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CONTROL OF LASER RADIATION PARAMETERS

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Measuring the shift of a femtosecond laser frequency comb by the interference method

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Abstract. **We have demonstrated the possibility of measuring the femtosecond laser frequency comb shift by the position of a Fabry–Perot interferometer's transmission bands with a statistical error of 10^{-2} and a systematic shift of 10^{-1}.**

Keywords: *femtosecond laser frequency comb, interferometer, transmission band.*

1. Introduction

Use of a femtosecond laser significantly simplified and improved the accuracy of measuring optical frequencies due to the broadening of the emission spectrum of a self-mode-locked laser generating octave-spanning frequency combs. Mixing low- and high-frequency wings of the spectrum made it possible to determine the shift of the frequency comb on the absolute frequency scale, to create a so-called *f–2f*-interferometer [1–3]. If the intermode frequency of the comb is locked to the frequency of the microwave standard and the comb shift is measured relative to the zero frequency, then the entire range of values of frequency components of a femtosecond laser is known with relative accuracy, defined by the frequency standard.

The use of a system, consisting of a frequency standard, a femtosecond synthesiser and an interferometer, allows one to create a uniform standard of time, frequency and length [4, 5]. At the same time, the authors of paper [6] demonstrated the possibility of measuring the carrier envelope offset frequency of the femtosecond laser frequency comb with a spectral bandwidth of less than an octave by using the interferometer. However, the problem of possible random and systematic errors due to various factors remained open.

In this paper we examine the effect of dispersion and misalignment of the interferometer mirrors on the shape and shift of the transmission bands of a femtosecond laser.

2. Main properties of radiation of a femtosecond laser and a Fabry–Perot interferometer

Radiation of a self-mode-locked laser can be represented in the form of electromagnetic waves at a frequency ω_0, close to the frequency corresponding to the maximum of the gain line, with an amplitude modulation whose period is equal to the round-trip transit time T for radiation in the cavity (for the time representation). In the spatial representation, the distance between pulses is equal to the round-trip path length of radiation in the cavity.

The emission spectrum of this laser is a set of equidistant frequencies:

$$\nu_m = m f_{\rm rep} + f_0, \qquad (1)$$

where the integer m is the mode number; f_0 is the laser frequency comb shift; $f_{\rm rep} = c/(2L)$ is the pulse repetition rate; c is the speed of light; L is the optical length of the laser cavity.

From a physical point of view, the comb shift f_0 is caused by the differences in the group and phase velocities [1, 2]. On the other hand, the comb shift f_0 can be explained by the fact that in the general case, there exists a phase difference between the oscillations at the carrier frequency ω_0 and the pulse envelope. If this difference varies from pulse to pulse by ϕ, then the shift of the laser frequency comb will be expressed as

$$f_0 = f_{\rm rep}\frac{\phi}{2\pi}. \qquad (2)$$

However, because the carrier frequency ω_0 is one of the components of the spectrum $\nu_{\rm m}$, the frequency comb shift f_0 will be absent only when, as follows from (1), the ratio of the carrier frequency to the repetition rate is an integer.

It should be noted, however, that in general, even in the absence of the frequency comb shift, the phase shift of the carrier frequency relative to the carrier envelope is possible, but this shift will be constant from pulse to pulse.

In the free lasing regime of a femtosecond laser there are fluctuations of the optical cavity length L, which leads to a corresponding instability in the repetition rate $f_{\rm rep}$. A number of factors, such as changing the pump power and the angular deviation of the cavity mirrors, influence the shift of the frequency comb f_0.

Dispersion of the optical elements of the Fabry–Perot interferometer in the first approximation can be described by introducing a transmission band shift g [7, 8]:

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$$\nu_n = \frac{(n+g)c}{2l}, \tag{3}$$

where the shift g is expressed in units of the intermode interval $c/(2l)$; l is the distance between the interferometer mirrors; and n is an integer.

3. Transmission of femtosecond laser radiation by a Fabry–Perot interferometer with the dispersion taken into account

Transmission of radiation of a femtosecond laser by a Fabry–Perot interferometer has been studied both theoretically and experimentally in several papers (see, for example, [4–6]).

