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Measurement of the mechanical deformations of an elastic spherical shell, filled with an incompressible fluid, with the help of a semiconductor laser autodyne

D.A. Usanov, An.V. Skripal', S.Yu. Dobdin

Abstract. **We report the possibility of measuring mechanical deformations of an elastic spherical shell filled with an incompressible fluid under the action of an air pulse. The value of the deformation was determined by the semiconductor laser autodyne signal using a wavelet transform. It is established that it correlates with the internal pressure.**

Keywords: *spherical shell, internal pressure, semiconductor laser, autodyne signal.*

Study of mechanical deformations of bodies, whose shells have a spherical shape, refers to the classical problems of the theory of shells, the solution of which is usually limited to a theoretical calculation using numerical methods [1, 2]. An example of a spherical object, the elastic properties of which are important to know, is the eyeball.

The authors of [3] present the results of the study of a scleral capsule of the eye with the help of an ophthalmic mechanograph designed for obtaining *in vivo* dependences of the strain on deformation. The values of the mechanical parameters of the sclera can be used to determine the intraocular pressure and biomechanical characteristics of the shells. Measuring intraocular pressure usually requires employment of direct contact between the eyeball and mechanical load, which necessitates anaesthetization of the patient's eye. In the well-known non-invasive methods used for measuring intraocular pressure, an air pulse is used as an external load. In this case, for contactless characterization of mechanical deformations, including in their dynamics, promising is the application of the method based on the use of a semiconductor laser autodyne. This method allows one to measure the magnitude of deformations not only at some point but also on the entire plane of the deformed surface.

The aim of this paper is to study experimentally, using a semiconductor laser autodyne, mechanical properties of a soft spherical shell filled with an incompressible fluid at different internal pressures.

An important parameter for studying the elastic properties of the shell is the value of deflection under external loading. Knowing this value will make it possible to determine the mechanical properties of the shell and the internal pressure. Deflection of a spherical shell is determined by the signal of a semiconductor laser autodyne. The normalised variable component of the autodyne signal can be represented as

$$P(t) = \cos\left[\theta + \frac{4\pi}{\lambda_0} Z(t)\right], \tag{1}$$

where θ is the phase incursion of the autodyne signal; λ_0 is the laser wavelength; t is the time interval of observation of the autodyne signal in different parts of the motion of the shell surface; $Z(t)$ is the function describing the longitudinal movement of the object.

To reconstruct the function $Z(t)$ from a semiconductor laser autodyne signal $P(t)$, advantage can be taken of the method presented in [4], where the function of the object motion $Z(t)$ is reconstructed by using the wavelet transform. In this case, the function $Z(t)$ can be written in the form

$$Z(t) = K_{\psi_1}^{-1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(a,b) \frac{1}{\sqrt{a}} \psi_1\left(\frac{t-b}{a}\right) \frac{\mathrm{d}a\mathrm{d}b}{a^2}, \tag{2}$$

where ψ_1 is the basic wavelet function, which is a derivative of the Gaussian function $[G(t) = \mathrm{e}^{-t^2/2}]$; $C(a,b)$ are the coefficients of the wavelet expansion of the function $Z(t)$ over the basis ψ_1; a is the scaling coefficient; b is the transfer coefficient;

$$K_{\psi_1} = 2\pi \int_{-\infty}^{\infty} \frac{|\psi_F(\omega)|^2}{|\omega|} \mathrm{d}\omega$$

is a constant defined by the basic wavelet function; ψ_F is the Fourier transform of the wavelet function ψ_1; ω is the frequency in the spectrum of the wavelet function ψ_1. Let us introduce into consideration the function $S(t)$, such that its wavelet spectrum to within a constant factor corresponded to the spectrum of the recovered signal (1):

$$S(t) = \frac{\mathrm{d}P(t)/\mathrm{d}t}{\pm\sqrt{1 - P^2(t)}}. \tag{3}$$

Taking into account the expression for the normalised component of the interference signal (1) and the function that characterises the longitudinal motion of the object (2), we obtain

