2 = 0$ at the boundaries of each kth cell ($k \leqslant N$) and with the maximum $|\Psi_k|^2_{\max}$ at its midpoint, the presence of a neutron being equally probable throughout the line of cells because of the identity of $|\Psi_k|^2$ in each of them.

The constant A in (2) is determined by the total number $N_{\rm Nt}$ of neutrons in the well: $A = (N_{\rm Nt}/N)(16/a^2\Lambda_{\rm dB})\times(1-\Lambda_{\rm dB}^2/2a^2)^{1/2}$. Therefore, if we fill the well with the neurons with $N_{\rm Nt} \sim N$, then their equal distribution over the cells will be most likely, which in a spatially structural relation can be likened to a one-dimensional neutron crystal (1D-CLPS). It is important to emphasise once again that the formation of such a crystal is caused by a peripheral effect of the walls of a well of infinite depth with zero boundary conditions rather than by the interaction between the constituent particles of the crystal. So in a sense, a probabilistic character should be attributed to this crystal-like structure, since even at $N_{\rm Nt} = N$ there is a finite probability of distributions with a few neutrons in the same cell and simultaneously with empty cells, which is quite similar to defects in an ordinary crystal. This analogy is even more obvious when $N_{\rm Nt} \neq N$. At the same time, it is worth noting that the laser methods of production of UCNs with a high concentration [10] eliminate the restrictions on the use of sufficient quantities of $N_{\rm Nt}$.

3. Evaluation of the CLPS description adequacy by the model of noninteracting neutrons

The model of noninteracting neutrons is only valid as an approximation used to generally establish the CLPS, because in reality neutrons with a finite magnetic dipole moment $\mu = -0.95\times10^{-24}$ erg G^{-1} interact with each other. Therefore, the concept of noninteracting particles and adequacy of the accepted model require evaluation.

The magnetic dipole–dipole binding energy ε_μ of the isolated pair of neighbouring neutrons with magnetic moments collinearly oriented along the z axis, which are located at the maxima of the neighbouring cells $|\Psi_k|^2_{\max}$, is as follows:

$$\varepsilon_\mu = \frac{48\mu^2}{\Lambda_{\rm dB}^3}\left(1-\frac{\Lambda_{\rm dB}^2}{2a^2}\right)^{3/2} \quad (4)$$

(collinearity of a freely orientable dipoles establishes spontaneously, because in this case the binding energy ε_μ is maximal).

The attraction of neutrons of this pair by the dipole–dipole force that can disrupt the crystal-like structure with the distribution $|\Psi_k(z)|^2$ (2) is prevented by the forces of corporate interaction with the walls of the well with zero boundary conditions. This counteraction of a purely quantum-mechanical nature is characterised by an energy

$$\varepsilon_\Psi = \frac{8\pi^2\hbar^2}{M\Lambda_{\rm dB}^2} \quad (5)$$

(M is the mass of the neutron), which is exactly equal to the work expended on the concentration of the Ψ-function of a neutron in a limited volume of the cell [4], i.e., as mentioned above, is essentially a consequence of the uncertainty relation.

The ratio of the characteristic energies of ε_Ψ (5) and ε_μ (4) is as follows:

$$\frac{\varepsilon_\Psi}{\varepsilon_\mu} = \frac{\pi^2\hbar^2}{6M\mu^2}\Lambda_{\rm dB}\left[1-\frac{1}{2}\left(\frac{\Lambda_{\rm dB}}{a}\right)^2\right]^{-3/2}. \quad (6)$$

In view of the limiting inequality (1), the value of the expression in square brackets (6) lies within

$$2.16 < \left[1-\frac{1}{2}\left(\frac{\Lambda_{\rm dB}}{a}\right)^2\right]^{-3/2} < \infty, \quad (7)$$

and accordingly

$$2.16\frac{\pi^2\hbar^2}{6M\mu^2}\Lambda_{\rm dB} < \frac{\varepsilon_\Psi}{\varepsilon_\mu} < \infty. \quad (8)$$

Thus,

$$\frac{\varepsilon_\Psi}{\varepsilon_\mu} > 2\times10^{15}\Lambda_{\rm dB}, \quad (9)$$

i.e., the energy ε_Ψ exceeds the magnetic dipole–dipole binding energy ε_μ by many orders of magnitude for neutrons of any energies [from extremely cold ($\Lambda_{\rm dB} \sim 10^{-5}$ cm) to relativistic ($\Lambda_{\rm dB} \sim 10^{-14}$ cm)], and the magnetic dipole–dipole interaction cannot be an obstacle to the existence of CLPSs.