For a Fabry–Perot on the assumption that the linewidth of each laser mode is much smaller than the interferometer bandwidth, the intensity $I_{\rm tr}$ of femtosecond radiation transmitted through a Fabry–Perot interferometer is determined by the transmittance $K_{\rm m}$ of all the modes of the laser and the intensity of its spectral components:

$$I_{\rm tr} = \sum_m I_m K_m, \;\; K_m = \{1 + p\sin^2[\pi(m+q-g)z]\}^{-1}. \tag{4}$$

Here, I_m is the intensity of the mth femtosecond laser mode incident on the interferometer;

$$q = \frac{f_0}{f_{\rm rep}} \tag{5}$$

is the relative shift of the laser frequency comb. Compared with the expression for an ideal interferometer [6], we introduced an additional transmission band shift of the comb of a femtosecond laser, g, arising from the dispersion of the interferometer. The quality factor of the interferometer is determined by the simple expression:

$$p = 4R(1-R)^{-2}, \tag{6}$$

where the effective reflection coefficient of the interferometer mirrors, $R = (R_1R_2)^{1/2}$, is the geometric mean of the reflection coefficients of mirrors R_1 and R_2. As previously [4–6], we consider the problem in the absence of losses.

The relative length z can be varied by changing the optical lengths of both the interferometer (l) and the laser cavity (L):

$$z = \frac{l}{L}. \tag{7}$$

It should be noted that when q and g are equal, expression (4) becomes equal to the analogous expression for an ideal interferometer in the absence of the frequency comb shift, i.e., the envelope maximum of the Fabry–Perot interferometer's transmission bands for femtosecond radiation in this case will coincide with a maximum of one of the fringes.

4. Experimental

In the experiment we used a femtosecond Ti:sapphire laser, which is a six-mirror ring resonator with chirped mirrors. The pulse duration was ~30 fs, the central wavelength was 0.82 μm, the output power was 550 mW for the pump power of 4.8 W (Verdi V8 laser, Coherent) and pulse repetition rate was about 495 MHz.

The laser radiation reflected from the semitransparent mirror was fed to the PD1 photodiode, which detected the

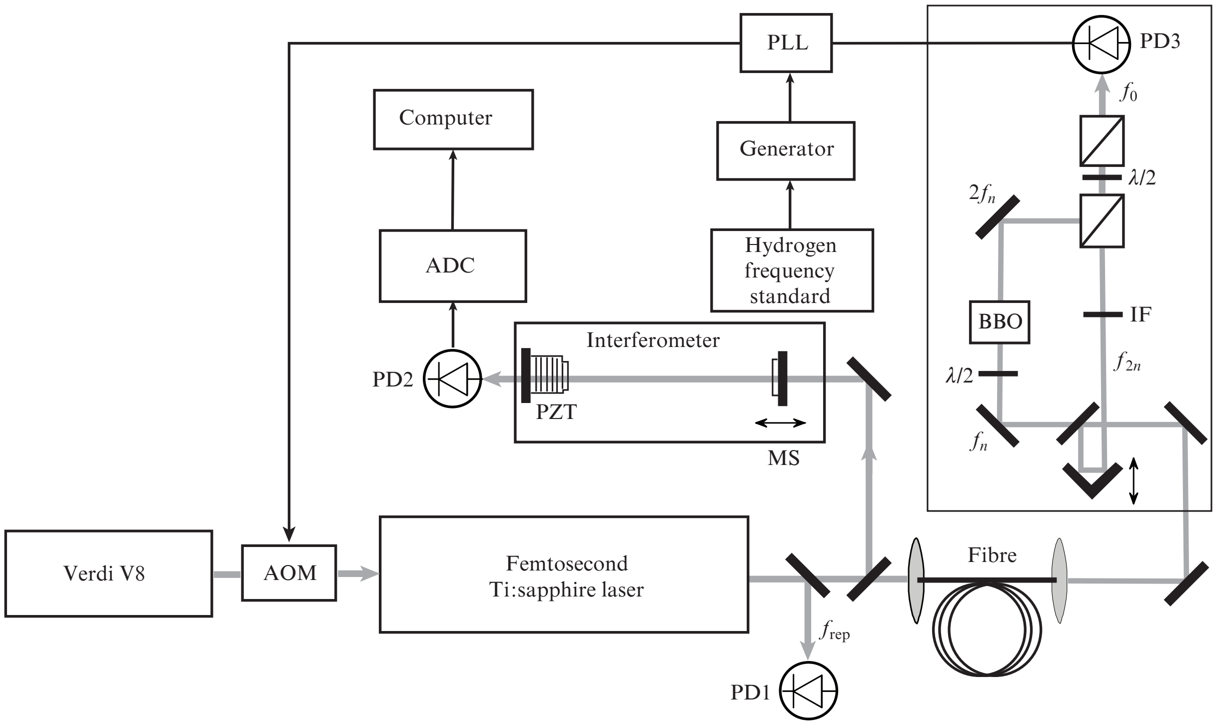


Figure 1. Block diagram of the experimental setup: (AOM) acousto-optic modulator; (PLL) phase-locked loop; (ADC) analogue-to-digital converter; (PZT) piezoceramic transducer; (MS) microscrew; (IF) interference filter.