$$S(t) = \frac{4\pi}{\lambda K_{\psi_1}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(a,b) \frac{1}{\sqrt{a}} \psi_2\left(\frac{t-b}{a}\right) \frac{\mathrm{d}a\mathrm{d}b}{a^2}, \tag{4}$$

$$C(a,b) = \int_{-\infty}^{\infty} \frac{\lambda}{4\pi} S(t) \frac{1}{\sqrt{a}} \psi_2\left(\frac{t-b}{a}\right) \mathrm{d}t, \tag{5}$$

where ψ_2 is the basic wavelet function, which is the second derivative of the Gaussian function.

D.A. Usanov, An.V. Skripal', S.Yu. Dobdin N.G. Chernyshevsky Saratov State University, ul. Astrakhanskaya 83, 410012 Saratov, Russia; e-mail: UsanovDA@info.sgu.ru

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Comparing the integral representations of the functions $Z(t)$ and $S(t)$, we see that they differ by basic wavelet functions and a constant factor $4\pi/\lambda$, but have the same coefficients of the wavelet expansion $C(a,b)$. Therefore, the expansion coefficients (5) resulting from the inverse wavelet transform can be used to reconstruct the function of object motion $Z(t)$, by substituting them into (2).

Thus, the solution to the problem of finding $Z(t)$ (inverse problem) will include the measurement of the autodyne signal $P(t)$, numerical calculation of its derivative $\mathrm{d}P(t)/\mathrm{d}t$, construction of the function $S(t)$ with the help of relation (3), derivation of the coefficients of the wavelet expansion $C(a,b)$ of the function $S(t)$, and then calculation of the function $Z(t)$, describing the longitudinal motion of the object, by using formula (2). The reconstructed function of the object motion $Z(t)$ can be used to determine the deflection at each point of a spherical shell.

Experiments were performed to measure mechanical deformations of spherical shells under the action of an air pulse at different internal pressures. The object of studies was a rubber ball with a diameter of 24 mm, filled with a gel with a density that is close to the density of intraocular fluid. Inside the ball, which can be treated as a model of the eyeball, we introduced a tube, the free end of which was coupled to an Y-shaped rubber hose. The free ends of the hose were attached to a blower and pressure gauge.

The ball's region to be analysed was exposed to 1-Hz air pulses from a membrane compressor operating at a power of 2 W and pressure of 0.01 MPa. The photograph of the experimental setup is shown in Fig. 1. Radiation of a semiconductor laser (*1*) pumped by a stabilised current source (*2*) was directed to the object (*3*), which was exposed to air pulses of the compressor (*4*). The object under study was fixed with the help of a special mechanism (*5*). Pressure inside the object was changed with a rubber bulb (*6*) and measured with a manometer (*7*). Part of the radiation reflected from the object returned to the resonator of the semiconductor laser; the change in the laser output power was recorded by an integrated photodetector. The signal from the photodetector was fed through an amplifier to the ADC (*8*), and then digitally stored in computer memory (*9*), where it remained for further analysis in the mathematical package MathCad.

In the experiment air pulses of the compressor were directed through a flexible hose and plastic tube to the surface of one of the objects, which created pressure on the shell from the outside. All experiments were performed using identical air pulses. The motion of the object's surface led to a change in the autodyne signal of the semiconductor laser. The value of the deflection in this case was determined by the autodyne signal using the method described above. To find the maximum deflection, the semiconductor laser, mounted on a translation stage, was moved by a microscrew (with a step of 1 mm). Such an approach made it possible to reconstruct the profile of the deformable surface.

Before reconstructing the function of motion $Z(t)$ we eliminated high-frequency components by smoothing the experimental curve with the help of the built-in function supsmooth of the mathematical package MathCad. Figure 2 shows the reconstructed profiles of the object's surface exposed to an air pulse for samples with different internal pressures. The pressure was measured with a manometer.