The energy of the magnetic dipole–dipole interaction decreases rapidly (as the cube of the distance), which makes it possible, in view of the very strong estimate (9), to neglect the interaction of more distant cells with the neutrons.

In addition to this it should be noted that the β^--radioactivity of the neutron with the inverse decay constant $\tau \approx 1300$ s requires a continuous extraneous replenishment of the numbers of neutrons in order to maintain the stationarity of the CLPS.

All the above indicates the adequacy of the model of noninteracting neutrons for establishing a stable CLPS and simultaneously points to the need to take into account the weak magnetic dipole–dipole interaction of neutrons when studying the properties of the CSPS that is already produced.

4. Expected properties of a neutron CLPS

An important feature of a CLPS is its resonant character: for a given geometry of the potential well the CLPS is feasible only for resonant neutrons with de Broglie wavelength $\Lambda_{\rm dB}$, satisfying equation (3).

Below are listed the expected properties of a neutron CLPS, which are subject to further theoretical and experimental studies.

(1) *A two- and three-dimensional CLPS*. The apparent possibility of transition from the 1D model to 2D and 3D struc-

tures is feasible when the right side of (2) is simply neglected. However, the simplicity and clarity of the one-dimensional model are lost in this case because of the probability of excitation of higher-order transverse modes.

We should also bear in mind the possibility of using potential wells for formation of a CLPS, which differ in nature and configuration from a rectangular parallelepiped with mirror reflection walls. In this case, formation of CLPSs not only of UCNs becomes promising.

(2) *Natural radioactivity of the neutron and the coherence length of de Broglie waves*. Boundedness of the neutron lifetime, caused by its β^--radioactivity, is the fundamental reason of nonmonokineticity of a neutron ensemble with relative dispersion

$$\frac{\Delta\Lambda_{\rm dB}}{\Lambda_{\rm dB}} \approx \frac{2\pi\hbar}{k_{\rm B}T\tau}, \tag{10}$$

and also determines the coherence length of the de Broglie wave propagating in a potential well along the z axis,

$$L_{\rm coh} \approx \frac{2\pi\hbar\tau}{M\Lambda_{\rm dB}}\left(1-\frac{\Lambda_{\rm dB}^2}{2a^2}\right)^{1/2}. \tag{11}$$

This sets a limit to the CLPS length:

$$L < \frac{L_{\rm coh}}{2} \approx \frac{\pi\hbar\tau}{M\Lambda_{\rm dB}}\left(1-\frac{\Lambda_{\rm dB}^2}{2a^2}\right)^{1/2}. \tag{12}$$

Inequality (12), which reduces to a simple evaluative rule $L \sim \Lambda_{\rm dB}^{-1}$ (numerical values in centimetres), in fact, does not impose any real limitations on UCNs with $\Lambda_{\rm dB} \approx 10^{-6}$ cm. However, other unaccounted sources of nonmonokineticity of the neutron ensemble (including technological reduction of the neutron lifetime, the thermal velocity dispersion, etc.), which limit the coherence length, may play a significant role.

(3) *Phonon field of a CLPS*. The existence of a finite energy value ε_Ψ (5) and inequality (9) indicates the rise of a restoring force when a neutron is displaced from its equilibrium position in the midpoint of the cell. This force, proportional in the first approximation to the displacement, can assign elastic properties with a finite Young's modulus to the CLPS and admit the possibility of propagation of longitudinal and transverse sound waves, described in the framework of the known phonon concept.