signal with the frequency of intermode beats f_{rep} (Fig. 1). The light beam reflected from the next semitransparent mirror was coupled to a Fabry–Perot interferometer, which was formed by flat semitransparent aluminium mirrors with transmittance ~0.1. Using a fine adjustment microscrew, the length of the interferometer was matched with the optical length of the laser cavity. Continuous tuning of the interferometer length, needed to record an interference pattern, was performed by applying a voltage to the piezoceramic transducer from a sawtooth voltage generator (not shown in Fig. 1). The space between the interferometer mirrors was protected from air flows with a housing, which reduces the fluctuations in the length of the interferometer. From the interferometer output, radiation was coupled to the PD2 photodetector and then through an analogue-to-digital converter to a computer for data recording and processing.

Laser radiation (350 mW) transmitted through the two semitransparent mirrors was focused by an aspherical lens into Femto White800 microstructured fibre, where the output emission spectrum was broadened to an octave. To detect the signal corresponding to the frequency comb shift f_0, we used a f–$2f$-interferometr based on the BBO crystal. The beat signal between the second harmonic of the nth mode in the IR region and the $2n$th mode in the 'green' part of the spectrum was detected by a PD3 photodetector and allowed one to obtain the frequency comb shift: $2f_n - f_{2n} = 2(f_0 + nf_{\text{rep}}) - (f_0 + 2nf_{\text{rep}}) = f_0$. For the obtained signal, the signal-to-noise ratio was ~45 dB in the band of width 100 kHz. The acousto-optic modulator made it possible to smoothly control the power of the pump laser and, consequently, the value of f_0. Stabilization of f_0 was performed by using a phase-locked loop unit with a reference signal from the generator stabilised by the hydrogen standard. The signal f_0 was recorded with a frequency meter (not shown in Fig. 1), whose reference signal was also fed from the hydrogen standard. The tuning region of the frequency comb offset was within 68 – 83 MHz, which corresponded to a range from 0.13 to 0.17 in terms of the relative values of q.

5. Experimental studies of a femtosecond laser frequency comb shift

When recording the interference patterns, we used precision scanning of the interferometer base, which was performed by applying a sawtooth voltage with a frequency of 10 Hz and an amplitude of ±200 V to the piezoceramic transducer. The number of points on the experimental curve was approximately 2000. Recording could be performed both with increasing or decreasing the length of the interferometer.

Studies have been performed for the ratio of the lengths of the interferometer and the laser cavity, $z = 1/2$. For each value of q the interference pattern was recorded 10 times while increasing and decreasing the length of the interferometer, which allowed us to estimate the statistical and systematic measurement error. Figure 2 shows a typical interference pattern.

The observed difference between experimental and calculated data is apparently due to the fact that we have not matched the phase fronts of the laser beam with the plane of the interferometer mirrors, and this led to an 'incomplete' interference and, consequently, to some discrepancy between the results of the experiment and calculation.

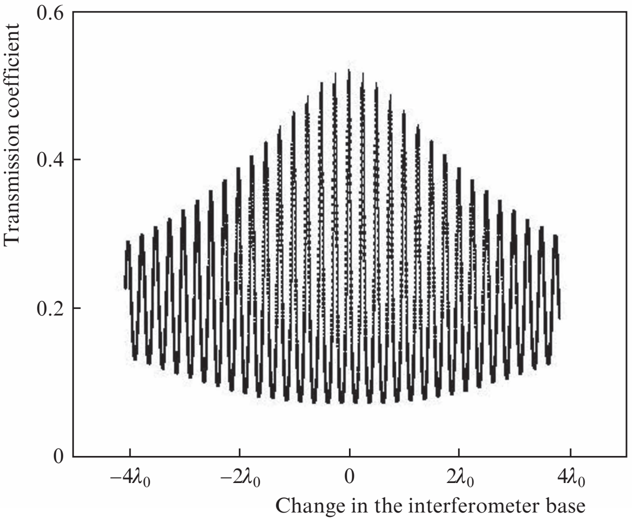


Figure 2. Transmission coefficient of femtosecond laser radiation by a Fabry–Perot interferometer at a relative length $z = 1/2$. Points are the experiment, the solid line is the calculation by formula (4) by fitting it to experimental data by the least squares method.