One can see from Figure 2 that the depth of deflection for the samples with different internal pressures is different. The measurement of deflection at various points on the surface allowed one to reconstruct its relief and to determine local features under the action of an air pulse. It follows from the measurements that the depth of deflection of the spherical shell correlates with the pressure inside the object. Therefore,

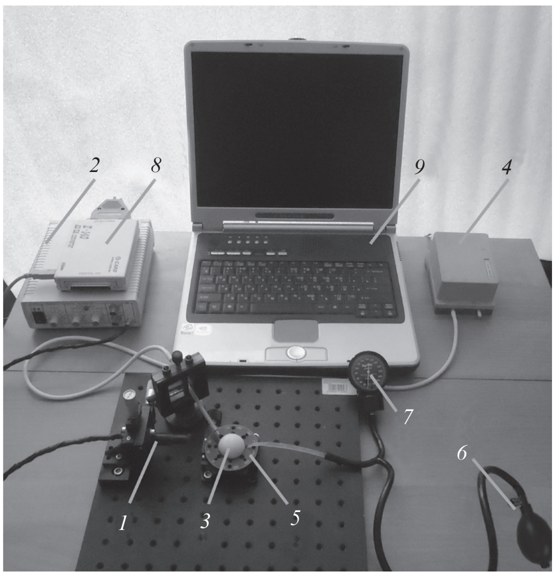

Figure 1. Photograph of the experimental setup: (*1*) semiconductor laser; (*2*) current source; (*3*) object; (*4*) compressor; (*5*) object lock mechanism; (*6*) rubber bulb; (*7*) gauge; (*8*) ADC; (*9*) computer.

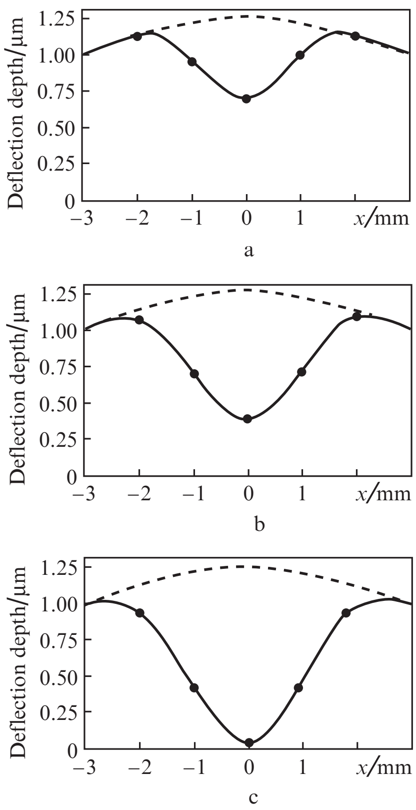

Figure 2. Profiles of the surface of the object with an internal pressure of 40 (a), 30 (b) and 20 Torr (c) reconstructed under the action of an air pulse. Dashed curves show the initial position of the shell.

the unknown pressure can be determined from the deflection of the shell itself.

Description of the behaviour of a spherical shell under the load can be used to obtain the biomechanical properties of biological objects *in vivo*. Investigation of the biomechanical characteristics of the eye (sclera), allows one to diagnose different disorders of its biomechanical status. Using a semiconductor laser autodyne increases the accuracy of determining the region and the magnitude of deflection as compared to the accuracy ensured by other known methods, whereas the lack of direct contact between the eye and a pressure gauge eliminates the need for anesthesia.

Thus, this study examined the possibility of using a semiconductor laser autodyne to measure mechanical deformations of spherical shells of objects with an internal pressure arising under the action of an air pulse. The use of a laser system allows the deflection at each point of the spherical shell to be determined with a high accuracy. It is shown that the value of deflection depends on the internal pressure. This dependence can be used, in particular, for contactless *in vivo* measurement of intraocular pressure.

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