(4) *Magnetic properties of a CLPS*. The probability density function $|\Psi(x,e,z)|^2$ (2) fixes the spatial position of the neutron magnetic dipoles in the CLPS, but leaves the orientation of the vectors of their moments uncertain. The longitudinal extraneous magnetic field removes this uncertainty by arranging all the partial moments collinearly to the field. As a result, there arises a macroscopic magnetic dipole with a total moment $\boldsymbol{P} = \boldsymbol{\mu}N_{\rm Nt}$ and hence with a macroscopic total spin $|\boldsymbol{S}| = N_{\rm Nt}/2$. In the case of collinear orientation, the energy of the dipole–dipole interaction reaches a maximum not only for a pair of neighbouring neutrons, but also for the entire macroscopic dipole produced. Therefore, the latter persists even after switching off an extraneous forcing magnetic field. Moreover, the strong inequality (9) admits the spontaneous rise of clusters with collinearly oriented partial magnetic moments without preliminary forcing by the magnetic field, as well as the further joining of individual clusters in a common macroscopic magnetic dipole with moment $\boldsymbol{P}$ and spin $\boldsymbol{S}$. These phenomena indicate a possible analogy with the behaviour of ferromagnetics. Precession of neutron dipoles in an extraneous magnetic field and precession waves, covering the entire CLPS, as well as possible emergence of conditions for manifestation of diamagnetic and paramagnetic properties in such a structure deserve separate consideration.

In forming a macroscopic total spin $\boldsymbol{S}$ of the CLPS, we may find the statistical properties of the latter (it obeys the Bose statistics for even $N_{\rm Nt}$ and the Fermi statistics for odd $N_{\rm Nt}$), which can manifest themselves, for example, in suspensions of CLPSs. The role of the CLPS topology is quite curious: if a rectilinear CLPS with a length sufficient to preserve the Ψ-mode ($L \gg \Lambda_{\rm dB}$) forms a ring by closing the ends with $z = 0$ and $z = L$, then the allowed $N_{\rm Nt}$ is inevitably always even, $\boldsymbol{P} = 0$ and $\boldsymbol{S} = 0$, and the statistics belongs to the Bose type (cf. [11]).

(5) *Heat exchange between neutrons and the walls of the potential well*. Upon reflection of UCNs with the temperature of the order of millikelvin from the walls of the well with a significantly higher temperature, we can expect the unwanted neutron heating. This is hindered by the elastic nature of collisions of neutrons with $\Lambda_{\rm dB} \sim 10^{-6}$ cm participating in every collision act with $\sim 10^5$ surface atoms of the solid wall with a total mass exceeding the mass of the neutron by $\sim 10^7$ times.

(6) *Propagation of electromagnetic waves in a CLPS*. The interaction of the magnetic wave vector with the neutron magnetic moments of the CLPS determines (in the range of photon energies of the order of 100 eV) the arising phenomena characteristic of wave propagation in any periodic structures (Brillouin energy bands, Bragg reflection, high-Q resonances, transmission of two-dimensional images over waveguide modes [12], etc.).

5. Conclusions

The discussion performed allowed us to establish the physical feasibility of stable macroscopic neutron associations (assumed noninteracting in a first approximation) under the collective extraneous ('corporate') forced influence of quantum-mechanical nature on the entire ensemble of neutrons as a whole. Such a 'corporate' influence is feasible, for example, by placing the entire ensemble of neutrons in the common potential well of sufficient depth. In this case, the cause of the 'corporate' impact is essentially the uncertainty relation.

The expected properties of one of the representatives of the neutron associations – a macroscopic CLPS, consisting of a set of ultracold neutrons in a common infinitely deep potential well with mirror reflecting walls, are similar to the known properties of conventional crystals and other spatially periodic structures. In particular, a CLPS with two neutrons can be treated in accordance with Section 1 as a neutron molecule $\rm Nt_2$ of macroscopic size of the order of 10^{-6} cm (but not as a heavy isotope of neutron $^2_0\rm Nt$, i.e., a bineutron).

Hypothetical properties of macroscopic neutron CLPSs not only encourage further research, but also tempt to invent exotic structural materials and devices on their basis. In this case, the loss of neutrons due to their inevitable decay should be compensated for by injecting neutrons by an extraneous source (just as the cathode emission adds to the number of electrons in vacuum electronic devices). The resulting protons and electrons must be removed by applying a weak electric field.

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