As shown in our previous studies [4–6], the interval between adjacent transmission fringes at a relative length $z \approx 1/2$ is equal to $\lambda_0/4$, where λ_0 is the average wavelength of the femtosecond laser. Because the optical length of the interferometer in our case is two times less than at $z = 1$, the effect of the air flows becomes smaller, which reduces the statistical error in measuring the shift of a femtosecond laser comb.

One of the fitting parameters of the problem is the relative linewidth of the femtosecond laser. The calculations were performed by assuming that the shape of the emission spectrum is Gaussian [6]. The spectrum was controlled by an optical spectrometer, made on the basis of a photodiode array. The results virtually coincided within the accuracy of the measurement error (Fig. 3). The FWHM of the laser spectrum was 25 nm.

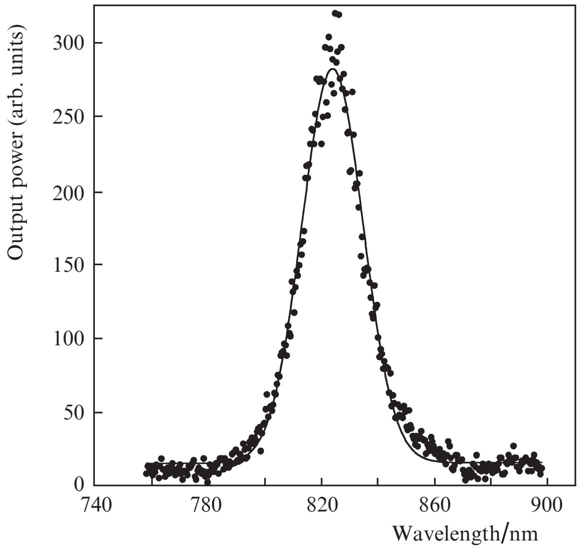


Figure 3. Emission spectra of the femtosecond laser, obtained using an optical spectrometer (points) and by fitting the calculated dependence (4) for the Gaussian shape of the spectrum to the experimental data shown in Fig. 2 (solid curve).

When the angular alignment of the interferometer mirrors was carried out, we searched for the maximum value of the parameter p. To assess the accuracy of alignment, we misaligned one of the interferometer mirrors both in the horizontal and vertical planes. In each angular position of one of the mirrors attached to the alignment head, ten recordings were performed. For each recording, we found the quality factor of the interferometer, p. The statistical error in determining p was less than 0.2, while the error of the angular position of the mirrors in our case was $\sim 2\times10^{-4}$ rad.

Figure 4 shows the dependence of the parameter p on the angular misalignment of the mirror in the horizontal plane. In the vertical plane, the dependence was similar. One can see that the values of p slightly differ for different directions of the mirror displacement. This is, apparently, due to a hysteresis of the piezoceramic transducer and nonparallel displacement of the mirror mounted on it.

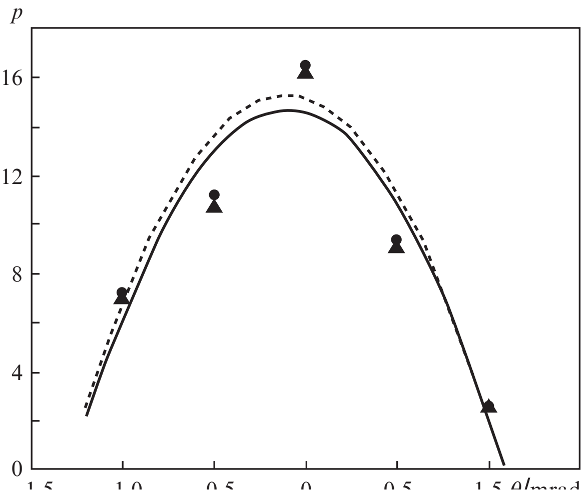
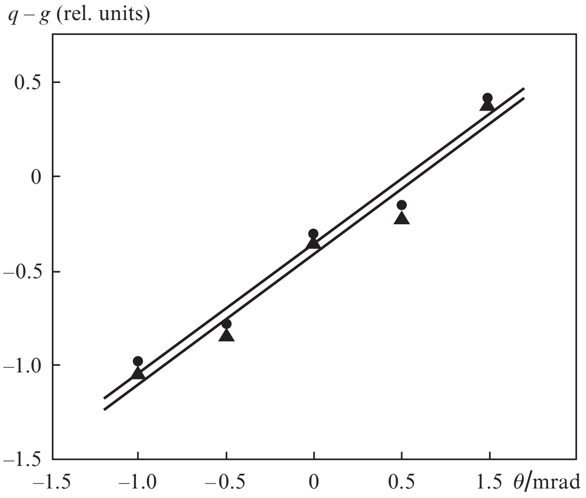


Figure 4. Dependence of the quality factor of the interferometer p on the angle of misalignment of its mirror θ in the horizontal plane in the case of recording an interference pattern with increasing (●, dashed line) and decreasing the interferometer base (▲, solid line).

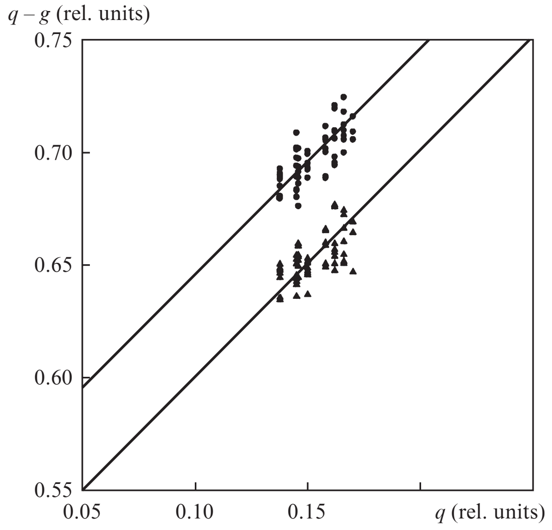
When one of the interferometer mirrors was misaligned by an angle $\theta = \pm1$ mrad, the relative shift of the transmission bands changed by ±0.5 (Fig. 5). As previously [6], for $z = 1/2$ the shift of the transmission bands was measured in units of $\lambda_0/4$. Obviously, with the error of the angular position of the interferometer mirror, equal to 2×10^{-4} rad, the relative systematic shift would be 10^{-1}.

The observed linear dependence of the shift on the angle θ is apparently related to nonparallel displacement of the mirror mounted on the piezoceramic transducer. This displacement leads to an asymmetry in the envelope of the comb of the transmission fringes, i.e., to the shift of the envelope of the interference pattern. Because of the hysteresis of the piezoceramic transducer for different directions of the mirror displacement, there is some discrepancy between the estimates of the shift of the transmission fringes (by 2×10^{-2}).

Figure 6 shows the transmission band shift (in other words, the values of $q - g$) as a function of the frequency q of the femtosecond comb shift. The measurements were performed under conditions when the angular drift of the mirrors is practically absent. It is obvious that the value of $q - g$ measured in our experiment depends not only on the dispersion of the interferometer, but also on the accuracy of the interferometer alignment. For a visual representation of the random error of each recording of the interference pattern, in which the recording was carried out by increasing the interferometer base, are marked by circles, whereas the points, in which the recording was carried out by decreasing the base, – by triangles.



Dependence of the relative calculated shift $q - g$ of the transmission band comb of femtosecond laser radiation by a Fabry–Perot interferometer on the angle θ in the case of recording an interference pattern with increasing (●) and decreasing the interferometer base (▲).



Dependence of the relative calculated shift $q - g$ of the transmission band comb of femtosecond laser radiation by a Fabry–Perot interferometer on the shift q obtained with increasing (●) and decreasing the interferometer base (▲).

A straight line corresponding to the slope $\partial(q - g)/\partial q = 1$ was fitted to experimental data by the least squares method. The absolute random error was less than 5 MHz, and the relative error did not exceed 10^{-2}. The difference between the experimental data for the forward and backward displacement of the interferometer mirror was 10 MHz, which in relative units corresponds to 2×10^{-2} and is consistent with the results shown in Fig. 5.

6. Conclusions

Measuring the shift of the transmission bands of a femtosecond laser by a Fabry–Perot interferometer with account for the dispersion of mirrors with a reflection coefficient ~0.9 shows that the random error in determining the relative shift of the femtosecond combs was equal to $\pm10^{-2}$. The systematic error in this case was estimated as $\pm10^{-1}$ and was, primarily, due to the error in the interferometer mirror alignment and nonparallel displacement of one of them while recording the interference pattern.

The improvement of the measurement accuracy (decrease in random and systematic errors) is caused, firstly, by the use of mirrors with a higher reflectivity; secondly, by the matching of the phase fronts of the beam with a Fabry–Perot interferometer; thirdly, by an increase in the angular alignment of the interferometer mirrors; and, fourthly, by a decrease in the angular misalignment of the mirror upon its displacement.

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